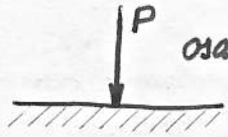
Vyšetrenie vnútorného stavu telesa zaťaženého.
Neex. abs. tuhé teleso.
Predpokladáme že teleso je v rovnováhe.

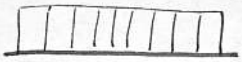
Vlastnosti materiálu.
dimenzovanie (optimálne riešenie)
Hook (17 stor.) . . . Euler . . .

Sily – objemové (vlastná váha)
– vzáj. pôsobenia

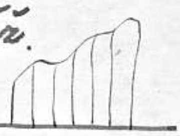


osamelá sila
kp Mp

spojité, rovnomerné zaťaž.



q kp/cm



momenty

trvalé
dočasné } „sily"

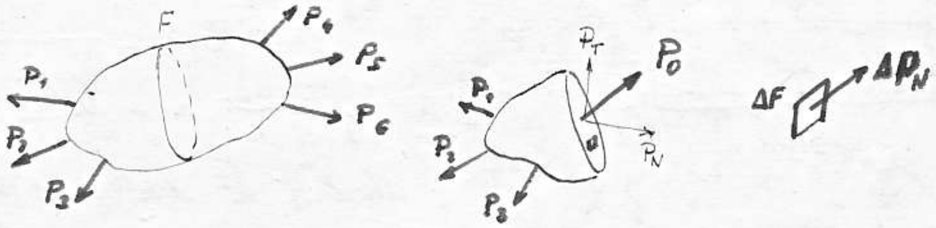
Statické sily pôsobia za stavu rovnováhy. | Dynamické sily – zrýchl.
Sila nemusí byť behom času konšt. [náhle zať., ráz, opakovaná premenná]

Teleso: zmena tvaru – deformácia < pružná / trvalá

• Vonkajšie sily musia byť v rovnov. s vnútornými.

Hl. ú.: určiť vnút. sily z vonkajších.

Rezy:



$P_T = P_0 . \sin\alpha \quad P_N = P_0 . \cos\alpha$

$$\sigma = \frac{\Delta P_N}{\Delta F}_{\Delta F \to 0} = \frac{dP_N}{dF} \sim \text{normálové napätie } (kp/cm^2 ; kp/mm^2)$$

$$\sigma . dF = dP_N \rightarrow P_N = \int_F \sigma \, dF .$$

Bernoulli-ho hypot.: sila sa rovnomerne rozloží po ploche.

$$\Downarrow \quad \sigma = \frac{P_N}{F}$$

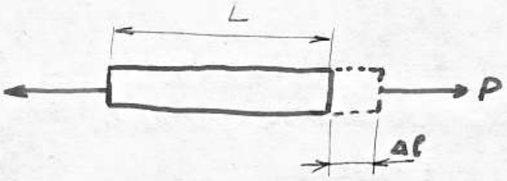
$$\tau = \frac{\Delta P_T}{\Delta F}_{\Delta F \to 0} = \frac{dP_T}{dF} \quad ; \quad \tau = \frac{P_T}{F} \sim \text{tangenc. napätie.}$$

V reze sme vonk. sily nahradili vnútornou silou (rovnováha)

Namáhanie v ťahu +

$$\sigma = \frac{P}{F} = \frac{4P}{\pi d^2}$$ (v krajných príp. σ je iný)

obrátené namáhanie v tlaku.



$$\varepsilon = \frac{\Delta l}{L}$$

$$\varepsilon = \frac{\sigma}{E}$$

E - modul pružnosti v ťahu. kp/cm^2

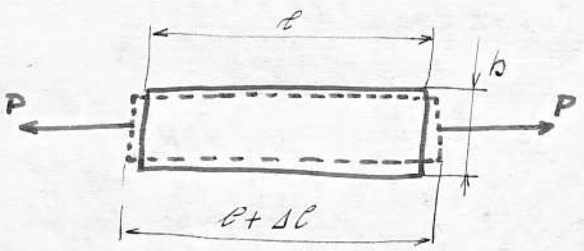
$E_{oc.} = 2{,}1 . 10^6 kp/cm^2$

$$\frac{\Delta l}{L} = \frac{\sigma}{E}$$

$$\Delta l = \frac{\sigma . L}{E} = \frac{Pl}{E.F}$$

EF - tuhosť telesa v ťahu.

Niektoré materiály nesledujú lineárnosť Hookovho zákona.
[liatiny, ... $\varepsilon = \frac{\sigma^n}{E}$]
U vláknovitých materiálov závisí aj od smeru sily.



$$\varepsilon_x = \frac{\Delta l}{L} = \frac{\sigma_x}{E}$$

$$\varepsilon_y = -\frac{\Delta b}{b} = -\underbrace{\mu}\frac{\sigma_x}{E}$$ / μ - Poissonov č.

súvislosť medzi zúžením (kontrakc.) a predĺž. materiálu.

$$\mu = \frac{\varepsilon_y}{\varepsilon_x} \qquad \mu = 0{,}25 \div 0{,}33$$

[v starých knihách $m = \frac{1}{\mu} = \frac{10}{3}$]

Zmena objemu : $\varepsilon_V = \frac{\Delta V}{V_0} = \frac{V - V_0}{V_0}$

$V_0 = L_0 . F_0$

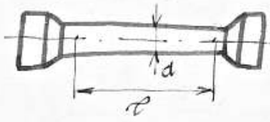
$\left.\begin{array}{l} L = L_0 + \varepsilon_x . L_0 = L_0(1+\varepsilon_x) \\ F = F_0(1 - \varepsilon_x . \mu)^2 \end{array}\right\} V$

$$V = F.L = l_0 F_0 (1+\varepsilon_x)(1-\mu\varepsilon_x)^2 =$$
$$= V_0(1+\varepsilon_x)(1 - 2\mu\varepsilon_x + \mu^2\varepsilon_x^2) =$$
$$= V_0[1 + \varepsilon_x - 2\mu\varepsilon_x - 2\mu\varepsilon_x^2 + \mu^2\varepsilon_x^2 + \mu^2\varepsilon_x^3] =$$
$$V = V_0[1 + \varepsilon_x(1-2\mu)]$$

$$\varepsilon_V = \frac{V_0[1+\varepsilon_x(1-2\mu)] - V_0}{V_0} = \varepsilon_x(1-2\mu)$$

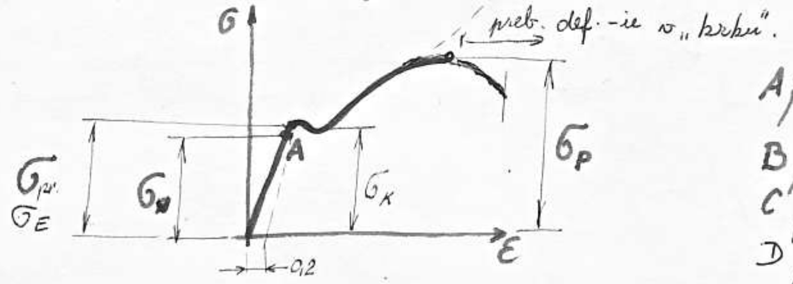
$\varepsilon_V = \varepsilon_x (1-2\mu) = \frac{\sigma_x}{E}(1-2\mu)$ ~ pri ťah. zväčš. objem.

Zkúšanie materiálu. Normaliz. rozmery skúšobného mat.



$l = 10.d$; $l = 5.d$

V diagramu sa vynáša napätie a rel. predĺž.



A; σ_U medza úmernosti
B; σ_E " pružnosti
C; [tečenie m] " klzu (prieťažnosti)
D; σ_P " pevnosti

U vš. mat. zostáva malá deformácia aj v A; (0,001%)

Mez klzu ... ost. def.: 0,2%

Pri tlaku u oc. rozdiel v diagr. nad „σ_U“ ...

σ_U

σ_K ~ trvalá def. Ťažnosť m.: $\delta = \frac{\Delta L}{L_0} . 100\%$

σ_P

$\delta = 5 \div 30\%$

Kontrakt materiálu: $\psi = \frac{F_0 - F}{F_0} . 100\%$ $\psi = 40 \div 70\%$

Nebezpečný stav $\sigma!$

Dyn. koef.
Únava

$k = \frac{\sigma!}{\sigma_{dov}}$

σ_d - dovolené náp.

$\sigma_{dov\,0,2} = \frac{\sigma_{02}}{k}$ (húževn. mat.)

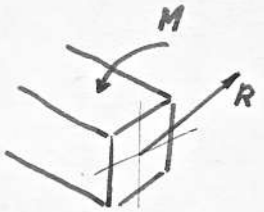
$\sigma_{dov\,(pevn.)} = \frac{\sigma_P}{k}$ (krehký mat.)

$k > 1$!!
$k = 1,5 \div 10$

Prúty: 

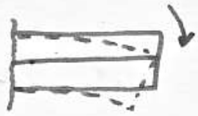
Steny, dosky:

Telesá:



ťah (platí, kým tel. je krátke)
tlak

sila v šmyku



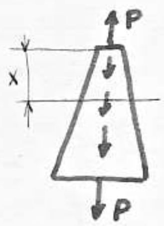
ohyb

krútenie

14. X. 1969.

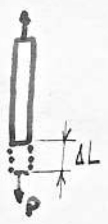
$$\sigma = \frac{P}{F} \qquad \varepsilon = \frac{\Delta l}{L} = \frac{\sigma}{E}$$

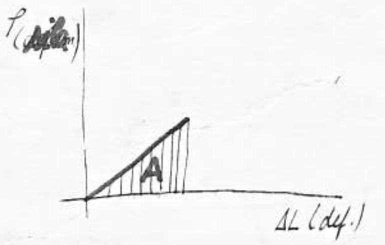
$$\Delta l = \frac{\sigma . L}{E} = \frac{PL}{EF}$$ – predĺž. telesa v ťahu



$$\sigma_x = \frac{P_{(x)}}{F_{(x)}} \qquad \Delta l = \int_0^L \frac{P_{(x)}}{E F_{(x)}} dx$$

Celá vn. práca sa mení na energiu (ktorá je nahromadená v telese) (teleso sa deformuje)





$$A = \frac{1}{2} P . \Delta L = \frac{1}{2} \cdot \frac{P^2 L}{EF} = U$$

$$u = \frac{U}{V} = \frac{1}{2} \frac{P^2 L}{EF . F . L} = \frac{1}{2} \frac{P^2}{EF^2} = \frac{\sigma^2}{2E}$$

energ. na jednotku objemu

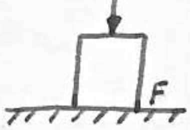
$$u = \frac{1}{2} \frac{\sigma}{E} \cdot \sigma = \frac{1}{2} \varepsilon . \sigma$$

Podmienka pevnosti : $\sigma_{max} \leq \sigma_{dov}$

$$\frac{P}{F} \leq \sigma_{dov} \quad \rightarrow \quad F \geq \frac{P}{\sigma_{dov}}$$

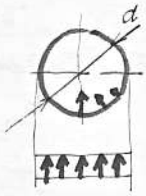
Pre dlhé prúty platia inakšie zákony.

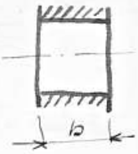
Tlak:



$$\sigma_{otlačenia} = \frac{P}{F} \leq \sigma_{dov.otl.}$$

v čapoch



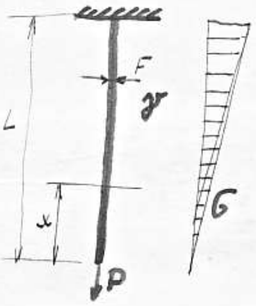


$$F = b.d$$

$$\sigma_{otl.} = \frac{P}{b.d} \leq \sigma_{otl.\,dov.}$$

(čap v pokoji!)

Nekonšt. sila. (vlastná váha prútov - spoj. zaťaženie; vlastná váha)



$$\sigma_{(x)} = \frac{G_{(x)}}{F} \quad / \quad G_{(x)} = x.F.\gamma$$

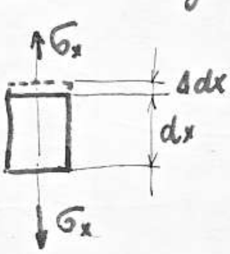
$$\sigma_{(x)} = \frac{x.\gamma F}{F} = x.\gamma$$

- napätie od vlastnej váhy nezávisí od plochy.

$$\sigma_{max} = \gamma.L$$

$$L_{kr} = \frac{\sigma_{p}}{\gamma}$$

Predĺženie od vl. váhy:



$$\varepsilon = \frac{\Delta dx}{dx} = \frac{\sigma_{(x)}}{E} = \frac{x.\gamma}{E}$$

$$\Delta dx = \frac{x.\gamma}{E}.dx$$

$$\Delta L = \int_0^L \Delta dx = \frac{\gamma}{E}\cdot\int_0^L x.dx = \frac{1}{2}\frac{\gamma}{E}L^2$$

upravením:

$$\Delta L = \frac{\gamma F L . L}{2EF} = \boxed{\frac{G.L}{2EF}}$$

Sila + vl. váha:

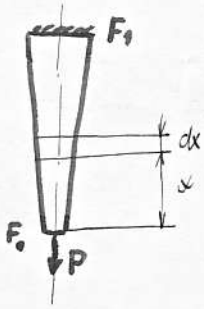
$$\sigma_x = \frac{P}{F} + \gamma x$$

$$\sigma_{max} = \frac{P}{F} + \gamma L \leq \sigma_{dov} \quad \rightarrow \quad F \geq \frac{P}{\sigma_{dov} - \gamma L}$$

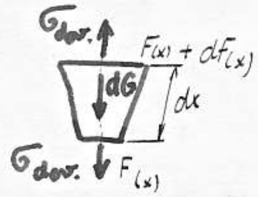
Celková deform.:

$$\Delta L = \frac{PL}{EF} + \frac{\gamma L^2}{2E} = \frac{L}{EF}\cdot\left(P + \frac{G}{2}\right)$$

Lano s konšt. napätím :



$\sigma = \sigma_{dov}$



$$dG = F_{(x)} \cdot dx \cdot \gamma$$

$$\sigma_{dov} \cdot (F_{(x)} + dF_{(x)}) = \sigma_{dov} \cdot F_{(x)} + \gamma \cdot F_{(x)} \cdot dx$$

$$\cancel{\sigma_{dov} F_{(x)}} + \sigma_{dov} dF_{(x)} = \cancel{\sigma_{dov} F_{(x)}} + \gamma \cdot dx \cdot F_{(x)}$$

$$\sigma_{dov} \cdot dF_{(x)} = \gamma \cdot F_{(x)} \cdot dx$$

$$\frac{dF_{(x)}}{F_{(x)}} = \frac{\gamma}{\sigma_{dov}} \cdot dx$$

$$\ln F_{(x)} + C = \frac{\gamma}{\sigma_{dov}} \cdot x$$

$$x = 0 : \quad F = F_0 \quad \longrightarrow \quad C = -\ln F_0$$

$$\ln F_{(x)} - \ln F_0 = \frac{\gamma}{\sigma_{dov}} x$$

$$\ln \frac{F_x}{F_0} = \frac{\gamma}{\sigma_{dov}} \cdot x$$

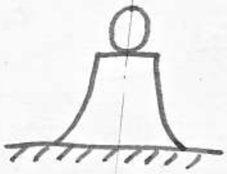
$$F_x = F_0 \cdot e^{\frac{\gamma x}{\sigma_{dov.}}}$$

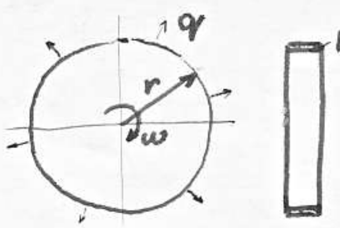
$$\sigma_{dov} = \frac{P}{F_0} \longrightarrow F_0 = \frac{P}{\sigma_{dov}}$$

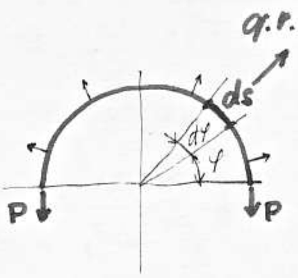
Deform. :

$$\varepsilon = \frac{\sigma_{dov}}{E} \longrightarrow \Delta L = \frac{\sigma_{dov} \cdot L}{E}$$

Tlak:

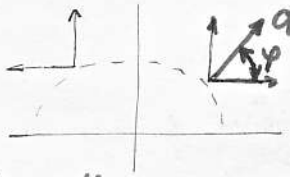






$q.ds$ | $ds = r.d\varphi$

elemént. sila : $q.r.d\varphi$



Zvislá sila : $q.r.d\varphi.\sin\varphi$

rovnováha síl: $$2P = 2\int_0^{\frac{\pi}{2}} q.r.\sin\varphi\, d\varphi = 2.q.r \quad \rightarrow P = q.r$$

nap. : $$\sigma = \frac{P}{F} = \frac{q.r}{F}$$

Sila P vzniká pri otáčaní.

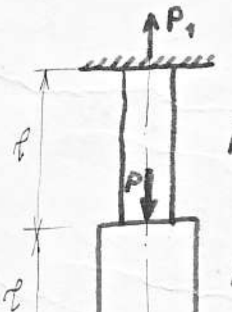
$q = m.\omega^2 r$ | m - hmota elementu, q - zaťaž (sila) na jedn. dĺžky

$q = \frac{F.1.\gamma}{g}.\omega^2 r$ | $v = \omega.r$

$q = \frac{F.\gamma}{g}.\frac{v^2}{r}$

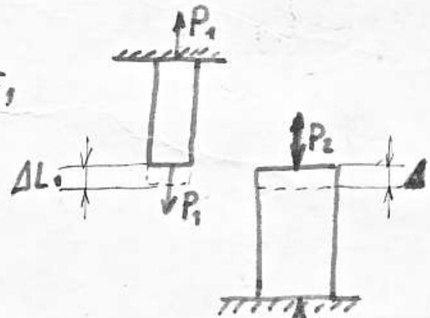
nap. $$\sigma = \frac{q.r}{F} = \frac{F.\gamma v^2}{F.g.\cancel{r}} = \frac{\gamma v^2}{g}$$

pri dimenzov.: $\sigma \leq \sigma_{dov}$ $\quad \frac{\gamma.v^2}{g} \leq \sigma_{dov.}$; z čoho dost. max. rychlosť.



$\Sigma P_i = 0$ $\quad P = P_1 + P_2$ $\quad$ raz staticky neurč. úloha.

Napíš. def. podmienku ; aby pr. bol riešiteľný:



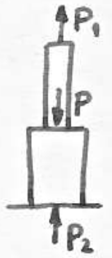
$\Delta L_1 = \Delta L_2$ — deformačná podmienka

$$\Delta L_1 = \frac{P_1 L}{E_1 F_1} \qquad \Delta L_2 = \frac{P_2 L}{E_2 F_2}$$

$$\left.\begin{array}{l} P = P_1 + P_2 \\ \frac{P_1 L}{E_1 F_1} = \frac{P_2 L}{E_2 F_2} \end{array}\right\} \text{rieš.}: \quad P_1 = P\frac{E_1F_1}{E_1F_1 + E_2F_2} \qquad \sigma_1 = \frac{P_1}{F_1}$$

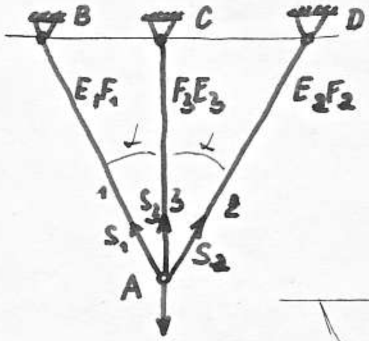
$$P_2 = P.\frac{E_2F_2}{E_1F_1 + E_2F_2} \qquad \sigma_2 = \frac{P_2}{F_2}$$

alternat. riešenie :



$\Delta L_0 = 0$ - mení deform.

$$\Delta L_0 = \frac{P_1 L}{E_1 F_1} - \frac{P_2 L}{E_2 F} = 0$$ ~ def. podm.



Určiť sily v prútoch.

1.) $\Sigma x = 0$

2.) $\Sigma y = 0$

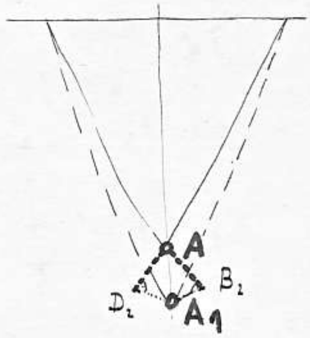
I.) $S_1 \cdot \sin\alpha - S_2 \cdot \sin\alpha = 0$

$$S_1 = S_2$$

$$S_3 + S_1 \cdot \cos\alpha + S_2 \cdot \cos\alpha = P$$

II.) $S_3 + 2 S_1 \cdot \cos\alpha = P$

deform. :



$\Delta L_1 = \overline{AB_2} = \overline{AA_1} \cdot \cos\alpha = \Delta L_3 \cdot \cos\alpha$

$\Delta L_2 = \overline{AD_2}$

$\Delta L_3 = \overline{AA_1}$

III.) $\dfrac{S_1 \cdot L_1}{E_1 F_1} = \dfrac{S_3 \cdot L_3}{E_3 F_3} \cdot \cos\alpha$ $\quad | \; L_3 = L_1 \cdot \cos\alpha$

$$\frac{S_1 L_1}{E_1 F_1} = \frac{S_3 . L_1}{E_3 F_3} \cdot \cos^2\alpha$$

$$S_1 = S_3 \frac{E_1 F_1}{E_3 F_3} \cos^2\alpha$$ / dos. do II

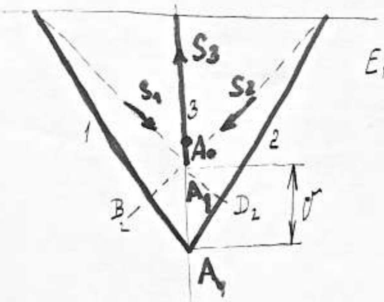
$$S_3 + 2 S_3 \frac{E_1 F_1}{E_3 F_3} \cos^3\alpha = P$$

$$S_3 = \frac{P}{1 + 2 \frac{E_1 F_1}{E_3 F_3} \cos^3\alpha} = P \frac{E_3 F_3}{E_3 F_3 + 2 E_1 F_1 \cos^3\alpha}$$ / dos. do III. rov. vyp. $S_1; S_2$

$$S_1 = P \frac{E_1 F_1 \cos^2\alpha}{E_3 F_3 + 2 E_1 F_1 \cos^3\alpha}$$

Sme predpokl. že α sa po deform. nezmenila.

Montážne napätie. (Nepresnosť δ)



$E_1F_1 = E_2F_2$

$\Sigma x \qquad S_1 \cdot \sin\alpha - S_2 \cdot \sin\alpha = 0$ I

$S_1 = S_2$

$\Sigma y \qquad S_3 - S_1\cos\alpha - S_2\cos\alpha = 0$

$S_3 - 2S_1\cos\alpha = 0$ II

$\overline{AB_2} = \Delta l_1 = \Delta l_3 \cos\alpha$

$\overline{AD_2} = \Delta l_2$

$\overline{A_0A_1} = \Delta l_3$

$$\Delta l_1 = \frac{S_1 l_1}{E_1F_1} \qquad l_1 = \frac{l_3}{\cos\alpha}$$

$$\Delta l_3 = \frac{S_3 l_3}{E_3F_3}$$

$$\delta = \overline{A_0A_1} + \overline{AA_1} = \Delta l_3 + \frac{\Delta l_1}{\cos\alpha}$$

$$S_3 - 2S_1\cos\alpha = 0$$

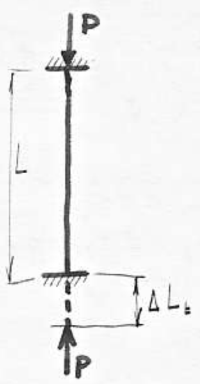
$$\delta = \frac{S_3 l_3}{E_3F_3} + \frac{S_1 l_1}{E_1F_1\cos\alpha} = \frac{S_3 l_3}{E_3F_3} + \frac{S_1 l_3}{E_1F_1\cos^2\alpha} \qquad / \text{dos na } S_1$$

Rieš:

$$S_3 = \frac{\delta \cdot E_3F_3}{l_3\left(1 + \frac{E_3F_3}{2E_1F_1\cos^3\alpha}\right)}$$

$$S_1 = \frac{S_3}{2\cdot\cos\alpha}$$

21. X. 1969.



$$\varepsilon = \frac{\Delta L}{L} = \alpha.(t_2 - t_1)$$

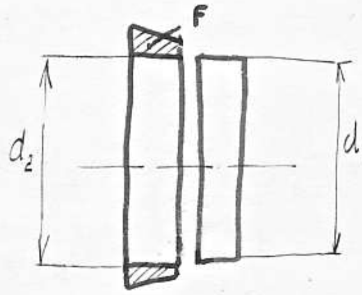
$$\Delta L_t - \Delta L_P = 0 \qquad \Big/ \ \Delta L_P = \frac{P.L}{EF}$$

$$\frac{PL}{EF} = \alpha.(t_2 - t_1).L$$

$$P = \alpha EF(t_2 - t_1)$$

$$\sigma = \frac{P}{F} = \alpha E(t_2 - t_1) \qquad / \ \alpha \text{ - koef. tepl. rozť.}$$

[pevné spojenie]

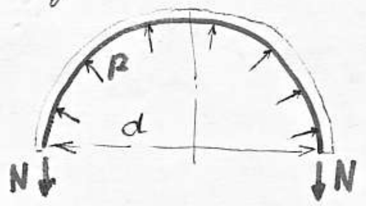


$$\Delta d = d_1 - d_2 = \frac{1}{m} d_1 \qquad \frac{1}{m} = \frac{1}{1000} \div \frac{1}{2000}$$

$$\left.\begin{aligned} \varepsilon &= \frac{\pi(d + \Delta d) - \pi.d}{\pi.d} = \frac{\Delta d}{d} = \frac{1}{m} \\ \varepsilon &= \frac{\sigma}{E} \rightarrow \sigma = \varepsilon.E \end{aligned}\right\}$$

$$\sigma = \frac{E}{m}$$

Aký zvierací tlak vznikne (pokrač. horného pr.):

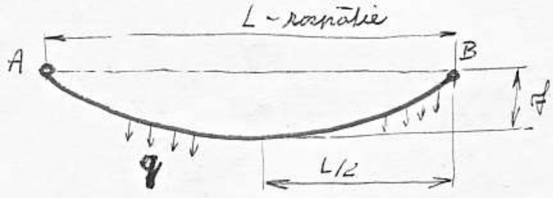


$$2N = p.d$$

$$p = \frac{2N}{d} \qquad \Big/ \ N = \sigma.F = \frac{EF}{m}$$

$$p = \frac{2EF}{m.d}$$

Pr.



$$\Rightarrow H.f = q\frac{L}{2}\cdot\frac{L}{4}$$

$$f = \frac{qL^2}{8H} \rightarrow H = \frac{qL^2}{8f}$$

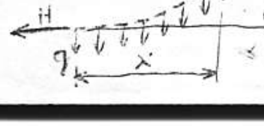
$$\sigma = \frac{H}{F} = \frac{qL^2}{8Ff} \le \sigma_{dov}$$

$$\sigma = \frac{\gamma.F.L^2}{8Ff} = \frac{\gamma}{8}\frac{L^2}{f}$$

Napät. lana je urč. prievesom a rozpätím.

keď $f < \approx 10\%L$ nová ťaha: spoj. ťaha po priamke lana ⇒ presná: kvadr. parab. [chyba v H je ↓ 1,3%]

zmena teploty!



$$y = \frac{qx^2}{2H}$$

$$\frac{dy}{dx} = \frac{qx}{H} = x\frac{8f}{L^2}$$

$$ds = \sqrt{dx^2 + dy^2} = dx\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \sqrt{1 + \frac{q^2x^2}{H^2}}\,dx$$

$$ds = \sqrt{1 + \frac{64f^2x^2}{L^4}}\,dx \approx 1 + \frac{32f^2x^2}{L^4} \qquad / \ 2\int_0^{L/2} \dots$$

$$s = L\left(1 + \frac{8}{3}\frac{f^2}{L^2}\right)$$

Chceme vypoč. silu H_2 v závese, keď sa mení zaťaženie a teplota.

$q_1 \rightarrow q_2$
$t_1 \rightarrow t_2$

$$\Delta S_t = \alpha (t_2 - t_1) \cdot L \quad \sim \text{predlž. zmenou teploty}$$

$$\Delta S_H = \frac{(H_2 - H_1) \cdot L}{EF} \quad \sim \text{predlž. zmenou zaťaženia}$$

$H_1 \rightarrow H_2$
$f_1 \rightarrow f_2$

$$S_2 = S_1 + \Delta S_H + \Delta S_t$$

$$S_1 = L\left(1 + \frac{8}{3}\frac{f_1^2}{L^2}\right)$$

$$\Downarrow$$

$$S_1 = L\left(1 + \frac{L^2 q_1^2}{24 H_1^2}\right) \qquad f_1 = \frac{L^2 q_1}{8 H_1}$$

$$L\left(1 + \frac{8}{3}\frac{f_2^2}{L^2}\right) = L\left(1 + \frac{8}{3}\frac{f_1^2}{L^2}\right) + \alpha (t_2 - t_1) \cdot L + \frac{H_2 - H_1}{EF} \cdot L$$

$$1 + \frac{L^2 q_2^2}{24 H_2^2} = 1 + \frac{L^2 q_1^2}{24 H_1^2} + \alpha (t_2 - t_1) + \frac{H_2 - H_1}{EF}$$

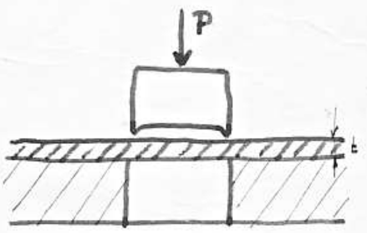
$$\frac{L^2 q_2^2}{24 H_2^2} - \frac{H_2}{EF} + \frac{H_1}{EF} - \frac{L^2 q_1^2}{24 H_1^2} - \alpha (t_2 - t_1) = 0 \qquad / \cdot H_2^2 EF$$

$$\frac{EFL^2}{24} q_2^2 - H_2^3 + H_1 H_2^2 - \frac{L^2 q_1^2}{24} H_2^2 EF - \alpha (t_2 - t_1) \cdot H_2^2 EF = 0$$

$$H_2^3 + \left[\frac{EF \cdot q_1^2 L^2}{24 H_1^2} + EF\alpha (t_2 - t_1) - H_1\right] H_2^2 - \frac{EFL^2 q_2^2}{24} = 0$$

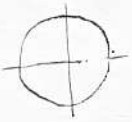
$$\Downarrow$$
$$H_2$$

Šmyk

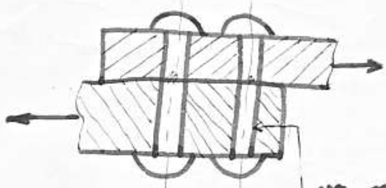


$$\tau \cdot F = P \qquad / F = \pi d \cdot t$$

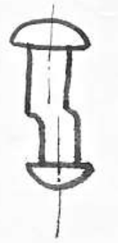
$$P \geq \tau_{perm.} \cdot F = \tau_p \cdot \pi \cdot d \cdot t$$



(nity, kliny)



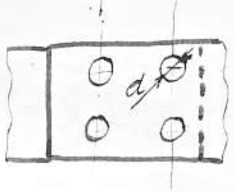
vz. napäť. otlač.

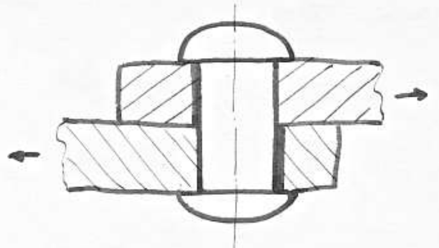


$$\tau = \frac{P}{F} \qquad / \quad F_1 = \frac{\pi d^2}{4}$$

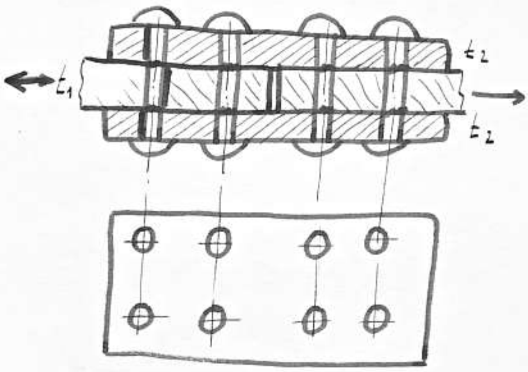
$$F = 4 \frac{\pi d^2}{4} = \pi d^2$$

$$\tau = \frac{P}{\pi d^2} \leq \tau_{dov}$$





$\sigma_{otl.} = \frac{P}{F} \leq \sigma_{dov.}$ $F_1 = d.t$



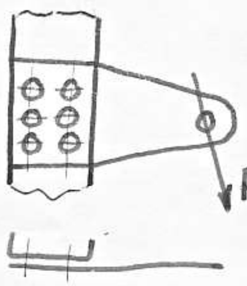
počet nitov : n
" strih : i

$F = F_1 . n . i$

$\tau = \frac{P}{F} \leq \tau_{dov}$

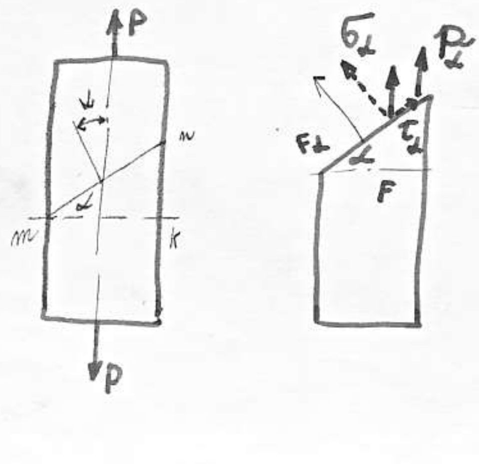
$\sigma_{otl.} = \frac{P}{n.t_1.d}$

$\sigma_{otl.} = \frac{P}{2 n t_2 d}$



Rozbor napätosti

jednoosový stav napätosti



$p_\alpha = \frac{P}{F_\alpha}$ $\quad / F_\alpha = \frac{F}{\cos\alpha}$

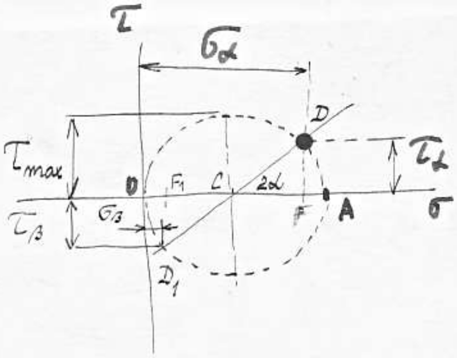
$p_\alpha = \frac{P.\cos\alpha}{F} = \sigma.\cos\alpha$

$\sigma_\alpha = p_\alpha.\cos\alpha = \sigma.\cos^2\alpha$ / kolmé nap. k rezu

$\sigma_\alpha = \sigma \frac{1+\cos 2\alpha}{2}$

$\tau_\alpha = p_\alpha.\sin\alpha = \sigma\cos\alpha\sin\alpha = \sigma_\alpha.\frac{\sin 2\alpha}{2}$

Grafické r.



Mohrova kružnica

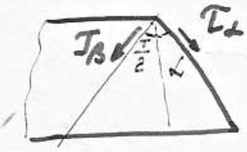
[Vynes. nap. σ (bod A)
opíšeme kružnicu.
cez stred uhol 2α.]

Dôk.: $\overline{OA} = \sigma \quad \overline{CD} = \frac{\sigma}{2} \quad \overline{OF} = \frac{1}{2}\overline{OA} + \overline{CD}.\cos 2\alpha$

$$\overline{OF} = \frac{\sigma}{2} + \frac{\sigma}{2}.\cos 2\alpha = \sigma_\alpha$$

$$\overline{DF} = \overline{CD}.\sin 2\alpha = \frac{\sigma}{2}.\sin 2\alpha = \tau_\alpha$$

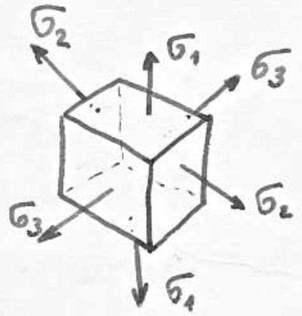
τ_{max} ... $\alpha = 45°$ .... $\tau_{max} = \frac{\sigma}{2}$



$$\tau_\beta = -\tau_\alpha$$

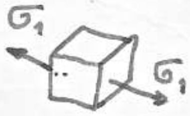
$$\sigma_\beta = \overline{OC} - \overline{CF_1} = \frac{\sigma}{2} - \frac{\sigma}{2}.\cos 2\alpha \quad \Big\} \quad \sigma_{\alpha,\beta} = \frac{\sigma}{2} \pm \frac{\sigma}{2}\cos 2\alpha$$

$$\sigma_\alpha + \sigma_\beta = \frac{\sigma}{2} + \frac{\sigma}{2}\cos 2\alpha + \frac{\sigma}{2} - \frac{\sigma}{2}\cos\alpha = \sigma$$ / súčet je konšt.

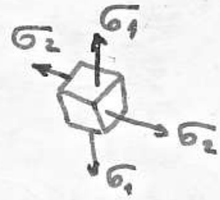


V hlavnej rovine nemí nap. tangenc.

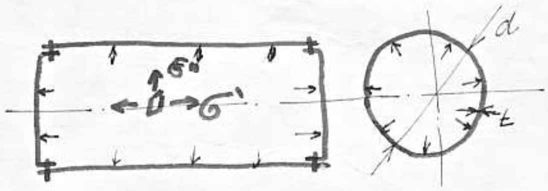
$\sigma_1 > \sigma_2 > \sigma_3$ ~ obecný stav nap.



$\sigma_2 = \sigma_3 = 0$

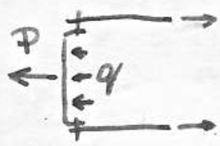


$\sigma_3 = 0$ „rovinná napätosť".



2 smerné ťahové nap.

$F' = \pi.d.t$



$$P = q \cdot \frac{\pi d^2}{4}$$

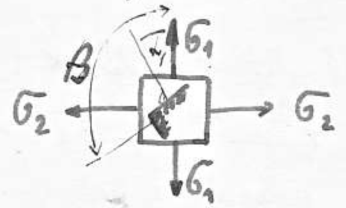
$$\sigma' = \frac{P}{F'} = \frac{q\pi d^2}{4\pi d t} = \frac{qd}{4t}$$

$$2N = q.d$$

$$\sigma'' = \frac{N}{F} = \frac{q.d}{2t}$$

3. XI. 1969.

Vzťahy pre nap. v obecnej rovine.



$\sigma_1 > \sigma_2 > \sigma_3$

——— opravil pán, ktorý nemal ani Lebo.

zistiť σ_α; τ_α

od σ_1 ..... $\sigma_\alpha' = \sigma_1 . \cos^2\alpha_1$ $\quad \tau_\alpha' = \frac{1}{2}\sigma_1 \sin 2\alpha_1$

od σ_2 .... $\sigma_\alpha'' = \sigma_2 \cos^2(90°+\alpha_1)$ $\quad \tau_\alpha'' = \frac{1}{2}\sigma_2 . \sin 2\alpha_1$

$\sigma_\alpha = \sigma_1 \cos^2\alpha_1 + \sigma_2 \sin^2\alpha_1$ $\quad \Big| \cos^2\alpha = \frac{1+\cos 2\alpha}{2} \quad \sin^2\alpha = \frac{1-\cos 2\alpha}{2}$

$$\underline{\sigma_\alpha = \frac{\sigma_1+\sigma_2}{2} + \frac{\sigma_1-\sigma_2}{2} . \cos 2\alpha_1} \quad ✓$$

$$\tau_\alpha = \frac{\sigma_1-\sigma_2}{2} . \sin 2\alpha_1 \quad ✓$$

V kolmej rovine: $\quad \beta = 90° + \alpha_1$

$$\sigma_\beta = \frac{\sigma_1+\sigma_2}{2} - \frac{\sigma_1-\sigma_2}{2} \cos 2\alpha_1$$

$$\tau_\beta = -\frac{\sigma_1-\sigma_2}{2} . \sin 2\alpha_1$$

$\tau_\beta = -\tau_\alpha$ $\quad$ šmyk. nap. v navzájom $\perp$ rovinách sú rovnaké ale opačn. znamienka.

$$\sigma_\alpha + \sigma_\beta = \frac{\sigma_1+\sigma_2}{2} + \frac{\sigma_1-\sigma_2}{2} . \cos 2\alpha + \frac{\sigma_1+\sigma_2}{2} - \frac{\sigma_1+\sigma_2}{2} \cos 2\alpha =$$
$$= \sigma_1 + \sigma_2$$

Súčet normál. nap. v 2 navzájom $\perp$ rovinách je rovný súčtu hlavných napätí: $\sigma_1 + \sigma_2$

---

maximum σ_α:

$$\frac{d\sigma_\alpha}{d\alpha} = -\sigma_1 \sin\alpha \cos\alpha + \sigma_2 \sin\alpha \cos\alpha = 0 \quad \rightarrow \sigma_1 = \sigma_2$$

$$\sin\alpha . \cos\alpha = 0 \quad \Rightarrow \alpha = 0$$

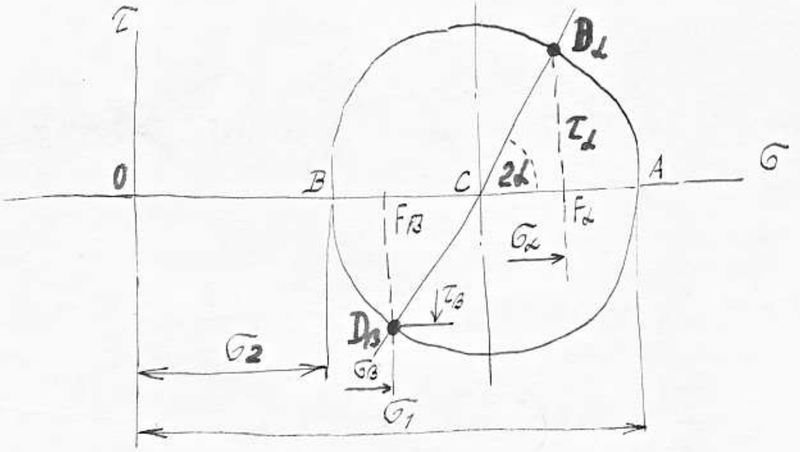
Norm. nap. maximum pre $\alpha = 0$; $\alpha = 180°$.

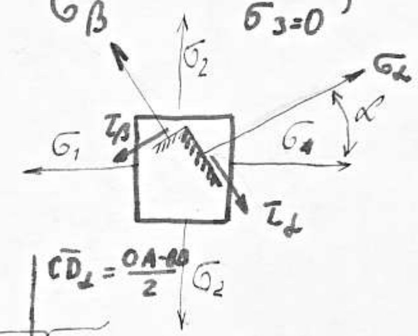
$\sigma_{\alpha\, max} = \sigma_1$ — hlavné (leží v hlavnej rovine)

Maxim. hodnota šmykového náp.:

$$\sin 2\alpha = 1 \qquad \max \tau_\alpha = \frac{\sigma_1 - \sigma_2}{2} \qquad (\alpha = 45^\circ)$$

Min. $\sigma_\alpha = \sigma_2$ ... hlavné.





$\sigma_1 > \sigma_2$ } známe hodn.
$\sigma_3 = 0$

Dôkaz:

$$\overline{OF_\alpha} = \overline{OC} + \overline{CF_\alpha} = \frac{\overline{OA} + \overline{OB}}{2} + CD_\alpha \cdot \cos 2\alpha \qquad \overline{CD_\alpha} = \frac{OA - OB}{2}$$

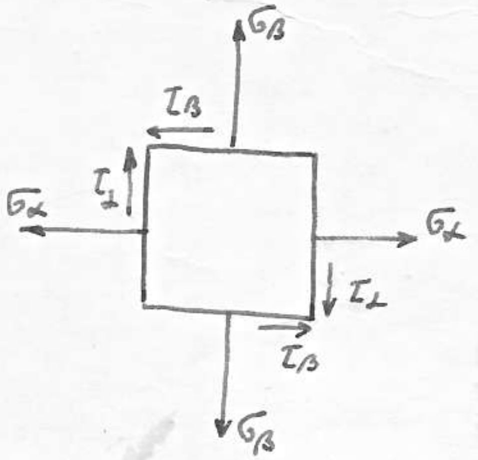
$$\overline{OF_\alpha} = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cdot \cos 2\alpha = \sigma_\alpha$$

$$\overline{OF_\beta} = \overline{OC} - \overline{CF_\beta} = \frac{\sigma_1 + \sigma_2}{2} - \frac{\sigma_1 - \sigma_2}{2} \cos 2\alpha = \sigma_\beta$$

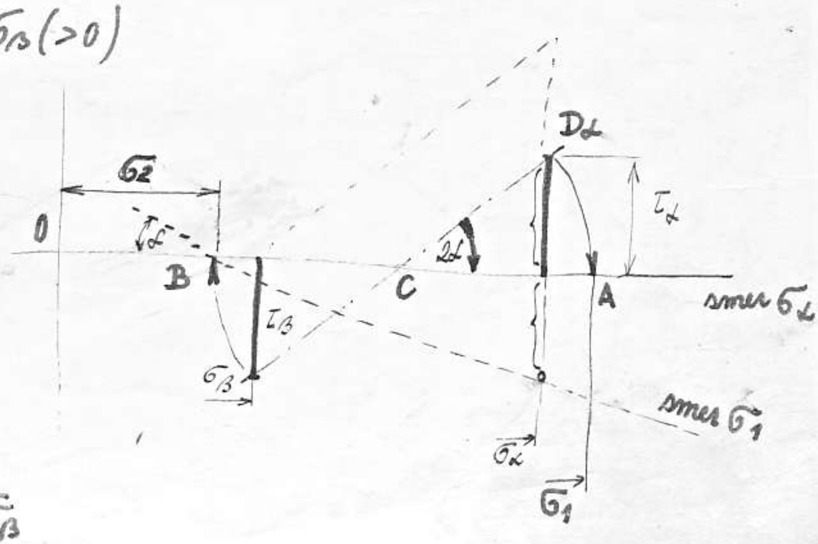
$$\overline{D_\alpha F_\alpha} = CD_\alpha \sin 2\alpha = \frac{\sigma_1 - \sigma_2}{2} \cdot \sin 2\alpha = \tau_\alpha$$

$\sigma_{max} = \sigma_1$ .... $\alpha = 0^\circ$
$\sigma_{min} = \sigma_2$ .... $\alpha = 90^\circ$ } $\tau_{max} = \dfrac{\sigma_1 - \sigma_2}{2}$ ..... $\alpha = 45^\circ$

Známe: $\sigma_{\alpha;\beta}$ $\tau_\alpha = -\tau_\beta$ / úloha: stanoviť hlavné napätia a uhol odklonu roviny.



$\sigma_\alpha > \sigma_\beta (>0)$



$$\operatorname{tg} 2\alpha = \frac{\overline{D_\alpha F_\alpha}}{\overline{F_\alpha C}} = -\frac{2\tau_\alpha}{\sigma_\alpha - \sigma_\beta}$$

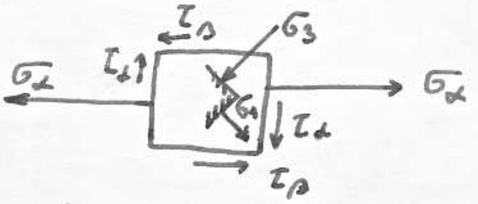
z obr.: $\sigma_1 = \overline{OA} = \overline{OC} + \overline{CA} = \dfrac{\sigma_\alpha + \sigma_\beta}{2} + \sqrt{\left(\dfrac{\sigma_\alpha - \sigma_\beta}{2}\right)^2 + \tau_\alpha^2}$ $\quad\Big|\quad \overline{CA} = \overline{CD}_\alpha = \sqrt{\left(\dfrac{\sigma_\alpha - \sigma_\beta}{2}\right)^2 + \tau_\alpha^2}$

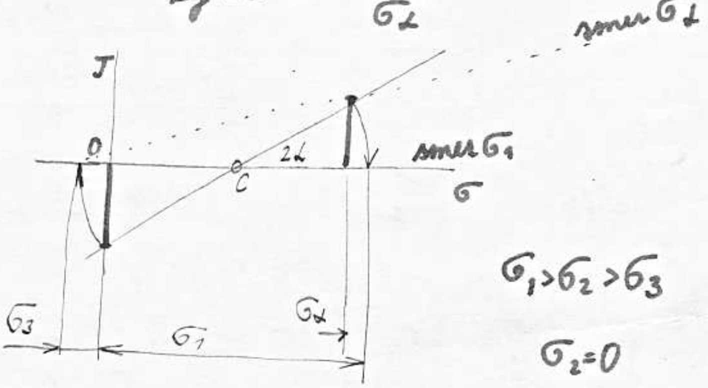
$$\sigma_2 = \frac{\sigma_\alpha + \sigma_\beta}{2} - \sqrt{\left(\frac{\sigma_\alpha - \sigma_\beta}{2}\right)^2 + \tau_\alpha^2}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_\alpha - \sigma_\beta}{2}\right)^2 + \tau_2^2}$$

Špec. prípad; keď 1 hl. nap. = 0 :

$\sigma_\beta = 0$, $\sigma_\alpha > 0$, $\tau_\alpha > 0$ $\qquad \sigma_{1,3} = \dfrac{\sigma_\alpha}{2} \pm \dfrac{1}{2}\sqrt{\sigma_\alpha^2 + 4\tau^2}$ $\qquad tg\, 2\alpha = -\dfrac{2\tau_\alpha}{\sigma_\alpha}$



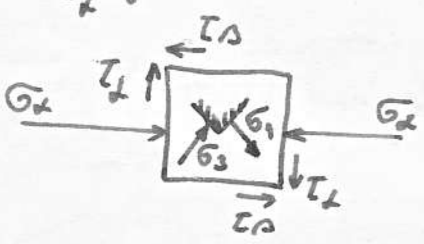


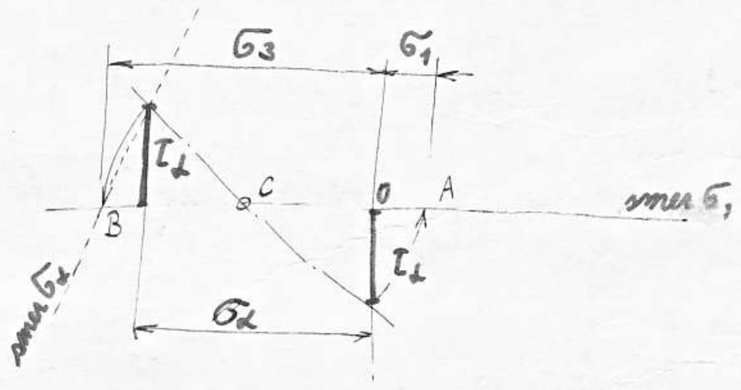
$\sigma_1 > \sigma_2 > \sigma_3$

$\sigma_2 = 0$

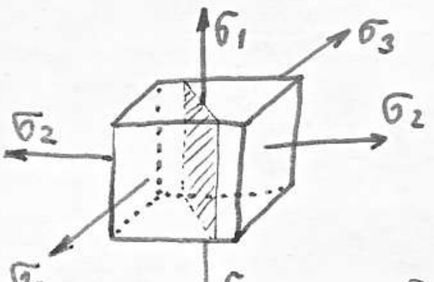
$\sigma_\alpha < 0$ $\qquad \sigma_\beta = 0$

$\tau_\alpha > 0$

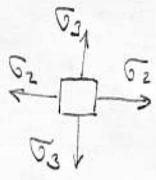


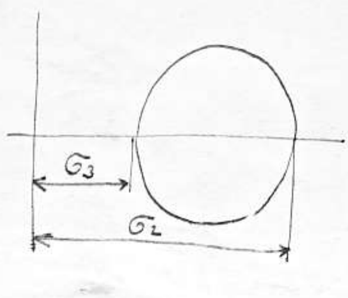


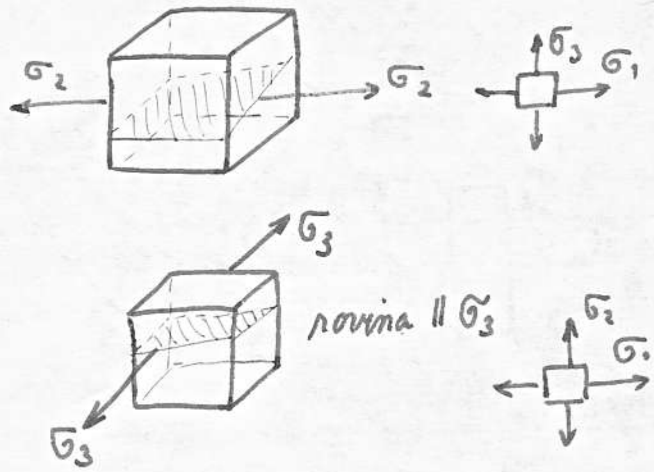
Priestorová napätosť

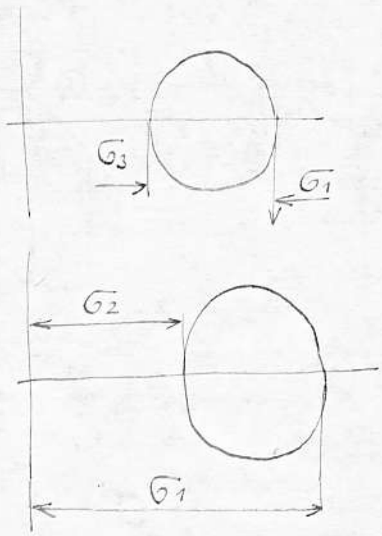


úloha: stanoviť napätie v ľubov. rovine.

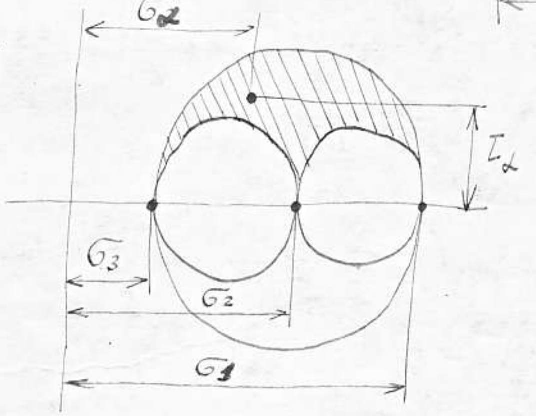




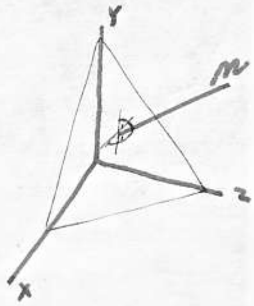




$\sigma_1 > \sigma_2 > \sigma_3$:



σ_α ; τ_α – nap. v obecnej rovine.



vzorce:

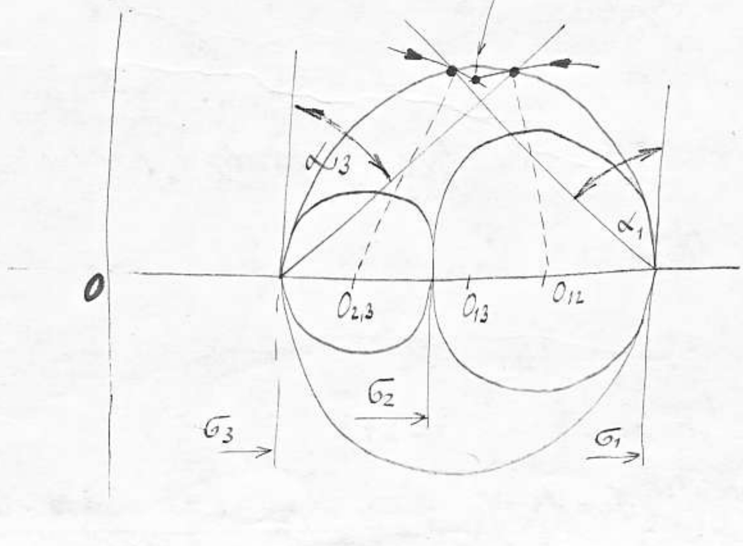
$$\sigma_\alpha = \sigma_1 \cos^2\alpha_1 + \sigma_2 \cos^2\alpha_2 + \sigma_3 \cos^2\alpha_3$$

$$\tau_\alpha = \sqrt{\sigma_1^2 \cos^2\alpha_1 + \sigma_2^2 \cos^2\alpha_2 + \sigma_3^2 \cos^2\alpha_3 - \sigma_\alpha^2}$$

z obr. : $\sigma_{\alpha\,max} = \sigma_1$ $\qquad \tau_{max} = \dfrac{\sigma_1 - \sigma_3}{2}$

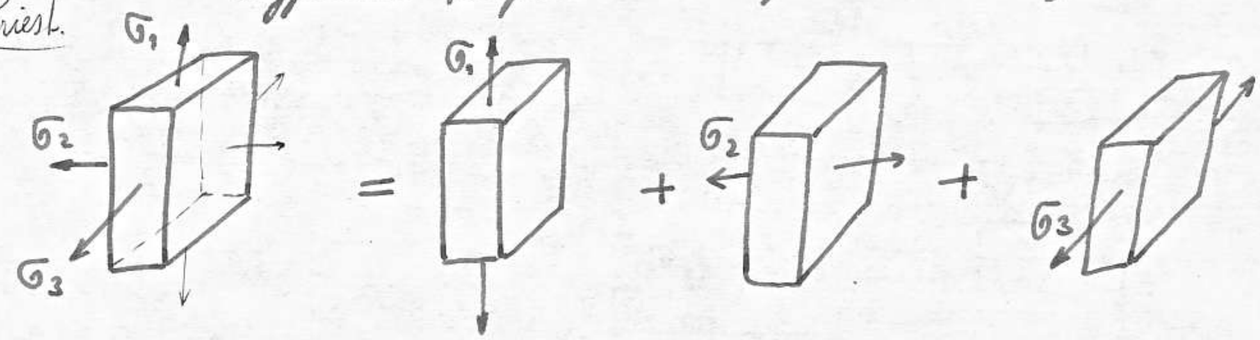
$\sigma_{\alpha\,min} = \sigma_3$

kl. bod. – predstavuji nam obecni rovinu.



Výpočet deformácií pri obecnej napätosti.

Priest.



$$\varepsilon_1' = \frac{\sigma_1}{E} \qquad \varepsilon_1'' = -\mu\frac{\sigma_2}{E} \qquad \varepsilon_1''' = -\mu\frac{\sigma_3}{E}$$

$$\varepsilon_1 = \varepsilon_1' + \varepsilon_1'' + \varepsilon_1''' = \frac{\sigma_1}{E} - \mu\left(\frac{\sigma_2}{E} + \frac{\sigma_3}{E}\right) \qquad \sim \text{ v smere } \sigma_1$$

$$\varepsilon_2 = \frac{\sigma_2}{E} - \mu\left(\frac{\sigma_1}{E} + \frac{\sigma_3}{E}\right)$$

$$\varepsilon_3 = \frac{\sigma_3}{E} - \mu\left(\frac{\sigma_1}{E} + \frac{\sigma_2}{E}\right)$$

Pri rovinnej napätosti:

$\sigma_3 = 0$

$$\left.\begin{aligned} \varepsilon_1 &= \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} \\ \varepsilon_2 &= \frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E} \end{aligned}\right\} \sigma_1; \sigma_2 \qquad \begin{aligned} \sigma_1 &= \frac{E}{1-\mu^2}(\varepsilon_1 + \mu\varepsilon_2) \\ \sigma_2 &= \frac{E}{1-\mu^2}(\varepsilon_2 + \mu\varepsilon_1) \end{aligned}$$

$$\varepsilon_3 = -\mu\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}$$

Priamková napätosť

$\sigma_2 = 0$

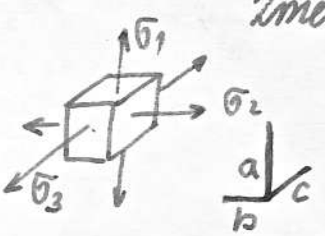
$\sigma_3 = 0$

$$\varepsilon_1 = \frac{\sigma_1}{E}$$

$$\varepsilon_2 = -\frac{\sigma_1}{E}\cdot\mu$$

$$\varepsilon_3 = -\mu\frac{\sigma_1}{E}$$

Zmena objemu pri priestorovej napätosti.



$$V_0 = a.b.c \; ; \; V_1 = (a+\Delta a)(b+\Delta b)(c+\Delta c) \; ;$$

$$V_1 \doteq abc + ab\Delta c + ac\Delta b + bc\Delta a = abc + \frac{abc}{c}\Delta c + \frac{abc}{b}\Delta b + \frac{abc}{a}\Delta a$$

$$V_1 \doteq abc\,(1 + \varepsilon_1 + \varepsilon_2 + \varepsilon_3)$$

Pomerná zmena objemu: $\vartheta = \frac{V_1 - V_0}{V_0} = \frac{abc(1+\varepsilon_1+\varepsilon_2+\varepsilon_3) - abc}{abc} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$

$$\vartheta = \frac{\sigma_1}{E} - \mu\left(\frac{\sigma_2}{E} + \frac{\sigma_3}{E}\right) + \frac{\sigma_2}{E} - \mu\left(\frac{\sigma_1}{E} + \frac{\sigma_3}{E}\right) + \frac{\sigma_3}{E} - \mu\left(\frac{\sigma_1}{E} + \frac{\sigma_2}{E}\right) =$$

$$= \frac{1}{E}\left[\sigma_1 - \mu(\sigma_2+\sigma_3) + \sigma_2 - \mu(\sigma_1+\sigma_3) + \sigma_3 - \mu(\sigma_1+\sigma_2)\right] =$$

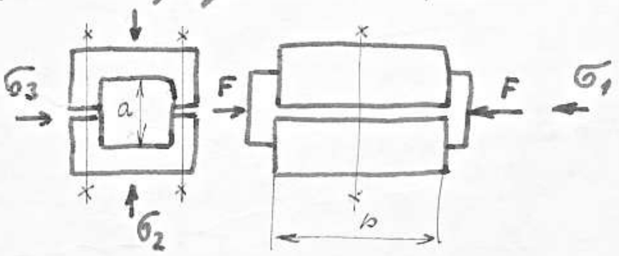
$$= \left(\frac{1-2\mu}{E}\right)(\sigma_1+\sigma_2+\sigma_3)$$

ak $\sigma_1 = \sigma_2 = \sigma_3 = \sigma$: $\vartheta = \frac{3(1-2\mu)}{E}\cdot\sigma$; $\Big\}\ \vartheta = \frac{\sigma}{K}$

$\frac{E}{3(1-2\mu)} = K$ ~ objemový modul pružnosti v ťahu.

$\sigma_m = \frac{\sigma_1+\sigma_2+\sigma_3}{3}$; $\varepsilon_m = \frac{\varepsilon_1+\varepsilon_2+\varepsilon_3}{3} = \frac{\sigma_m}{3K}$

Príklad: Žel. teleso so ○ prier. je stlač. silou P a zovreté tak že sa a nemôže meniť. Stanoviť nap. v spojovacích skrutkách:



$\sigma_1 = \frac{F}{a^2}$

$$\varepsilon_2 = \frac{\sigma_2}{E} - \mu\left(\frac{\sigma_1}{E} + \frac{\sigma_3}{E}\right) = 0 \quad \Big|\ \sigma_3 = \frac{\sigma_2}{\mu} - \sigma_1$$

$$\varepsilon_3 = \frac{\sigma_3}{E} - \mu\left(\frac{\sigma_1}{E} + \frac{\sigma_2}{E}\right) = 0 \quad \Big|\ \sigma_3 = \mu(\sigma_2+\sigma_1)$$

$$\frac{\sigma_2}{\mu} - \sigma_1 = \mu(\sigma_2+\sigma_1)$$

$$\sigma_2 = \sigma_1\frac{\mu}{1-\mu}$$

$2P = \sigma_2\cdot a\cdot b \rightarrow P = \sigma_2\cdot\frac{ab}{2}$ $\quad \sigma_s = \frac{P\cdot 4}{\pi d^2} = \frac{\sigma_2\cdot ab}{2\cdot\pi d_s^2} = \frac{2\sigma_2 ab}{\pi d_s^2}$

$\sigma_2 = 0{,}43\cdot\sigma_1$; $\quad \sigma_s = \frac{0{,}86\,F\cdot b}{a\cdot\pi\cdot d_s^2}$

Vnútorná energia (deformačná práca) pri obecnej napätosti:

Specif. energia pri lin. nap.: $u = a = \dfrac{\sigma^2}{2E} = \dfrac{\sigma \cdot \sigma}{2E} = \dfrac{\sigma \varepsilon}{2}$

obecná nap.: $u = \dfrac{\sigma_1 \varepsilon_1}{2} + \dfrac{\sigma_2 \varepsilon_2}{2} + \dfrac{\sigma_3 \varepsilon_3}{2}$ / dos do p. za ε_i

$$u = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_1\sigma_3 + \sigma_2\sigma_3)\right] \quad \text{spec. vn. en.}$$

u < zložka potrebná na zmenu objemu
" " na " tvaru.

práca $= \frac{1}{2}\sigma\varepsilon$ (Hookov z.)

predpokladáme: $\sigma_m = \dfrac{\sigma_1 + \sigma_2 + \sigma_3}{3}$ $\quad \varepsilon_m = \dfrac{\sigma_m}{3K}$

$$u_V = \frac{\sigma_m \cdot \varepsilon_m}{2} \cdot 3 = \frac{\sigma_m^2}{2K} = \frac{(\sigma_1 + \sigma_2 + \sigma_3)^2}{18K} = \frac{(1-2\mu)}{6E}(\sigma_1 + \sigma_2 + \sigma_3)^2$$

$$K = \frac{E}{3(1-2\mu)}$$

$$u_t = u - u_V = \frac{1+\mu}{E}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_1\sigma_3)$$

Prostý ťah: $\sigma_2; \sigma_3 = 0$

$\sigma_1 = \sigma_{osové}$

$u_V = \dfrac{1-2\mu}{6E} \cdot \sigma_0^2$ ~ zmena objemu

$u_t = \dfrac{1+\mu}{3E} \cdot \sigma_0^2$ ~ zmena tvaru.

$$u = u_t + u_V = \frac{\sigma^2}{2E}$$

$$u_V + u_t = \frac{1}{6E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + 2\sigma_1\sigma_2 + 2\sigma_2\sigma_3 + 2\sigma_1\sigma_3\right]\left[1-2\mu\right] + \frac{1+\mu}{E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_1\sigma_3 - \sigma_3\sigma_2\right]$$

Teórie pevnosti

Pri posudz. rozmerov d. konštr. je potr. vedieť, pri akej σ nast. nebezp. stav. N.s - u krehkých mat. porušenie
húževnaté mat. - deformácia.

Pri prostom ťahu σ_{neb} urč. experimentálne.

Stanovíme $\sigma_{dov.}$ podľa miery bezpečnosti. $\sigma_{dov} = \frac{\sigma_P}{k} = \frac{\sigma_K}{k'}$

$$\sigma_{max} \leq \sigma_{dov}$$

Obecné napn. - experimentálne určiť nemôžeme.

Preto pre posúd. n.s. pri pried. nap. boli zost. hypotézy. (ideálne - zrovnávacie nap ~ v závislosti pri danej zať., ktoré by sa dalo zrovnať so stavom pri priamk. zať.)

1. - Teória najväčš. norm. napätia. (približná)

je rozhodujúce najväčš. norm. nap., < nap. pri prostom ťahu.

$\sigma_1 > \sigma_2 > \sigma_3$ ťah $\sigma_1 < \sigma_{dov}$ - ťah.

$\sigma_3 < \sigma_{dov\,tlak}$ - tlak.

Dobré výsledky len pri ťahu krehkých mat.
Nepouž. sa!

2. - T. najv. pomerného predĺženia.

N.s. nastane pri hodnote najv. pomernej deformácie.

Pri pos. pevn. nesmie byť najv. pom. def pri ob. nap. > ako def. pri lin. def.

$$\varepsilon_{max} = \varepsilon_1 = \frac{\sigma_1}{E} - \mu\left(\frac{\sigma_2}{E} + \frac{\sigma_3}{E}\right)$$

pre priamkový stav napätosti: $\varepsilon_{dov} \leq \frac{\sigma_{dov}}{E}$

$$\varepsilon_{max} \leq \varepsilon_{dov}$$

$$\frac{\sigma_1}{E} - \mu\left(\frac{\sigma_2}{E} + \frac{\sigma_3}{E}\right) \leq \frac{\sigma_{dov}}{E}$$

$$\sigma_1 - \mu(\sigma_2 + \sigma_3) \leq \sigma_{dov}$$ / σ_{dov} - ťahacia dovolená

σ_{ZR} - zrovnávacie napätie:

$$\sigma_{ZR} \leq \sigma_{dov.}$$

Osvedč. sa pre krehké mat.

3. T. najv. šmykového napätia.

N.s – pri max. hodnote šm. nap.

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} \leq \tau_{dov} = \frac{\sigma_{dov}}{2} \quad / \ \tau_{dov} \text{ – pri lin. nap.}$$

$$\underbrace{\sigma_1 - \sigma_3}_{} \leq \sigma_{dov}$$

$$\sigma_{2R} \leq \sigma_{dov}$$

len pre mat, kde pevnosť rovnaká pri tlaku a ťahu

– pre húževnaté mat. –

[nepočíta so stredným napätím, so čísel. veľkosťou!]

4. T. pretvarnej práce.

príčina N.s. je tá časť pot. en. nahromadenej v jedn. objeme, ktorá spôs. zmenu tvaru.

$$u_t \leq u_{t\,dov} \qquad / \ u_{t\,dov} \text{ – lin. stav napätosti.}$$

$$\frac{1+\mu}{3E}\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_1\sigma_3 - \sigma_2\sigma_3\right) \leq u_{t\,dov.} = \underbrace{\frac{1+\mu}{3E}\sigma_{dov.}^2}_{\text{pre prostý ťah}}$$

$$\sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_1\sigma_3 - \sigma_2\sigma_3} \leq \sigma_{dov.}$$

$$\sigma_{2R} \leq \sigma_{dov.}$$

$$\sqrt{\tfrac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2\right.} \leq \sigma_{dov.}$$

– pre húževnaté mat. ~~dobre~~ najlepšie súhlasí so skúškami –

Obecne hľadáme σ_{2R}; porovnávame s $\sigma_{t.}$; $\sigma_{\check{s}}$; ...

Guľe $R_1 R_2$ $\quad \sigma_{max} = 0{,}388 \sqrt[3]{PE^2 \frac{(R_1 + R_2)^2}{(R_1 \cdot R_2)^2}}$ 

dotyk. plocha: $Ax^2 + By^2 = C$ priblíž. (def.) $\qquad \sigma_{max} = \frac{3}{2}\frac{P}{\pi a b}$

$$\sigma_{max} = \alpha \sqrt[3]{4A^2 \cdot P \cdot E^2}$$

Pre vyp. plochy, aj priamk., bodový dotyk, A, B, $= \frac{1}{2R_1} + \frac{1}{2R_2}$

$\frac{A}{B}$	1	0,9	0,8	0,7	0,6	0,5	0,4	0,3	0,2	0,15	0,1	0,05	0,02	0,01	0,007
α	0,388	0,4	0,42	0,44	0,468	0,49	0,536	0,6	0,716	0,8	0,97	1,280	1,8	2,241	3,202

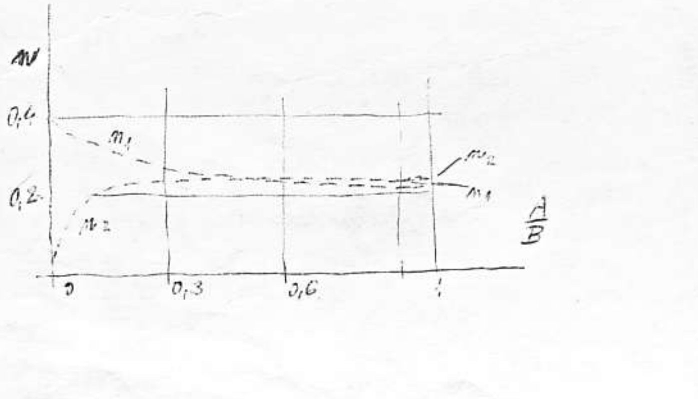
Výpočt. nap. podľa energ. teor.

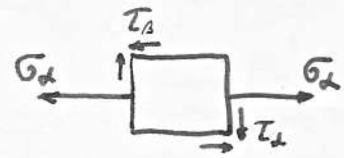
vo vnútri mat. (najv.) $\quad \sigma_r = 0{,}6\,\sigma_{max}$

stred. elips. dotyku $\quad \sigma_v = m_1 \sigma_{max}$

koniec veľkej poloosi $\quad \sigma_v = m_2 \sigma_{max}$

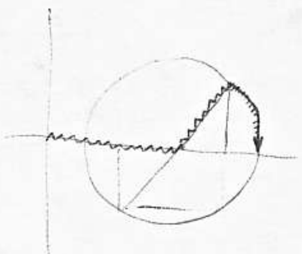
porovnať s $\sigma_{dov,\,ťah}$



2.) 

$$\sigma_{ZR} = \sigma_1 - \mu(\sigma_2 + \sigma_3)$$

$$\sigma_{1,2} = \frac{\sigma_\alpha + \sigma_\beta}{2} \pm \frac{1}{2}\sqrt{(\sigma_\alpha - \sigma_\beta)^2 + 4\tau^2}$$



$\sigma_\beta = ?$ $\Rightarrow$ $$\sigma_{1,2} = \frac{\sigma}{2} \pm \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

$$\sigma_{ZR} = \underbrace{\frac{\sigma}{2} + \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}}_{\sigma_1} - \cancel{2}\cdot\frac{\mu}{2}\left(\sigma - \cancel{\tfrac{1}{2}}\sqrt{\sigma^2 + 4\tau^2}\right) =$$

$$= \frac{\sigma}{2}(1-\mu) + \frac{1}{2}(1+\mu)\sqrt{\sigma^2 + 4\tau^2} \qquad / \mu = 0{,}3$$

$$\sigma_{ZR} = 0{,}35\sigma + 0{,}65\sqrt{\sigma^2 + 4\tau^2}$$

podmienka pevnosti: $\sigma_{ZR} \leq \sigma_{dov}$

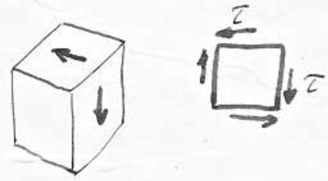
pre húževnaté materiály

3.) $$\left.\begin{aligned}\sigma_{ZR} &= \sigma_1 - \sigma_3 \\ &\sigma_1 - \sigma_2\end{aligned}\right\} \quad \sigma_{ZR} = \sqrt{\sigma^2 + 4\tau^2}$$

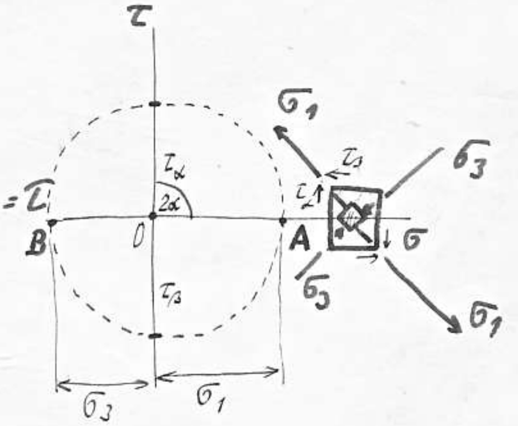
4.) $$\sigma_{ZR} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_1\sigma_3 - \sigma_2\sigma_3} \qquad / \sigma_3 = 0 \quad \sigma_1 =$$

$$\sigma_{ZR} = \sqrt{\sigma^2 + 3\tau^2} \leq \sigma_{dov}$$

Čistý šmyk



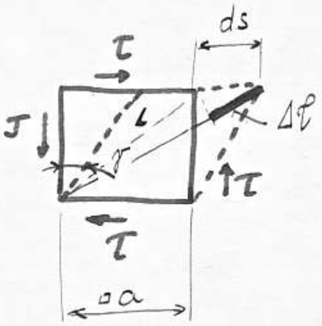
$\sigma_\alpha = 0$
$\sigma_\beta = 0$
$\tau_\alpha = -\tau_\beta = \tau$



$\sigma_1 = \tau \qquad \sigma_3 = -\tau \qquad \alpha = 45°$

Aj u šm. vzniká deformácia (posúvanie)

→



$\frac{\Delta s}{a} = tg\,\gamma \doteq \gamma$ ~ pomerné posunutie - skos

$\Delta\ell = \Delta s . \cos 45°$

$\frac{\Delta L}{L} = \varepsilon = \frac{\Delta L}{\frac{a}{\sin 45°}} = \frac{\Delta L . \sin 45°}{a} = \frac{\Delta s}{a} . \cos 45 . \sin 45°$

$\varepsilon = \gamma . \cos 45° . \sin 45° = \frac{\gamma}{2} = \varepsilon$

$\varepsilon = \frac{\sigma_1}{E} - \mu \frac{\sigma_3}{E}$ $\quad | \; \sigma_1 = \tau \quad \sigma_3 = -\tau$

$\varepsilon = \frac{\tau}{E} - \mu \frac{-\tau}{E} = \frac{\tau}{E}(1+\mu) = \varepsilon$

$\tau = \frac{E}{2(1+\mu)} . \gamma$ $\quad | \; \frac{E}{2(1+\mu)} = G$ ~ modul pružnosti vo šmyku $[kp/cm^2]$

pomerné posunutie (skos) : $\gamma = \frac{\tau}{G}$ ~ Hook-ov zákon pre šmyk

2., Teor. : $\sigma_1 - \mu(\sigma_2 + \sigma_3) \leq \sigma_{dov}$ $\quad | \; \sigma_1 = \tau \quad \sigma_2 = 0 \quad \sigma_3 = -\tau$

$\tau + \mu\tau = \sigma_{dov}$

$(1+\mu)\tau = \sigma_{dov}$ $\qquad \tau_{dov} = \frac{\sigma_{dov}}{1+\mu} = (0{,}7 \div 0{,}6)\sigma_{dov}$

3., Teor. : $\sigma_1 - \sigma_3 = \sigma_{dov}$

$\tau + \tau = \sigma_{dov}$

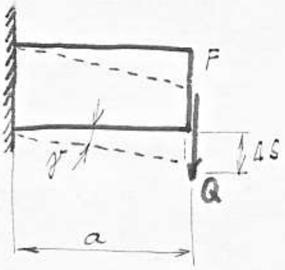
$\tau_{dov} = \frac{\sigma_{dov}}{2} = 0{,}5\,\sigma_{dov}$

4., Teor. : $\sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_1\sigma_3 - \sigma_2\sigma_3} = \sigma_{dov}$

$\sqrt{3\tau^2} = \sigma_{dov}$

$\tau_{dov} = \frac{\sigma_{dov}}{\sqrt{3}} \doteq 0{,}6\,\sigma_{dov}$

W



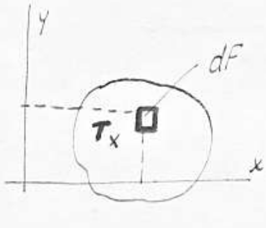
$$\Delta s = a.\gamma = \frac{\tau}{G}.a \qquad / \ \tau = \frac{Q}{F}$$

$$\Delta s = \frac{Q.a}{G.F} \qquad / \ G.F \text{ - tuhosť telesa vo šmyku}$$

$$A = U \qquad / \ A = Q.\Delta s.\frac{1}{2}$$

$$\frac{1}{2} Q.\Delta s = \frac{1}{2}\frac{Q.Q.a}{GF} = \frac{1}{2}\frac{Q^2 a}{G.F}$$

energia na jedn. objemu: $u = \frac{U}{V} = \frac{1}{2}\frac{Q^2 a}{G F.F.a} = \frac{1}{2}\frac{Q^2}{F^2 G} = \frac{1}{2}\frac{\tau^2}{G}$

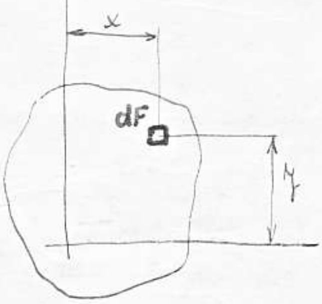


$$S_x = \int_F dF.y = \text{statický moment plochy k osi } x$$

$$S_y = \int_F dF.x \qquad [cm^3;\ mm^3]$$

$$F.y_T = S_x \quad \rightarrow \quad y_T = \frac{\int_F dF.y}{F}$$

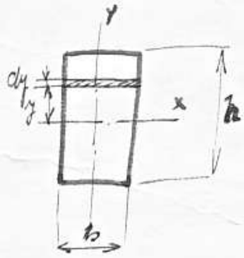
$$F.x_T = S_y \quad \rightarrow \quad x_T = \frac{\int_F dF.x}{F}$$



$$\int_F dF.y^2 = J_x \quad \text{- moment zotrvačnosti plochy k osi } x$$

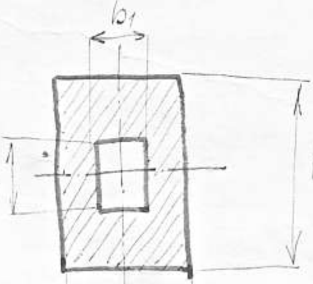
$$\int_F dF.x^2 = J_y \qquad [cm^4;\ mm^4]$$

Pr.:



$$J_x = \int_F dF.y^2 \qquad | \ dF = b.dy$$

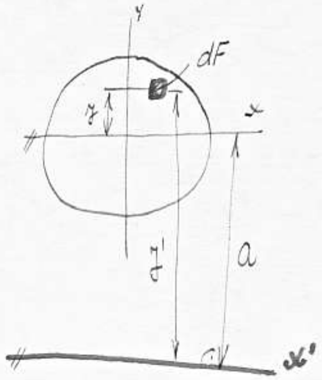
$$J_x = 2\int_0^{h/2} b.y^2 dy = \frac{b.h^3}{12} \qquad J_y = \frac{h.b^3}{12}$$



$$J_x = \frac{1}{12}.bh^3 - \frac{1}{12}b_1 h_1^3 = \frac{1}{12}(bh^3 - b_1 h_1^3)$$

Polomer zotrvačnosti: $i_x = \sqrt{\frac{J_x}{F}}$; $i_y = \sqrt{\frac{J_y}{F}}$

$$i_x^2.F = J_x \qquad i_y^2.F = J_y$$



I_x - dané $I_{x'} = ?$

$$I_x = \int_F y^2 \, dF$$

$$I_{x'} = \int_F y'^2 \, dF \qquad / \; y' = a + y$$

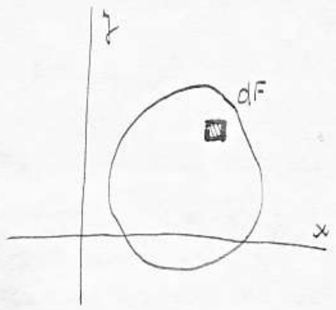
$$I_{x'} = \int_F (a+y)^2 \, dF = \int_F (a^2 + 2ay + y^2) \, dF = a^2 \int_F dF + 2a \underbrace{\int_F y \, dF}_{0} + \underbrace{\int_F y^2 \, dF}_{I_x}$$

$$I_{x'} = a^2 F + I_x$$

Steinerova veta

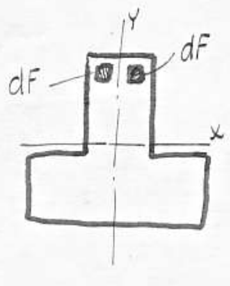
↓

$$I_x = I_{x'} - a^2 F$$



$$\int_F x \cdot y \, dF = I_{xy}$$

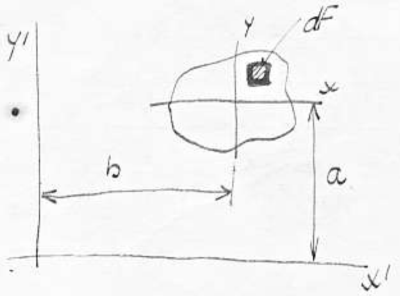
~ deviačný moment [cm^4]



Ak os je osou symetrie $I_{xy} = 0$

Pre $I_{xy} = 0$ sú hlavné osy

Ak os ide ťažiskom je hlavná centrálna os



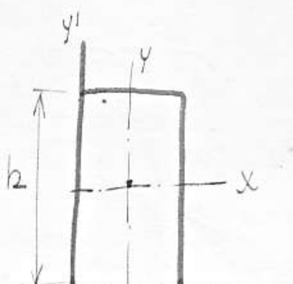
$y' = y + a$

$x' = x + b$

$$I_{x'y'} = \int_F y' \cdot x' \, dF$$

$$I_{x'y'} = I_{xy} + a.b.F$$

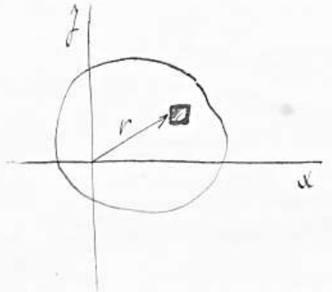
Pr.:



$$I_x = \frac{1}{12} b h^3$$

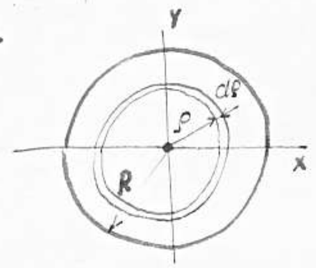
$$I_{x'y'} = \underbrace{I_{xy}}_{0} + \frac{h}{2} \cdot \frac{b}{2} \cdot \overbrace{b.h}^{F} = \frac{h^2 b^2}{4}$$

Polárny moment zotrvačnosti :



$$J_p = \int_F r^2 dF \qquad / r^2 = x^2 + y^2$$

$$J_p = \int_F x^2 dF + \int_F y^2 dF = J_x + J_y$$



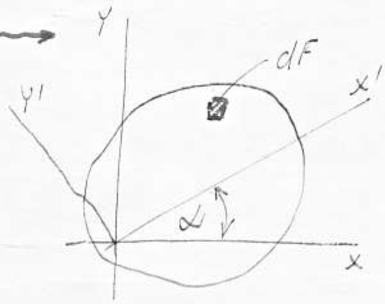
$$dF = 2\pi . \rho . d\rho$$

$$J_p = \int_0^R \rho^2 . 2\pi \rho d\rho = 2\pi \int_0^R \rho^3 . d\rho = \frac{\pi R^4}{2} = \frac{\pi d^4}{32}$$

$$J_p = J_x + J_y$$

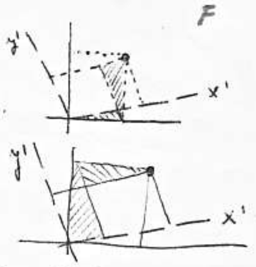
$$J_p = 2 J_x = 2 J_y$$

$$J_x = \frac{J_p}{2} = \frac{\pi R^4}{4} = \underline{\frac{\pi D^4}{64} = J_y}$$



$$J_{x'} = \int_F y'^2 dF \qquad J_{y'} = \int_F x'^2 dF \qquad J_{x'y'} = \int_F x'y' dF$$

$$\left.\begin{array}{l} x' = x \cos\alpha + y \sin\alpha \\ y' = y . \cos\alpha - x . \sin\alpha \end{array}\right\} \text{dosad.}$$



$$J_{x'} = \frac{J_x + J_y}{2} + \frac{J_x - J_y}{2} . \cos 2\alpha \overset{?}{-} J_{xy} . \sin 2\alpha$$

D: $J_x \quad J_y \quad J_{xy} \quad \alpha$

H: $J_{x'} \quad J_{y'} \quad J_{x'y'}$

$$J_{y'} = \int (y \cos\alpha - x \sin\alpha)^2 dF = \int y^2 \cos^2 dF + \int x^2 \sin^2 dF - 2 \int x y \cos \sin dF$$

$$J_{y'} = J_y \cos^2\alpha + J_x \sin^2\alpha - 2 J_{xy} \cos\alpha \sin\alpha$$

$$/ \cos^2\alpha = \frac{1 + \cos 2\alpha}{2} \qquad \sin^2\alpha = \frac{1 - \cos 2\alpha}{2}$$

$$J_{y'} = \frac{J_x + J_y}{2} - \frac{J_x - J_y}{2} \cos 2\alpha \overset{?}{+} J_{xy} . \sin 2\alpha$$

$$J_{x'y'} = \cos^2 J_{xy} + \cos \sin (J_y - J_x) - \sin^2 J_{xy}$$

$$J_{x'y'} = \frac{J_x - J_y}{2} . \sin 2\alpha + J_{xy} . \cos 2\alpha \qquad / \text{keď } J_{x'y'} = 0 \text{ dost. hl. os.}$$

<u>Hlavná os</u> : $\alpha_0 = ? \quad J_{x'y'} = 0$:

$$\frac{J_x - J_y}{2} . \sin 2\alpha_0 + J_{xy} . \cos 2\alpha_0 = 0 \quad \Downarrow$$

$$\operatorname{tg} 2\alpha_0 = - \frac{2 J_{xy}}{J_x - J_y}$$

$$J_1 = \frac{J_x+J_y}{2} + \frac{1}{2}\sqrt{(J_x-J_y)^2+4J_{xy}^2}$$

$$J_2 = \frac{J_x+J_y}{2} - \frac{1}{2}\sqrt{(J_x-J_y)^2+4J_{xy}^2}$$

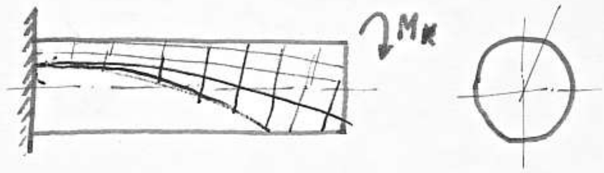
} momenty zotrv. k hlavným osiam

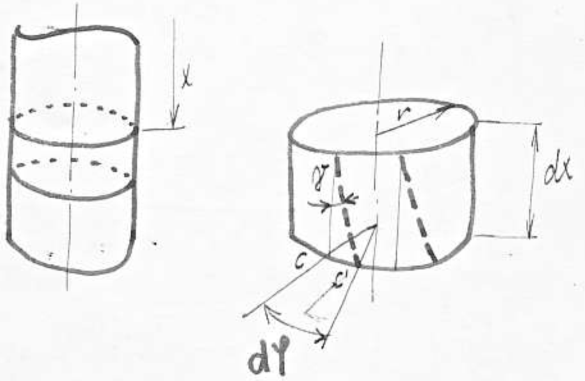
$J_1 ; J_2$ analog. ako $\sigma_1 ; \sigma_2$ z $\sigma_\alpha ; \sigma_\beta ; \tau$ ($J_x ; J_y ; J_{xy}$)

Podľa Mohr. kružnice.

Po sčítaní predch.: $\underline{\underline{J_{0x'} + J_{0y'} = J_{0x} + J_{0y}}}$

Krut



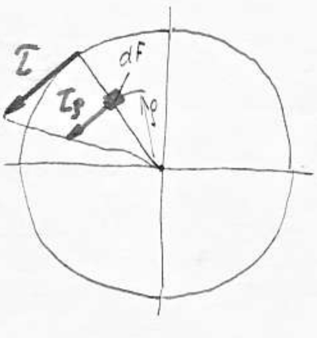


pootočenie jednotkovej dĺžky

$$\gamma = \frac{CC'}{dx} = \frac{r\,.\,d\varphi}{dx} = r\,.\,\vartheta$$

$$\gamma = \frac{\tau}{G} \quad \sim \text{šmyk}$$

$$\tau = r\,.\,\vartheta\,.\,G$$



$$\tau_\rho = \rho\,.\,\vartheta\,.\,G$$

$$M_K = \sum M_\tau$$

$$dT = \tau_\rho\,.\,dF = \boxed{\rho\,.\,\vartheta\,.\,G\,.}\,dF$$

$$dM_\tau = dT\,.\,\rho = \rho^2\,.\,\vartheta\,.\,G\,.\,dF$$

$$M_\tau = \int_F dM_\tau = G\,.\,\vartheta\int_F \rho^2\,.\,dF = G\,.\,\vartheta\,.\,J_P = M_k \Downarrow \vartheta$$

$$\vartheta = \frac{M_K}{G\,.\,J_P}$$

/ J_P – moment polárny zotrvačnosti.

$$\vartheta = \frac{\tau}{r\,.\,G} = \frac{M_K}{G\,.\,J_P}$$

$$\tau = \frac{M_K\,.\,r}{J_P}$$

Modul prierezu v krute : $W_p = \frac{J_p}{r}$

$$\tau = \frac{M_k}{W_p} \leqq \tau_{dov.}$$

⟶ $J_p = \frac{\pi d^4}{32}$ $\qquad W_p = \frac{J_p}{\frac{d}{2}} = \frac{\pi d^3}{16}$ ~ modul prierezu v krute.

$$\tau_{max} = \frac{M_k}{W_p} \leqq \tau_{dov}$$

$$\tau_{dov} \geqq \frac{M_k \cdot 16}{\pi d^3} \longrightarrow d = \sqrt[3]{\frac{16 \cdot M_k}{\pi \cdot \tau_{dov}}}$$

⟶ $\vartheta = \frac{d\varphi}{dx} = \frac{M_k}{G \cdot J_p}$

$\rightarrow d\varphi = \frac{M_k}{G \cdot J_p} \cdot dx$

$$\varphi = \int_l \frac{M_k \cdot dx}{G \cdot J_p} = \frac{M_k \cdot l}{G \cdot J_p} = \varphi$$

/ $G \cdot J_p$ – tuhosť t. v krute.

$[W_k$; J_p ! 18. XI. 1969.

Pot. energ. alebo deformačná práca pri krútení (kruh. prier.)

Predp.: zaťaž. pôs. staticky, keď sa hriadeľ otočí o φ vykoná pritom prácu, ktorú možno graf. znázor.:

$$A = U = \frac{1}{2} M_k \cdot \varphi = \frac{M_k \cdot l}{G \cdot J_p} \cdot \frac{1}{2} M_k ;$$

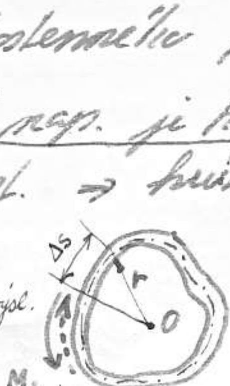
$$A = \frac{1}{2} \cdot \frac{M_k^2 \cdot l}{G \cdot J_p}$$

M_K

Krútenie tenkostenného prierezu, nekruhového:

Predp.: šmykové nap. je konšt. po hrúbke t. Na prierez pôs. hnací moment ⇒ krútiaci moment. $\Sigma M_i = 0$

$M_{h.vypl} = M_{K\,vypl.}$



Vyberieme element a zakreslíme naň pôs. sily.

→

$$dT = \boxed{\Delta s . t}^{dF} . \tau$$ / označ: $t.\tau = q$ šmykový tok

moment šmykového toku (resp. element. krútiaci moment) k ľubov. bodu 0

$$dM_k = dT . r = \Delta s . \underbrace{t . \tau}_{q} . r$$

$$\Delta M_q = q . \boxed{\Delta s . r}$$

$\Delta s . r$ ~ je dvojnásobok plochy (vyšr. čiastí) je to plocha: (strednica ; a Δs).

Keď $t \neq konšt.$; musíme výsledky spočítavať:

$$M_k = \sum_{i=1}^{m} \Delta M_k = \sum_{i=1}^{m} q . \Delta s . r$$

$$M_q = \sum_{i=1}^{m} q . \Delta s . r$$

$\Rightarrow M_k = q . 2F = M_q$

kde F je plocha ohraničená strednicou prierezu a polomermi r.

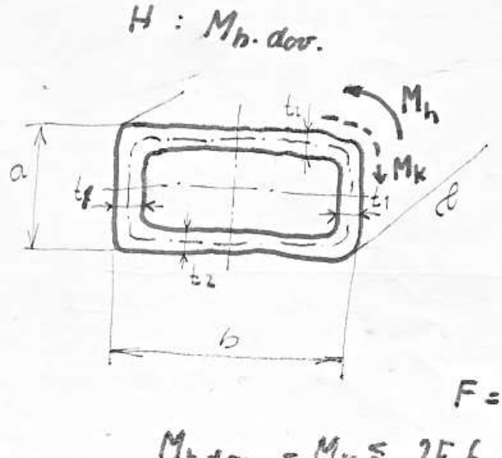
Záver: $\tau_{max_{skut.}} = \frac{M_k}{2F . t_{min}} \leq \tau_{dov.}$ ~ Bredtov vzorec.

$$q = \frac{M_k}{2F}$$

Príklad: Rúrkový nosník je namáhaný krútiacim momentom M_{k}. Vyp. jeho dov. hodnotu, keď sú D: $\tau_{dov} = 500$

$t_1 = 0,4 cm$ $a = 10 cm$ $l = 1 m$
$t_2 = 0,2 cm$ $b = 20 cm$ $G = 8 . 10^5$

H : $M_{h.dov.}$



I. $\Sigma M_i = 0$ $\quad M_h - M_k = 0$

$M_h = M_k$

II. z pevnostnej podm.

$\tau_{max_{sk}} \leq \tau_{dov}$

$M_k = \ldots$

$$F = (a - t_2) . (b - t_1) = 192 cm^2$$

$$M_{h dov} = M_k \leq 2F . t_{min} . \tau_{dov} = 384 kp\ cm$$

Uhol skrútenia (tenkost., nekruhov. prierezu)

sa stanoví z deform. podmienky za predpokladu nezborenia prierezu.

Pre kruhový prierez: $\varphi = \frac{M_k \cdot l}{G \cdot J_p}$

Pre nekruhový prierez: $\vartheta = \frac{M_k \cdot l}{G \cdot J_p'}$ / v literatúre $J_p' = J_k$ moment zotrvačnosti pri krútení.

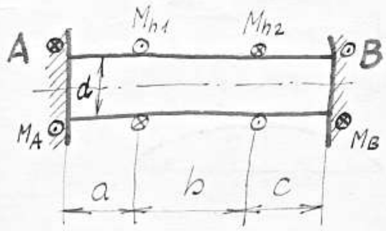
pre t = konšt. bol odvodený: $J_k = \frac{4F^2 t}{s}$ / s- dĺžka strednice

pre $t \neq$ konšt.:

$$J_k = \frac{4F^2}{\int_s \frac{ds}{t}}$$

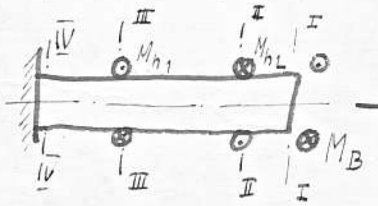
Statický neurč. úlohy v krútení. (pre ľubov. prierez)

Zad.: D: M_{h1} M_{h2} d a, b, c H: M_a M_b (reakcie) priebehy M_k; φ; τ



1.) 1 st. podm. $\Rightarrow$ 1x stat. neurč. $\Rightarrow$ 1 def. podm.

2.) čiastočne uvolniť hriadeľ - prev. na stat. urč. úlohu.



~ mysl. rezy

I.) $\Sigma M_i = 0$ ~ stat. podm. rovnov. $\xrightarrow{\odot} \oplus$, $\otimes$

$$M_B - M_{h2} + M_{h1} - M_A = 0$$

II.) $\varphi_{AB} = 0$ $\varphi_{I I'} + \varphi_{II II'} + \varphi_{III III'} + \varphi_{IV IV'} = 0$

Rieš. def. podm.:

II.

$$\varphi_{x-x'} = \frac{M_k(x-x') \cdot l^{(x-x')}}{G_{(x-x')} \cdot J_{p(x-x')}}$$ J_p = konšt / d = konšt.

$$\frac{M_B \cdot c}{G \cdot J_p} + \frac{(M_B - M_{h2}) \cdot b}{G \cdot J_p} + \frac{(M_B - M_{h2} + M_{h1}) \cdot a}{G \cdot J_p} + 0 = 0$$

$\Downarrow M_B$ — dos. do stat. podm. $\rightarrow M_A$

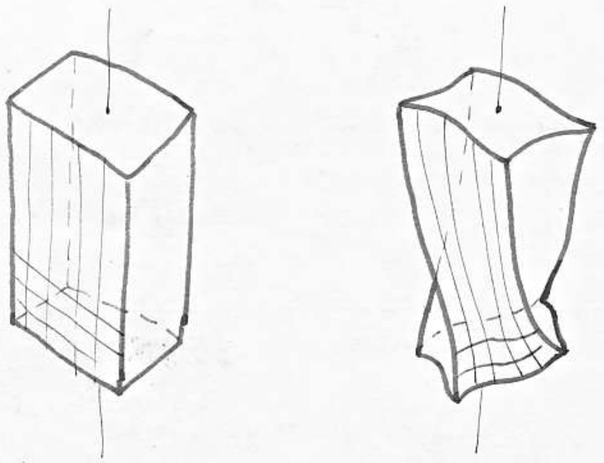
Keď máme známe vš. vonk. zaťaž. sily — pristúpime k vyčísl. napätí, zostroj. grafov krút. momentov, čiast. deform. po dĺžke hriadeľa ....

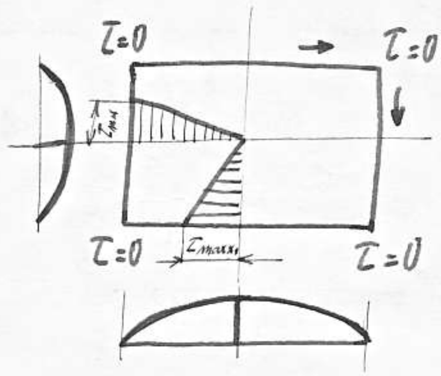
Keď znamienka sú + ; orientácie ostanú.

Krútenie prútov nekruhového prierezu.

N. prier. účinkom krút. sa zborí: najmenej sa def. prút pozdĺž hrán, najviac uprostred. → na veľkosť a rozloženie τ. V rohoch $\tau = 0$; uprostred strán najväčšie.

Z uv. vyplýva že pre výpočet τ a φ nemôžu platiť vzťahy odvodené pre kruh. prierez. Hodnoty τ a φ sa poč. približne na základe exper. skúšok, ktoré sú tabuľkované vo forme koeficientov α; β.





$$\tau_{max(sk)} = \frac{M_k}{W_k} \leq \tau_{dov} \qquad I_k = \alpha \cdot b^4 \qquad \alpha = f\left(\frac{h}{b}\right) \quad 0{,}11 \div 1{,}12$$

$$\varphi = \frac{M_k \cdot l}{G \cdot I_k} \qquad W_k = \beta \cdot b^3 \qquad \beta = f\left(\frac{h}{b}\right) \quad 0{,}208 \div 1{,}15$$

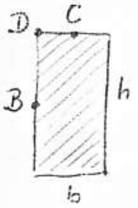
Skrutková valcová pružina. (tesne vinutá)

(zjednoduš. výpočet pre malé stúpanie)

Výpoč. namáhania pružín si vyžad. osobitn. stanoviskom. (zložitosť namáhan.; rozmanitosť tvarov pružín)

Zanedbáme vplyv norm. zložky sily P a ohybového momentu vzhľ. na malý pomer drôtu k polomeru vinutia: $d \ll R$; malý uhol α.

Krútenie nekruhových prierezov



$J_k = \alpha b^4 \ cm^4$

$W_k = \beta b^3 \ cm^3$

$\tau_B = \frac{M_k}{W_k}$

$\tau_C = \gamma \tau_B$

$\tau_D = 0$

h/b	α	β	γ
1,0	0,14	0,208	1,0
1,5	0,294	0,346	0,859
2,0	0,457	0,493	0,795
3,0	0,79	0,801	0,753
4,0	1,123	1,150	0,745

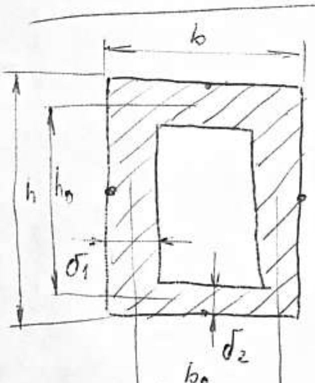
$J_k = \frac{(m - 0,63) b^4}{3}$

$W_k = \frac{(m - 0,63) b^3}{3}$

$\tau_B = \frac{M_k}{W_k}$

$\tau_C = 0,74 \tau_B$

$m = \frac{h}{b}$			
6,0	1,789	1,789	0,743
8,0	2,456	2,456	0,742
10,0	3,123	3,123	0,742



$$J_k = \frac{h_0^2 b_0^2 \delta_1 \delta_2}{h \delta_2 + b \delta_1 - \delta_1^2 - \delta_2^2}$$

$W_{k1} = 2 h_0 b_0 \delta_1 \qquad \tau_1 = \frac{M_k}{W_{k1}}$

$W_{k2} = 2 h_0 b_0 \delta_2 \qquad \tau_2 = \frac{M_k}{W_{k2}}$

Zhustenie napätia vo vnút. rohoch.

$\alpha_r = 1,74 \sqrt[3]{\frac{\sigma_{max}}{r}}$ | r – polomer zaokruhl.

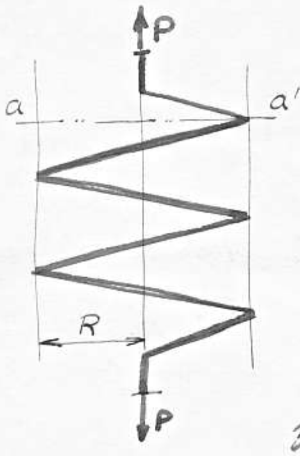
⊥ I [$\qquad J_k = \eta \frac{1}{3} \sum h \delta^3$

L ..... $\eta = 1,00$

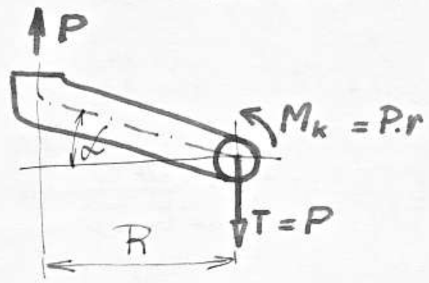
I ..... $\eta = 1,20$

⊥ ..... $\eta = 1,15$

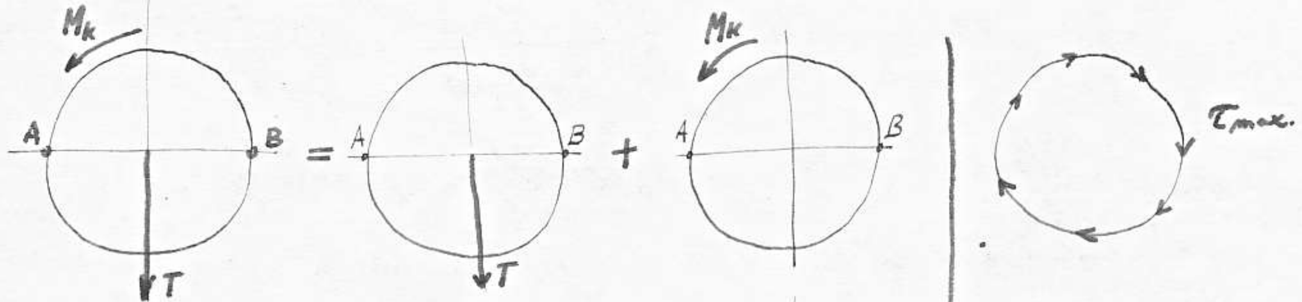
[..... $\eta = 1,12$



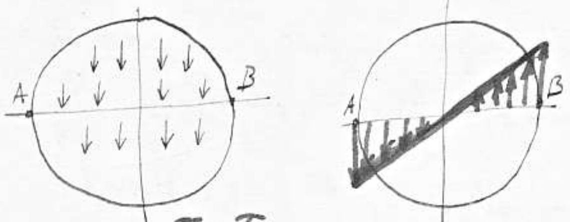
REZ a-a'



Účinky vn. síl. veličín v rámci vnút. prierezov superponujme.



$$\tau_{výsl} = \tau_T + \tau_{MK} = \tau_1 + \tau_2$$



$\tau_1 = \tau_T$

(rozloží sa rovnomerne)

$$\tau_1 = \frac{T}{F}$$

$$\tau_2 = \frac{M_k}{W_p}$$

$$\tau_{výsl.A} = \tau_A = \tau_1 - \tau_2$$

$$\tau_{výsl\,B} = \tau_B = \tau_1 + \tau_2$$

Pozn. : vplyv krút. momentu na šmykové nap. je rozhodujúci, ak máme pri presnejš. výp. $\tau_{max.}$ uvážiť nerovn. rozlož. po priereze resp. iné účinky ; vyjadríme s koeficientom k.

$$k = f\left(\frac{R}{d}\right)$$

$$\tau_{max} = k \cdot \frac{M_k}{W_o} \leq \tau_{dov.}$$

Z literatúry : $k = \left(\frac{4m-1}{4m-4} - \frac{0{,}615}{m}\right)$ / $m = \frac{2R}{d}$

Bežné hodnoty m:

m	k
4	k = 1,4
10	k = 1,14

Potenc. energ. pružiny:

pri odv. uvážime len krútenie pružiny:

$A = U$

$M_k = P.R$

$$A = U = \frac{M_k^2 . l}{2G . J_p}$$

l - rozvinutá dĺžka pružiny:

$$l = 2\pi R . n$$

n - počet účinných závitov

$$J_p = \frac{\pi d^4}{32}$$

$$A = U = \frac{1}{2} \frac{P^2 R^2 . 2\pi R n}{G . \frac{\pi d^4}{32}} = \frac{32 P^2 R^3 n}{G d^4}$$

Deformácia (zmena dĺžky) pružiny

Vplyvom skrút. pružiny pôsobište sa posunie o f. Tento posuv sa dá riešiť s pomocou def. práce a potenciálnej energie.

$$\left.\begin{aligned} A &= U \\ A &= \frac{1}{2} P . f \\ U &= \frac{32 . P^2 R^3 n}{G d^4} \end{aligned}\right\}$$

$$A = U$$

$$\frac{1}{2} P f = \frac{32 P^2 R^3 n}{G d^4}$$

$$\boxed{f = \frac{64 P R^3 . n}{G d^4}}$$

Za pomoci f. môžeme určiť pružinovú konštantu.

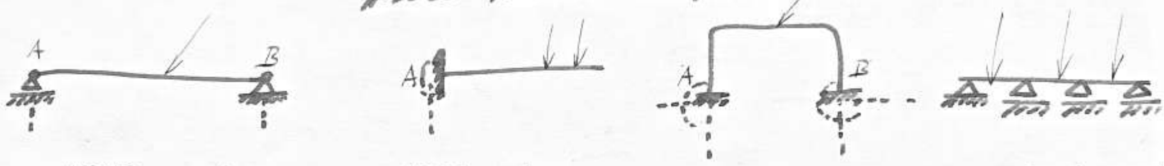
$$c = \frac{P}{f} = \frac{P . G . d^4}{64 P R^3 n} = \frac{G . d^4}{64 R^3 n} \quad [kp/cm]$$

$$c = \frac{J_p . G}{l R^2}$$

Ohyb.

Všeob. pojmy: nosník – prút ktorého priečne rozmery sú malé oproti jej dĺžky.

uloženie nosníkov – rôzne, podopreté väzbami, ktoré udob. stupne voľnosti:



VRSS st.u VRSS st.u VRSS 2x st. neurč.

Staticky určité úlohy. —

vonkajšie sily – považ. nie len vonk. zaťaženie, ale aj <u>väzbové reakcie</u>! Spôsobujú ohyb nosníka, ktorý závisí na uložení nosn.; na veľkosti a vzájomnej polohe síl, na tvare a veľkosti prierezu.

čistý ohyb – vznikne vtedy keď jediným vonkajším účinkom vonk. síl. z jednej strany uvaž. rezu je silová dvojica – moment, táto je v rovnováhe s výsledným účinkom vn. síl. (Jediné silová dvojica – moment). Obe tieto dvojice pôsobia prehnutie pôvodne priameho nosníka.

rovinný ohyb. – pokiaľ rovina vonk. síl (⇒ aj vn. síl) obsahuje niektorú z hlavn. centr. osí zotrvačnosti, ide o tzv. rov. ohyb.

def. nosníka – je potom rovinná krivka totožná s rovinnou ohýbajúcim momentom.

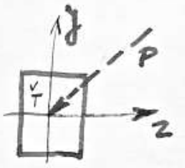
Je vtedy potr. rozlíšiť: a) čistý rov. ohyb.
b.) rov. ohyb za prítomnosti ohyb. momentu M a priečnych síl. – Vznikne keď v ľubov. reze nosníka pôsobí okrem vn. momentu ohybového aj posúvajúca sila.

Šikmý (priestorový) ohyb – v prípade že rovina ohybového momentu (i rov. vonk. síl) obsahuje iba os prúta (ani 1 z hlav. centr. osí).

Tento prípad môžme považovať za kombináciu dvoch jednod. prípadov rov-oh.

Po zať. nosníka priehybová čiara nosníka je priestorová krivka.

⇝

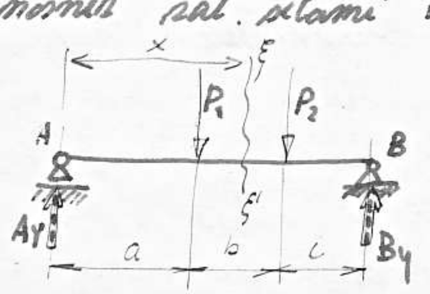


Vnútorné sily – pojem ohybového momentu a posúvajúcej sily.

Po vyp. väzb. reakc. keď poznáme vš. vonk. sily, jednoducho môžme vyčísliť vnút. sily v ľubov. reze – pomocou metódy rezu.

Pr: nosník zať. silami P_1 ; P_2 D: $P_1 P_2$ a b c ℓ   H: A B $M(x)$

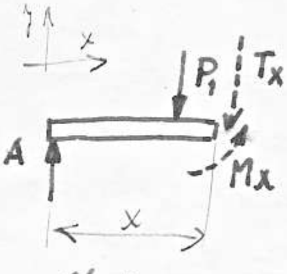
$T(x)$ – priečna sila v priereze x



1.) RRSS → stat. urč.

2.) $\left.\begin{array}{l} \sum P_i = 0 \\ \sum M_{iA} = 0 \end{array}\right\} A ; B$

3.) vyčísl. vnút. síl. veličín v ľ. reze



I.) $\overbrace{A - P_1}^{R_{vonk}} = \overbrace{T(x)}^{R_{vnút}}$ / posuvný výsledný účinok

Keď celý nosník je v rovnov. a platia stat. podm. rovnov., musí byť aj časť nosníka v rovnováhe s vnútornými silami:

II.) $\sum (M_i)_{\xi\xi_1} = 0$ $-Ax + P_1(x-a) + M(x) = 0$

$M(x) = -P_1(x-a) + Ax$

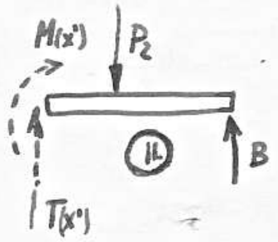
Dost. sme výsledný otáčavý účinok vnút. síl.

pravá strana výsl. ot. úč. vonk. síl.

$M(x)$ – ohybový moment. – dostaneme sčítaním dielčich momentov vonkajších síl z jednej strany rezu.

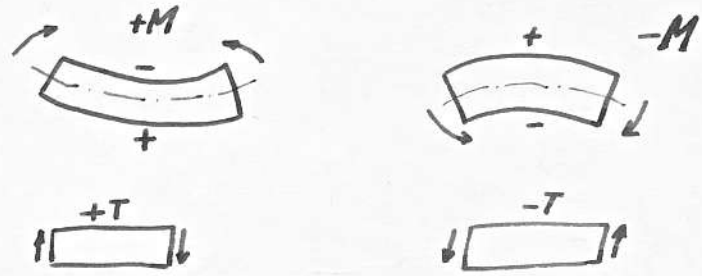
Keď vyšetrujeme pravú, druhú časť nosníka, dostaneme pre ohybový moment a posúvajúcu silu také isté veličiny ako pri vyšetr. ľavej časti; však ich smysly budú opačné. (zákon akc. – reakc.)

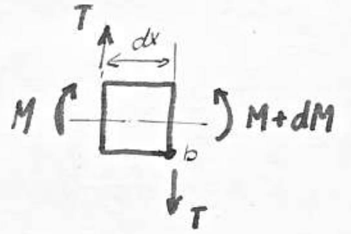
$$M(x) = M(x)'$$
$$T(x) = T(x)'$$



25. XI. 1969.

označenie.



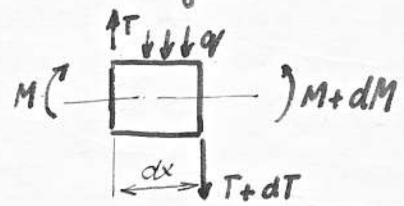


$\Sigma M_b = \dots$ $\quad M + dM = M + T.dx$

$$dM = T.dx \quad \longrightarrow \quad T = \frac{dM}{dx}$$

Schwedler-ova veta

Žuravský

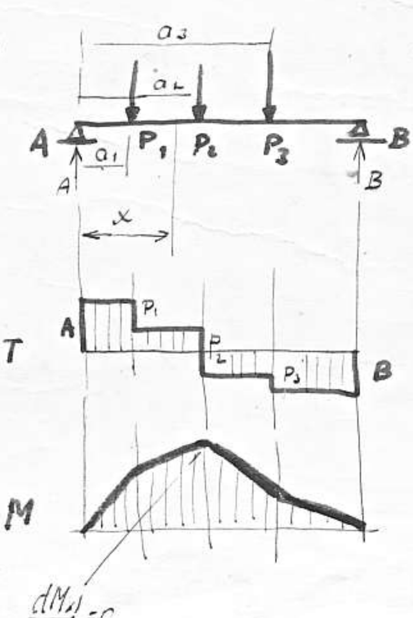


$\Sigma y \dots$ $\quad T - q.dx - (T + dT) = 0$

$$dT = -q.dx$$

$$-q = \frac{dT}{dx} \left(= \frac{d^2M}{dx^2} \right)$$

Pr.



Stanoviť namáh. a deform. nosníka.

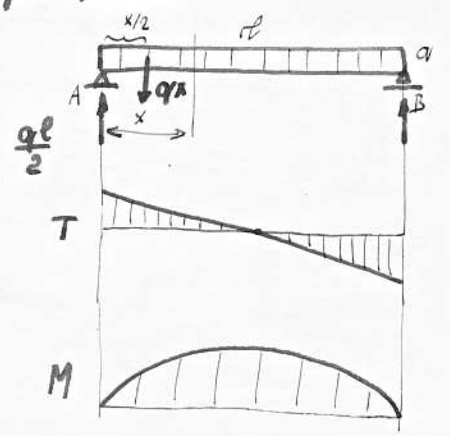
$$\left.\begin{aligned} \Sigma Y &= 0 \\ \Sigma M &= 0 \end{aligned}\right\} \Rightarrow A; B \qquad (1)$$

$$\left.\begin{aligned} x &< a_1 & T &= A \\ a_1 < x &< a_2 & T &= A - P_1 \\ a_2 < x &< a_3 & T &= A - P_1 - P_2 \\ a_3 < x &< l & T &= A - P_1 - P_2 - P_3 = B \end{aligned}\right\} T$$

$$\left.\begin{aligned} x &< a_1 & M(x) &= A.x \\ a_1 < x &< a_2 & M(x) &= A.x - P_1(x - a_1) \\ a_2 < x &< a_3 & M(x) &= A x - P_1(x - a_1) - P_2(x - a_2) \end{aligned}\right\} M$$

Moment sa mení lineárne.

Spojité zaťaž.:



$$\frac{q\cdot l}{2} = A = B$$

$$T_{(x)} = \frac{ql}{2} - q\cdot x$$

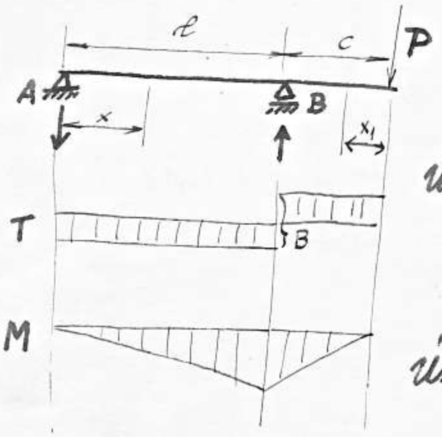
$$M_{(x)} = \frac{ql}{2}\cdot x - \frac{qx^2}{2} = \frac{q}{2}(lx - x^2)$$

$\frac{dM_{(x)}}{dx} = 0$ ⇒ extrém $\Big|\ (lx - x^2)'$

$l - 2x = 0 \Rightarrow x = \frac{l}{2}$

max. moment.

$$M_{max.} = \frac{ql^2}{8}$$



$A = \frac{P.c}{L}$ $\quad B = \frac{P(l+c)}{L}$

Úsek A ÷ B $\quad T_{(x)} = -A = -\frac{P.c}{L}$

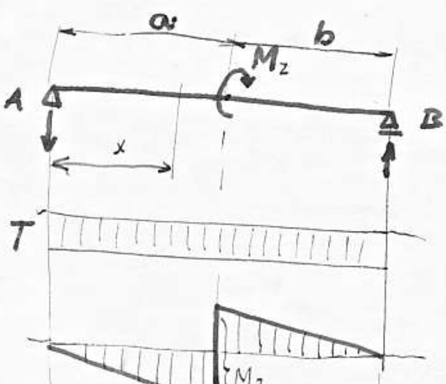
$M_{(x)} = -A.x = -\frac{P.c.x}{L}$

Úsek B ÷ P $\quad T_{(x_1)} = P$

$M_{(x_1)} = P.x_1$

Momenty z oboch strán sú rovnaké:

$\underset{x=L}{M_{(x)}} = -\frac{P_c}{L}\cdot L = -P.c \qquad \underset{x_1=c}{M_{(x_1)}} = -P.c$



$M_z = A.L \to A = \frac{M_z}{L} = -B$

$T_{(x)} = -A = -\frac{M_z}{L}$

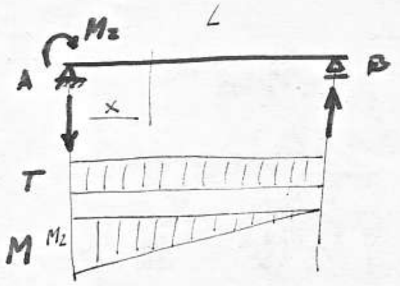
$M_{(x)} = -A\cdot x$

$x<a$

$M_{(x)} = -A.x + M_z = -\frac{M_z}{L}.x + M_z$

$x>a$

keď $x = L \qquad M_{(x)} = -\frac{M_z}{L}.L - M_z = 0$

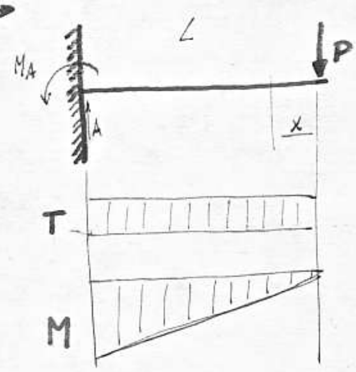


$A = \frac{M_z}{L} = -B$

$T_{(x)} = -A \qquad M_{(x)} = M_z - A \cdot x$

$dM = T.dx$

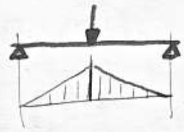
$M = \int T.dx$

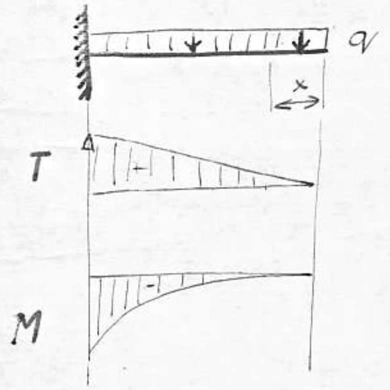


$T_{(x)} = P$

$M_{(x)} = -P.x \quad / \quad M_{max} = P.L$

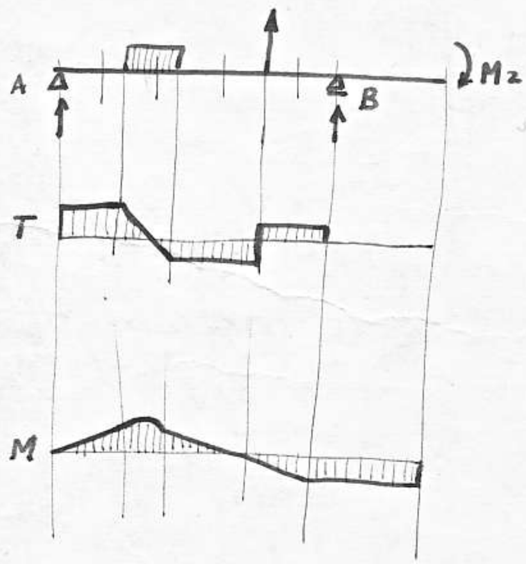
= polovica :



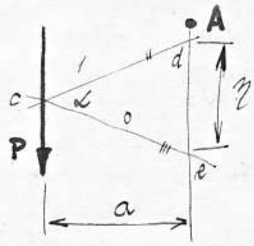


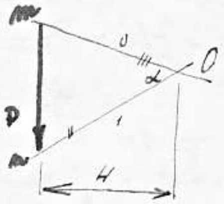
$T_{(x)} = q.x$

$M_{(x)} = q\,x \cdot \frac{x}{2} = q\,\frac{x^2}{2}$



$\left.\begin{array}{l} \sum Y = 0 \\ \sum M = 0 \end{array}\right\} A ; B$

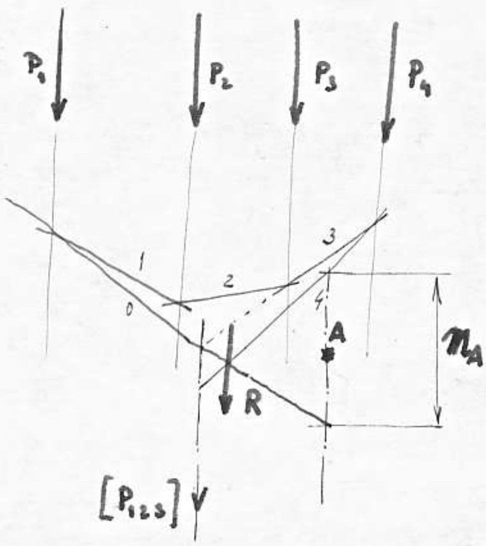


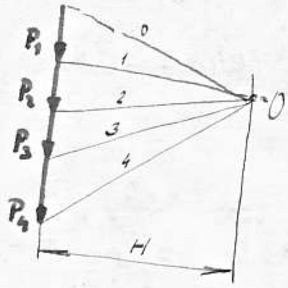


$M_A = P.a$

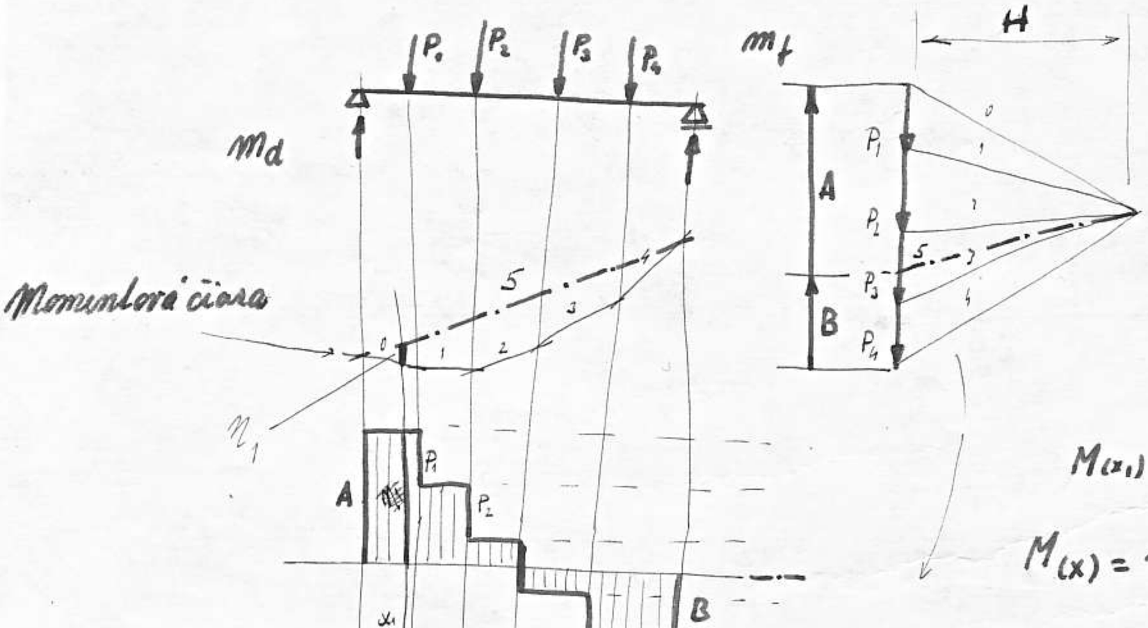
$\Delta\ Omm \sim cde\ \Delta$

$$\frac{H}{a} = \frac{P}{\eta} \rightarrow \eta H = P.a = M_A$$





$M_A = \eta_A . H$



$M_{(x_1)} = \eta_1 . H$

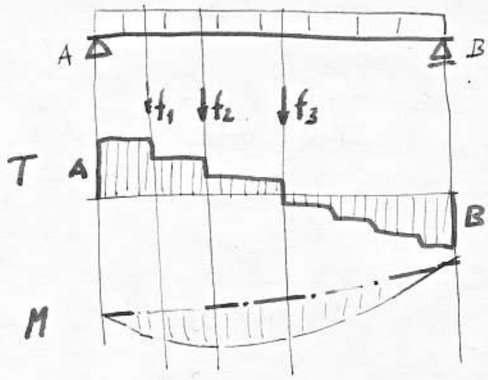
$M_{(x)} = \eta_{(x)} \cdot H$

Merítka : m_d m_f $m_M = H \cdot m_d \cdot m_f$

m_f - merítko síl
m_d - " vzd.
m_M - " momentu

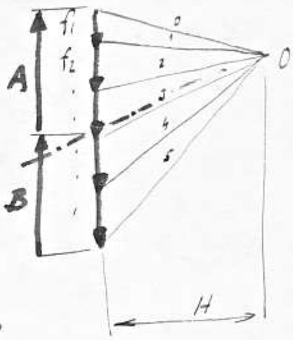
$$M_{(x)} = \eta_{(x)} \cdot m_M = \eta_{(x)} . H . m_d . m_f$$

spojité zať.



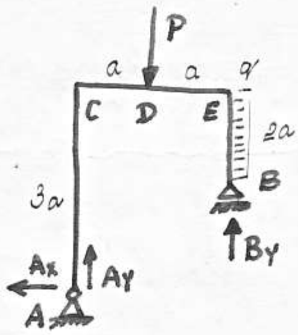
~ prevedieme na predošlú úlohu.
Sila v ťažisku obrázka.

$f_1 = q \cdot a$
$f_2 = \ldots$
. . . .



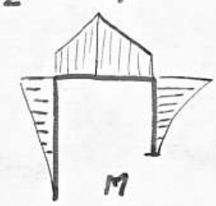
Keď element. úseky sú malé,
AB prejde na priamku, obraz moment. čiary na parabolu.

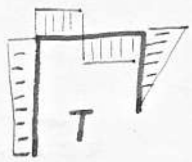
~~>

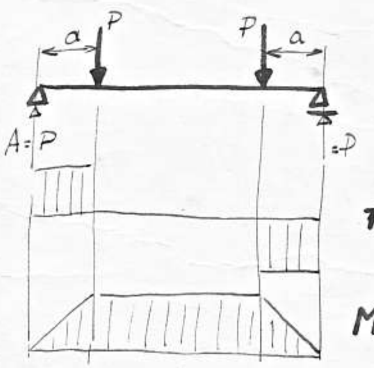


$$\left.\begin{array}{l} \Sigma Y = 0 \\ \Sigma X = 0 \\ \Sigma M = 0 \end{array}\right\} \quad \begin{array}{l} A_x = 2a \cdot q \\ A_Y = \frac{P}{2} - 2aq \\ B_Y = \frac{P}{2} + 2aq \end{array}$$

E - B
E - D
D - C
C - A



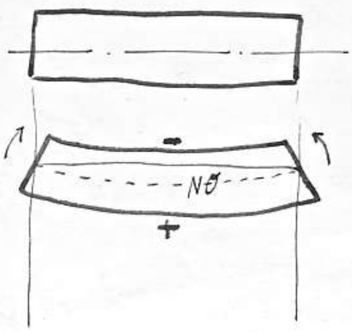




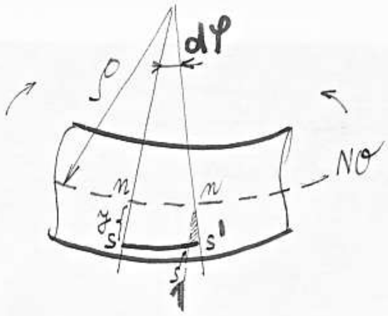
2.XII. 1969.

$$\frac{dM_x}{dx} = T$$

NO - vlákno bez predĺženia



→



$$\varepsilon = \frac{s_1 s'}{ss_1} = \frac{s_1 s'}{n.n}$$

$$ss' = y.d\varphi$$

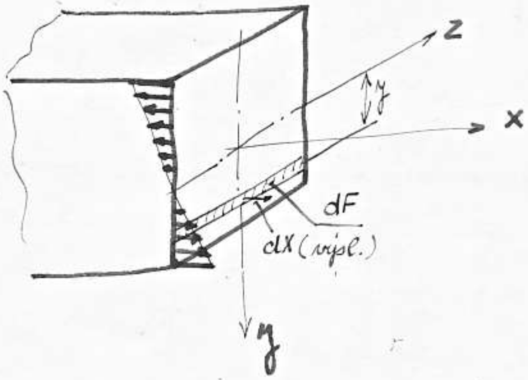
$$nn = \varrho.d\varphi$$

$$\varepsilon = \frac{y\,d\varphi}{\varrho.d\varphi} = \frac{y}{\varrho}$$

z H. z. :

$$\varepsilon = \frac{\sigma}{E} = \dots = \frac{y}{\varrho} \rightarrow \boxed{\sigma = \frac{E}{\varrho}.y}$$ / lineárne rozloženie

$$\sigma = k.y$$



$$\circlearrowleft\ \circlearrowright \equiv \rightleftarrows$$

$$\sum X = 0$$

$$dX = dF.\sigma_{(x)} = \frac{E}{\varrho}.y.dF$$

$$\int dX = \int \frac{E}{\varrho}.y.dF = 0$$

$$\underbrace{\frac{E}{\varrho}}_{konšt.}\int_F \underbrace{y\,dF}_{stat.\ mom.} = 0$$ $\Rightarrow$ neutrálna os prech. ťažiskom n

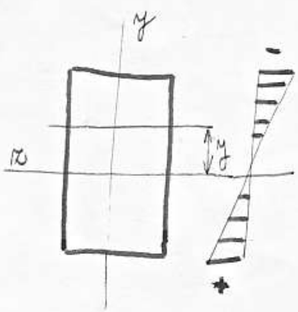
$$\sum M_z = 0$$

$$dM_z = dX.y = \frac{E.y}{\varrho}.dF.y = \frac{E}{\varrho}.dF.y^2$$

$$M_z = \int_F dM_z = \frac{E}{\varrho}.\int_F dF.y^2 = M_{vonk.;\ ohybový}$$

$$M = \frac{E}{\varrho}.\underbrace{\int_F dF.y^2}_{mom.\ zotrv.\ k\ z\ (prierezu)} = \frac{E.J_z}{\varrho} \rightarrow \frac{1}{\varrho} = \frac{M}{J_z.E} \Rightarrow$$

$$\sigma = \frac{M.y.E}{E.J_z} = \boxed{\frac{M.y}{J_z} = \sigma}$$

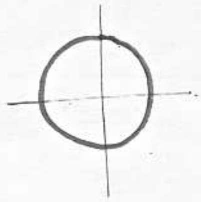


$$\sigma_{max} = \frac{M.y_{max}}{J_z}$$ / $$\frac{J_z}{y_{max}} = W_z$$ modul prierezu v ohybe

$$\boxed{\sigma = \frac{M}{W_z}} \leq \sigma_{dov}$$

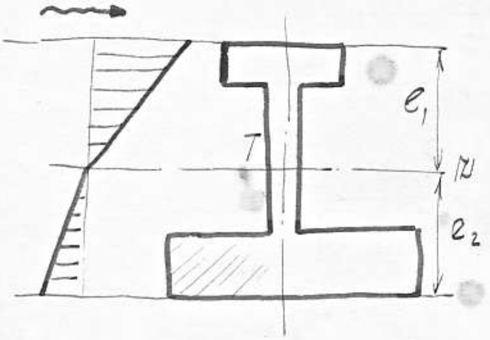
$$J = \frac{1}{12}bh^3$$

$$W = \frac{J}{\frac{h}{2}} = \frac{1}{6}bh^2$$



$$J = \frac{\pi d^4}{64}$$

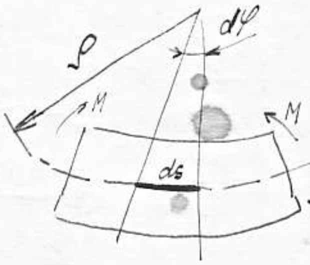
$$W = \frac{J}{\frac{d}{2}} = \frac{\pi d^3}{32}$$



$$W_1 = \frac{J_z}{e_1}$$

$$W_2 = \frac{J_z}{e_2}$$

Práca pri deform.; vn. energia.



$$A = U$$

$$dA = M \cdot d\varphi \cdot \frac{1}{2}$$

$\rho\, d\varphi = ds \rightarrow d\varphi = \frac{ds}{\rho}$

$ds \doteq dx$

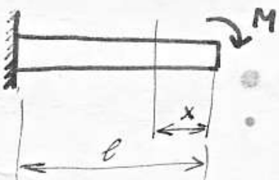
$\left.\right\} d\varphi \doteq \frac{dx}{\rho}$

$$\left.\right\} dA = \frac{1}{2} M \frac{dx}{\rho} \qquad \Big| \; \frac{1}{\rho} = \frac{M}{E.J}$$

$$U = \int \frac{1}{2} M \cdot dx \cdot \frac{M}{EJ} = \frac{1}{2} \int \frac{M_x^2}{EJ} \cdot dx$$

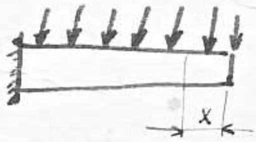
tuhosť v ohybe

Pr.:



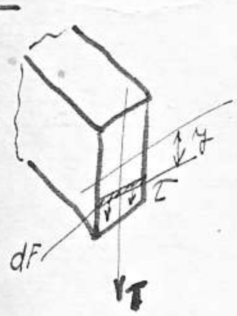
$$M_{(x)} = M$$

$$U = \frac{1}{2} \int_0^l \frac{M^2}{E.J} \cdot dx = \frac{1}{2} \frac{M^2 l}{E.J}$$

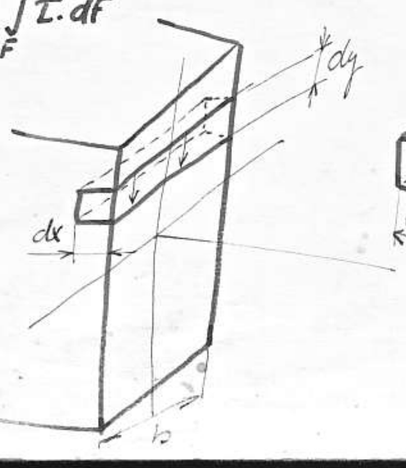


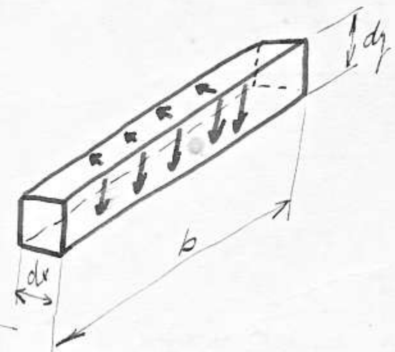
$$M_x = \frac{1}{2} q x^2$$

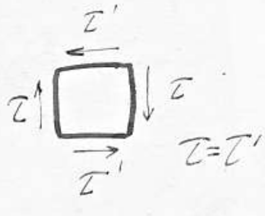
$$U = \frac{1}{2} \int \frac{1}{4} \frac{q^2 x^4}{E.J} \cdot dx$$

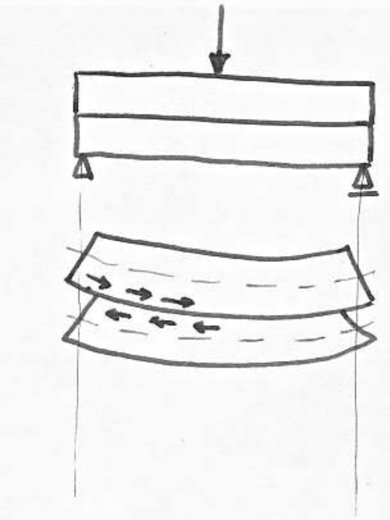


$$T = \int_F \tau . dF$$

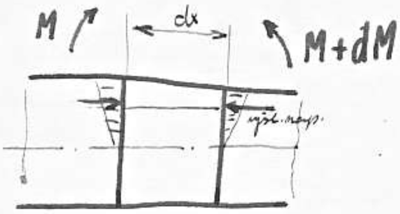


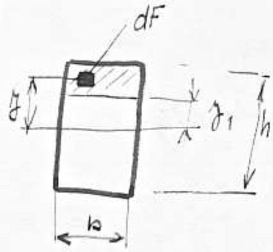






→ vznik vodor. nap. τ'



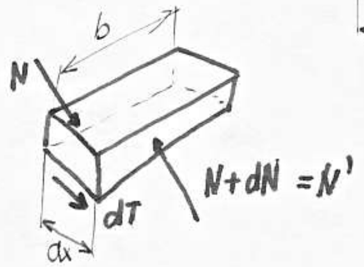


$$\sum x = 0$$

$$dT = \tau \cdot b \cdot dx$$

$$dN = dF \cdot \sigma = \frac{M \cdot y}{J} \cdot dF$$

$$N = \int dN = \int_{y_1}^{h/2} \frac{M \cdot y}{J} \cdot dF$$

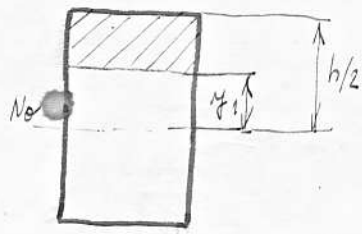


$$N' = \int_{y_1}^{h/2} \frac{(M+dM)y}{J} \cdot dF$$

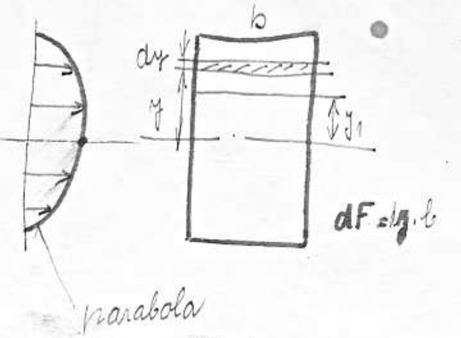
$$N + dT = N' \quad \ldots\ldots \int_{y_1}^{h/2} \frac{My}{J} dF + \tau b dx = \int_{y_1}^{h/2} \frac{M+dM}{J} y \, dF$$

$$\tau b dx = \int_{y_1}^{h/2} \left(\frac{M+dM}{J} \cdot y \, dF - \frac{M}{J} y \cdot dF \right) = \int_{y_1}^{h/2} \frac{dM \cdot y}{J} \cdot dF$$

$$\tau = \frac{dM}{dx} \cdot \frac{1}{J \cdot b} \cdot \int_{y_1}^{h/2} y \cdot dF = T \cdot \frac{1}{b \cdot J} \cdot \int_{y_1} y \, dF$$



$$\tau = \frac{T}{b \cdot J} \cdot S$$ / S - stat. moment nad vyšetř. plochou

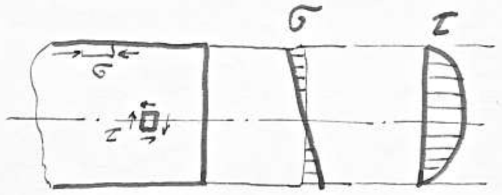


$$\tau_{y_1} = \frac{T}{b \cdot J_z} \int_{y_1}^{h/2} y \cdot b \cdot dy = \frac{T}{J_z} \cdot \left[\frac{y^2}{2} \right]_{y_1}^{h/2} = \frac{T}{J_z} \left[\frac{h^2}{8} - \frac{y_1^2}{2} \right]$$

$$\tau_{max\ y_1=0} = \frac{Th^2}{8 J_z} = \frac{12 h^2}{8 b h^3} T = \frac{12}{8bh} T = \frac{3}{2} \frac{T}{bh} = \frac{3}{2} \frac{T}{F}$$

nap. o km)

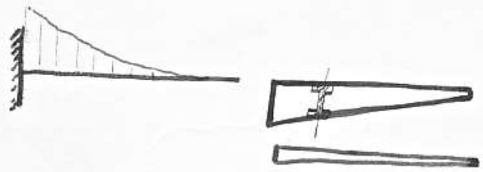
V ohybe vyskyt. aj norm. aj tang. napätie – slož.



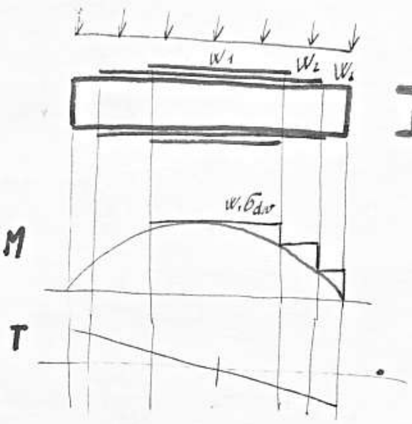
$$\sigma_{kom.\,3} = \sqrt{\sigma^2 + 4\tau^2} \leq \sigma_{dov.}$$

$$\sigma_{kom.\,4} = \sqrt{\sigma^2 + 3\tau^2} \leq \sigma_{dov.}$$

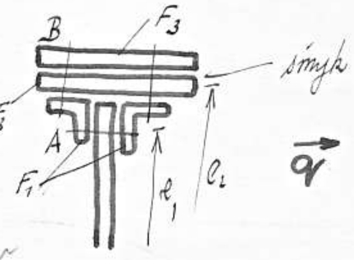
$[\tau < \sigma]$

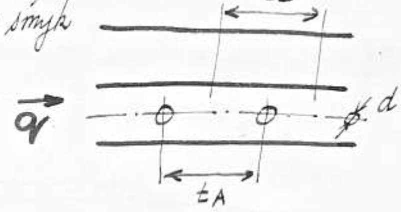


din. nosník



$$M_{dov} = W . \sigma_{dov}$$





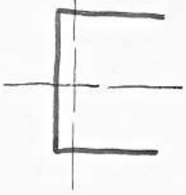
$$\tau = \frac{T.S}{J.b}$$ stat. mom. / $\tau . b = q$ ~ šmykový tok [kp/cm]

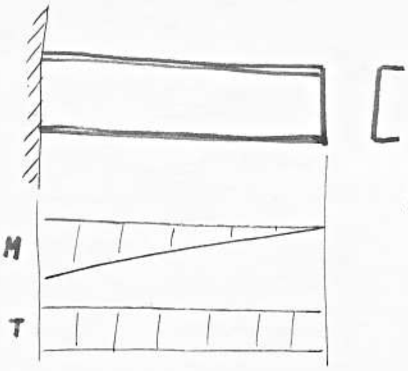
$$q = \frac{T.S}{J} \Rightarrow P_A = q . t_A \rightarrow \tau = \frac{P_A}{F} \leq \sigma_{dov.}$$

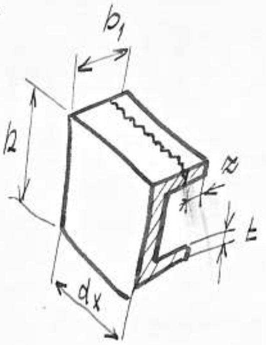
stat. mom. – $S_{\text{pre nity } A} = F_1 . e_1 + F_2 e_2 + F_3 e_3$ (stat. mom.)

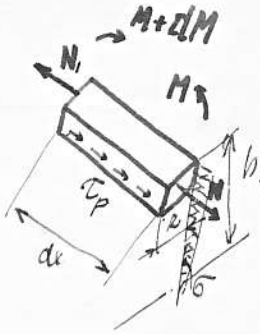
[Rozstup nitov sa podľa napätia môže meniť]

Nesymetrické profily :









$\Sigma x = 0 \qquad N_1 - N = dT$ | $dT = dx \cdot t \cdot \tau_p$

$N = \sigma \cdot t \cdot z$ (tenkostenný profil, σ považ. za konšt.)

$N_1 = \sigma_1 \cdot t \cdot z$

$$\sigma = \frac{M}{J} \cdot y_{max} = \frac{M}{J} \cdot \frac{h}{2}$$

$$\sigma_1 = \frac{M+dM}{J} \cdot \frac{h}{2}$$

$$\tau_p \cdot dx \cdot t = N_1 - N = \left(\frac{M+dM}{J} - \frac{M}{J}\right)\frac{h}{2} \cdot t \cdot z$$

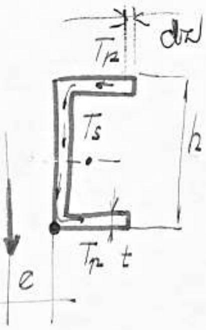
$$\tau_p \cdot dx \cdot t = \frac{dM}{J} \cdot \frac{h}{2} \cdot t \cdot z$$

$$\tau_p \cdot t \cdot dx = \frac{dM}{J} \cdot \frac{h}{2} \cdot t \cdot z$$

$$\tau_p \cdot t = \frac{dM}{dx} \cdot \frac{h \cdot t \cdot z}{2J} = \frac{T}{J} \boxed{\frac{h}{2} \cdot z \cdot t}$$

$$\tau_p \cdot t = \frac{T}{J} \cdot S_y$$

$$q_p = \frac{T}{J} \cdot S_y \quad / \text{ šmykový tok}$$



stred šmyku

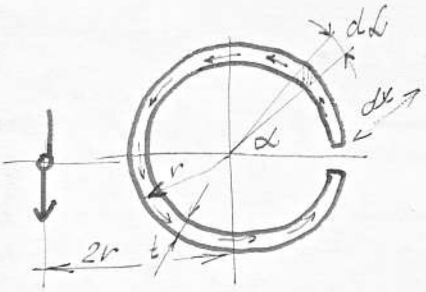
$$T_p = \int_0^{b_1} q_p \, dz = \int_0^{b_1} \frac{T}{J} \cdot \frac{h}{2} \cdot z \cdot t \cdot dz = \frac{Th}{J4} b_1^2 t \quad (\text{pomocná sila})$$

$T_s = T$ (šmyková sila)

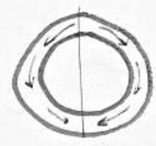
$T_s \cdot e = T_p \cdot h$ (momenty aby sa rovnali)

$$e = \frac{T_p h}{T_s} = \frac{T h^2 b_1^2 t}{4JT} = \frac{h^2 b_1^2 t}{4J}$$

Aby profil nebol namáh. na krut, sila musí prechádzať stredom šmyku.



$$q_\alpha = \frac{T \cdot S_\alpha}{J}$$ | S_α - stat. moment plochy.



$$dF = r \cdot d\alpha \cdot t$$

$$J_p = \int_F r^2 dF = \int_0^{2\pi} r^3 t \cdot d\alpha = 2\pi r^3 t$$

$$J_x = \frac{J_p}{2} = \pi r^3 t$$

$$S_\alpha = \int_0^\alpha y \cdot dF = \int_0^{\alpha} r \sin\alpha \cdot r \cdot d\alpha \cdot t = r^2 t \int_0^\alpha \sin\alpha \, d\alpha$$

$$S_\alpha = r^2 t (1 - \cos\alpha)$$ $\Big/ \int_0^\alpha \sin\alpha \, d\alpha = [-\cos]_0^\alpha = [-\cos\alpha + 1]$

Alebo:

$$\tau_\alpha \cdot t\, d\alpha = \int_0^\alpha \overbrace{\frac{dM}{J} \cdot r \sin\alpha}^{\sigma_\alpha} \cdot \overbrace{t r \, d\alpha}^{dF}$$

$$q_\alpha = \int_0^\alpha \frac{T}{J} \cdot r^2 \sin\alpha \, t \, d\alpha$$

$$q_\alpha = \frac{T t r^2}{J} [1 - \cos\alpha]$$

$$eT = \int_0^{2\pi} r \cdot q_\alpha \cdot r \, d\alpha$$

$$eT = r^2 \int_0^{2\pi} \frac{T t r^2}{\pi r^3 t} [1 - \cos\alpha] \, d\alpha$$

$$e = \frac{r}{\pi} \int_0^{2\pi} (1 - \cos\alpha) \, d\alpha$$

$$e = \frac{r}{\pi} (2\pi - 0) = 2r$$

$$q_\alpha = \frac{T r^2 t (1 - \cos\alpha)}{\pi r^3 t}$$

$$q_\alpha = \frac{T}{\pi r} [1 - \cos\alpha] \longrightarrow M_t = \int_0^{2\pi} q_\alpha \cdot ds \cdot r = r^2 \int_0^{2\pi} \frac{T}{\pi r} [1 - \cos\alpha] =$$

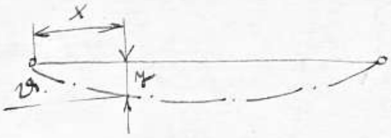
$ds = r \cdot d\alpha$

$$M_t = 2T \cdot r.$$

Podm. rovnov.: $eT = 2Tr \rightarrow \underline{\underline{e = 2r}}$ (v) musí pôs. sila aby profil nebol namáhaný na krut.

~> Keď je v profile otvor, okraj sa musí zosilniť, aby prenášal šmykový tok. (Otvory nie sú výhodné pri dimenzovaní)

Deformácia nosníka



$$y = f(x)$$

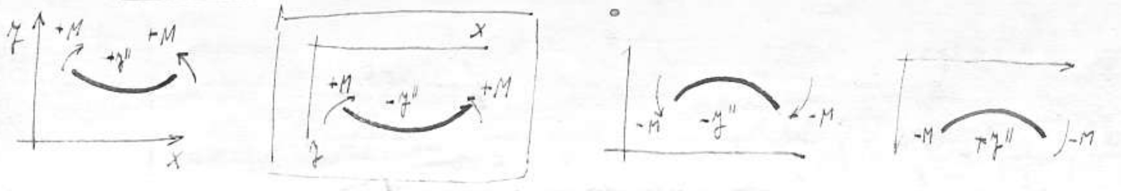
$$\operatorname{tg} \vartheta = \frac{dy}{dx} \doteq \vartheta$$

$$\frac{1}{\rho} = \frac{M}{E \cdot J}$$

$$\frac{1}{\rho} = \pm \frac{y''}{(1 + y'^2)^{3/2}}$$

$$\left. \begin{aligned} y' < 1 \rightarrow\ & y'' \ll 1 \\ & y'^2 \doteq 0 \end{aligned} \right\} \Rightarrow \frac{1}{\rho} = \pm y''$$

$$\underline{\underline{y'' = \pm \frac{M}{E \cdot J}}}$$ ~ zjednodušená dif. rovn. elastickej čiary.



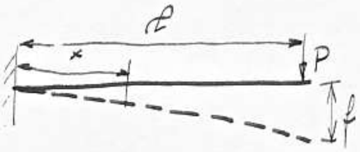
$$\Bigg\} \; y'' = -\frac{M}{E J}$$

$$y'' = -\frac{M_{(x)}}{EJ}$$

$$y' = -\int \frac{M}{EJ}\,dx + C_1 \quad = \vartheta$$

$$y = -\int dx \int \frac{M_{(x)}}{EJ}\,dx + C_1 x + C_2$$ ~ rovnica krivky

Príklad:



$$y'' = -\frac{M_{(x)}}{EJ} \quad \Big| \quad M_{(x)} = -P(l-x)$$

$$y'' = \frac{P(l-x)}{E.J} \Rightarrow \quad y''EJ = P(l-x) \quad /\text{int}$$

$$EJ.y' = P\left(lx - \frac{x^2}{2}\right) + C_1 \quad /\text{int}$$

$$EJ.y = P\left(\frac{lx^2}{2} - \frac{x^3}{6}\right) + C_1 x + C_2$$

vyp. konšt.: 1.) $x=0 \; \dots \; y=0$

2.) $x=0 \; \dots \; \vartheta = 0 = y'$

$$P\left(l.0 - \frac{0}{2}\right) + C_1 = 0 \rightarrow \quad C_1 = 0$$
$$C_2 = 0$$

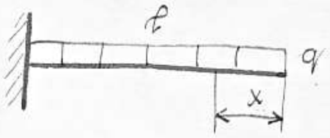
$$y' = \vartheta = \frac{P}{E.J}\left(lx - \frac{x^2}{2}\right) = \frac{Plx}{2EJ}\left(2 - \frac{x}{l}\right)$$

$$y = \frac{P}{EJ}\left(\frac{lx^2}{2} - \frac{x^3}{6}\right) = \frac{Plx^2}{6EJ}\left(3 - \frac{x}{l}\right)$$

$x = l$

$$f = \frac{P.l^3}{6EJ}(3-1) = \frac{Pl^3}{3EJ}$$

$$\vartheta = \frac{Pl^2}{2EJ}$$



$$M_{(x)} = -\frac{qx^2}{2} \quad \Big| \quad y'' = \frac{qx^2}{2EJ} \quad \Big| \quad EJ.y'' = \frac{qx^2}{2}$$

$$EJ.y' = \frac{qx^3}{6} + C_1$$

$$EJ.y = \frac{qx^4}{24} + C_1 x + C_2$$

1.) $x = L \; \dots \; y = 0$

$x = L \; \dots \; y' = 0$

$$\frac{ql^3}{6} + C_1 = 0 \rightarrow C_1 = -\frac{ql^3}{6}$$

$$\frac{ql^4}{24} + \left(-\frac{ql^4}{6}\right) + C_2 = 0 \rightarrow \quad C_2 = \frac{ql^4}{8}$$

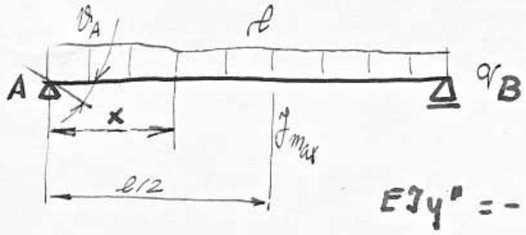
$$y' = \frac{1}{EJ}\left(\frac{qx^3}{6} - \frac{ql^3}{6}\right)$$

$$y = \frac{1}{EJ}\left(\frac{qx^4}{24} - \frac{ql^3}{6}x + \frac{ql^4}{8}\right)$$

$$y_{max\,(x=0)} = \frac{1}{EJ}\cdot\frac{ql^4}{8}$$

$$\vartheta_{max\,(x=0)} = -\frac{ql^3}{6EJ}$$

$x_1 = \frac{l}{2}$: $\quad y_{max} = \frac{Pl^3}{48EJ}$



$$A = \frac{ql}{2}$$

$$M_{(x)} = \frac{ql}{2}x - \frac{qx^2}{2} \qquad y'' = -\frac{M(x)}{EJ}$$

$$EJy'' = -\frac{q}{2}(lx - x^2)$$

$$EJy' = -\frac{q}{2}\left(l\frac{x^2}{2} - \frac{x^3}{3}\right) + C_1$$

$$EJy = -\frac{q}{2}\left(l\frac{x^3}{6} - \frac{x^4}{12}\right) + C_1x + C_2$$

Podm.: $x = 0 \quad y = 0 \quad \rightarrow C_2 = 0$

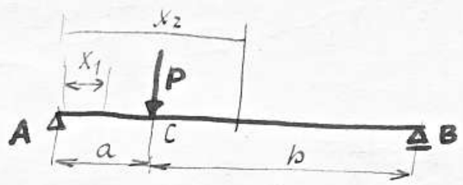
$x = l \quad y = 0 \quad \rightarrow \frac{q}{2}\left(\frac{l^4}{6} - \frac{l^4}{12}\right) + C_1 l = 0 \rightarrow C_1 = \frac{ql^3}{24}$

$$\vartheta = y' = -\frac{q}{2EJ}\cdot\left(\frac{lx^2}{2} - \frac{x^3}{3}\right) + \frac{ql^3}{24}$$

$$y = -\frac{q}{2EJ}\left(\frac{lx^3}{6} - \frac{x^4}{12}\right) + \frac{ql^3x}{24EJ}$$

$$y_{max}\,[l/2] = \frac{5}{384}\cdot\frac{ql^4}{EJ}$$

$$\vartheta_{A\,(x=0)} = \frac{ql^3}{24EJ}$$



$$A = \frac{Pb}{l} \qquad B = \frac{P.a}{l}$$

úsek A:C

$$M_{x_1} = Ax_1 = \frac{Pb}{l}x_1$$

$$EJy_1'' = -\frac{Pb}{l}x_1$$

$$EJy_1' = -\frac{Pb}{l}\cdot\frac{x_1^2}{2} + C_1$$

$$EJy_1 = -\frac{Pb}{l}\frac{x_1^3}{6} + C_1x_1 + C_2$$

úsek C:B

$$M_{x_2} = A.x_2 - P(x-a) = \frac{Pb}{l}x_2 - P(x_2-a)$$

$$EJy_2'' = -\frac{Pb}{l}x_2 + P(x_2-a)$$

$$EJy_2' = -\frac{Pb}{l}\frac{x_2^2}{2} + \frac{P(x_2-a)^2}{2} + C_3$$

$$EJy_2 = -\frac{Pb}{l}\frac{x_2^3}{6} + \frac{P(x_2-a)^3}{6} + C_3x_2 + C_4$$

1.) $x_1 = 0 \dots y_1 = 0$

2.) $x_2 = L \dots y_2 = 0$

3.) $x_1 = x_2 = a \dots y_1' = y_2'$

4.) $x_1 = x_2 = a \dots y_1 = y_2$

$y_1 = y_2 \;|\; -\frac{Pb}{l}\frac{a^3}{6} + C_1a + C_2 = -\frac{Pb}{l}\frac{a^3}{6} + C_3a + C_4$

$y_1' = y_2' \;|\; -\frac{Pb}{l}\cdot\frac{a^2}{2} + C_1 = -\frac{Pb}{l}\frac{a^2}{2} + C_3 \quad \rightarrow C_1 = C_3$

$\Big\}\; C_2 = C_4$

$x_1 = 0,\ y_1 = 0 \;|\; C_2 = 0$

$x_2 = L,\ y_2 = 0 \;|\; C_3 = \frac{Pb}{6l}(l^2 - b^2) = C_1$

$\Big\}\; C_4 = 0$

A:C $\quad \vartheta = y' = \frac{1}{EJ}\left[\frac{Pb}{6l}(l^2-b^2) - \frac{Pb}{l}x_1^2/2\right]$

$\quad y = \frac{1}{EJ}\left[\frac{Pb}{6l}(l^2-b^2) - \frac{Pb}{6l}x_1^3\right]$

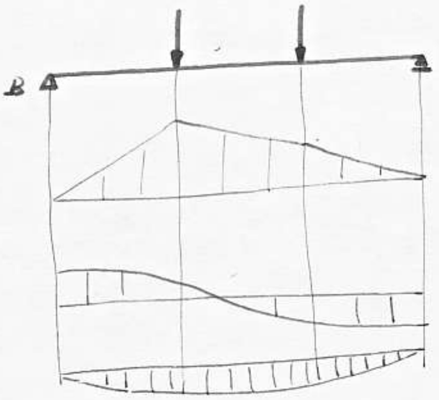
C:B $\quad \vartheta = y' = \frac{1}{EJ}\left[\frac{Pb}{6l}(l^2-b^2) - \frac{Pb}{l}\frac{x_2^2}{2} + \frac{P(x_2-a)^2}{2}\right]$

$\quad y = \frac{1}{EJ}\left[\frac{Pb}{6l}(l^2-b^2) - \frac{Pb}{l}x_2^3/6l + \frac{P(x_2-a)^3}{6}\right]$

PREDNÁŠKA z P. 16. XII. 1969.

Metóda momentovej plochy (Mohr)

Štálovi D 321.



$M(x) = q_f$ – považujeme za spoj. zaťaž. nosníka.

T_f

M_f

$$\frac{d^2 M_f}{dx^2} = q_f$$

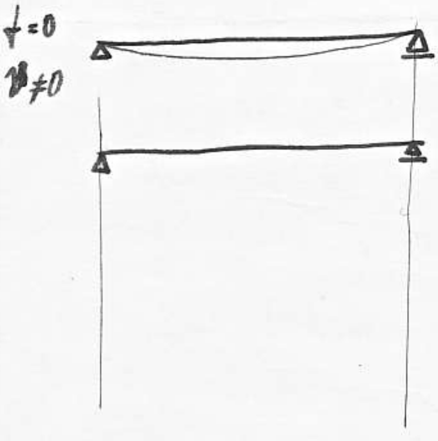
$$q_f = M(x) = \frac{d^2 M_f}{dx^2}$$

z predošl. predn.: $EJ \frac{d^2 y}{dx^2} = M(x)$

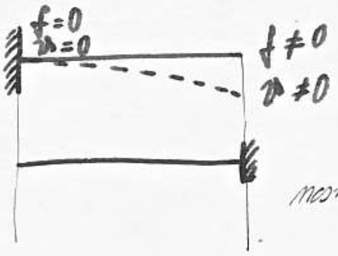
$$EJ \frac{d^2 y}{dx^2} = \frac{d^2 M_f}{dx^2}$$

$$EJ \cdot \frac{dy}{dx} = \frac{dM_f}{dx} = T_f \longrightarrow \frac{\partial y}{dx} = \vartheta = \frac{T_f}{EJ} \quad \|$$

$$EJ\, y = M_f \longrightarrow y = \frac{M_f}{EJ} \quad \|$$

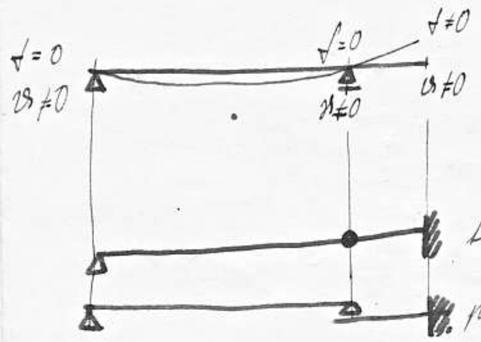


v podporách je nulový moment



nosník skutočný

nosník pri počítaní



skut.

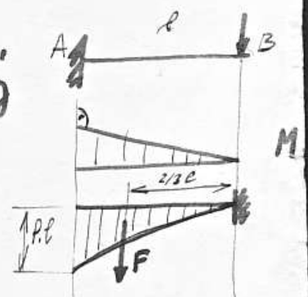
teor.

pri poč.

Zosk. nosník skutočný
" moment. plochy

M(x) považ. za zaťaž.

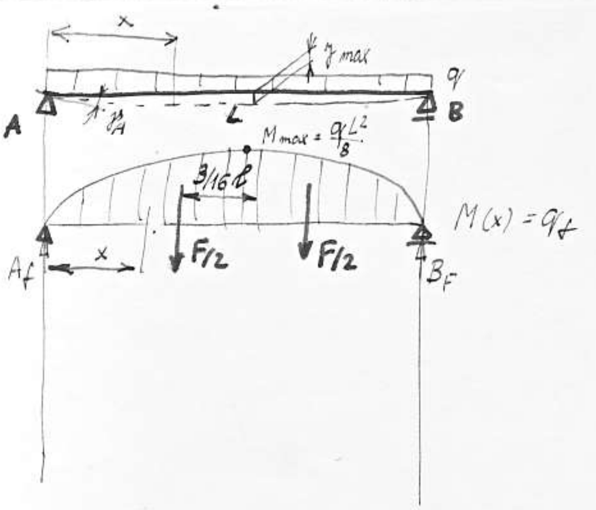
sila vyrieš. teor. nosn.
dáva uhol natoč.



$$F = \frac{PL}{2} L = \frac{PL^2}{2}$$

$$T_{f_B} = F = \frac{PL^2}{2} \qquad \vartheta_{max} = \frac{T_{fB}}{EJ} = \frac{PL^2}{2EJ} \qquad y_{max} = \frac{M_{fB}}{EJ} =$$

$$y_{max} = \frac{2}{3} L \cdot \frac{PL^2}{2EJ}$$



z plochy paraboly:

$$F = \frac{2}{3}\cdot\frac{qL^2}{8}\cdot L = \frac{qL^3}{12} \rightarrow \frac{F}{2} = \frac{ql^3}{24}$$

$$A_f = \frac{F}{2} = \frac{qL^3}{24}$$

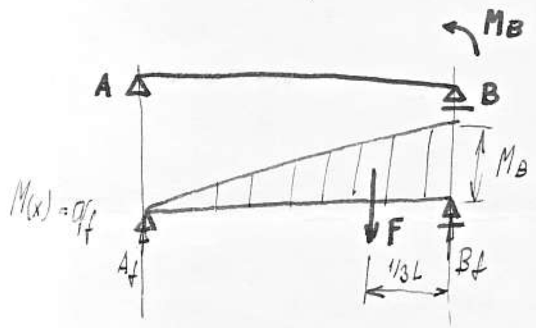
$$M_{fc} = A_f\cdot\frac{l}{2} - \frac{F}{2}\cdot\frac{3}{16}L = \frac{ql^3}{24}\cdot\frac{L}{2} - \frac{qL^3}{24}\cdot\frac{3}{16}\cdot L$$

$$M_{fc} = qL^4\left(\frac{1}{48} - \frac{1}{128}\right) = \frac{5}{384}qL^4$$

$$y_{max} = \frac{M_{fc}}{EJ} = \frac{5}{384EJ}\cdot qL^4$$

$$\varphi_c = \frac{T_c}{JE} = \frac{A_f - F/2}{EJ} = 0$$

$$\vartheta_A = \frac{T_{fA}}{EJ} = \frac{A_f}{EJ} = \frac{qL^3}{24EJ}$$



$\vartheta_A = ?$ $\vartheta_B = ?$

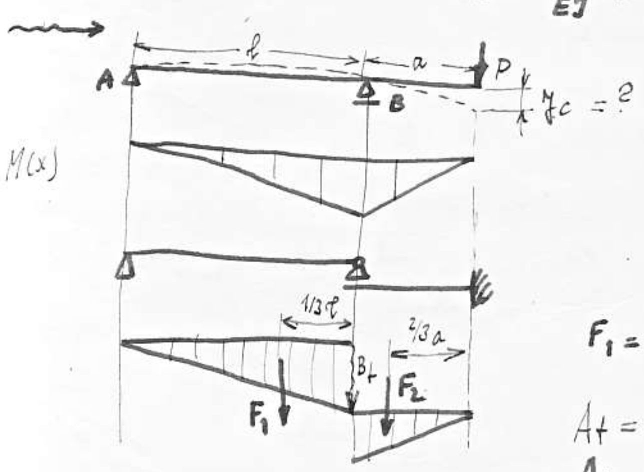
$$F = \frac{M_B . L}{2}$$

$$F.\frac{2}{3}L - B_f L = 0 \longrightarrow B_f = \frac{2}{3}F = \frac{2}{3}\cdot\frac{M_B L}{2} = \frac{M_B . L}{3}$$

$$A_f = \frac{1}{3}F = \frac{M_B L}{6}$$

$$\vartheta_A = \frac{T_{fA}}{EJ} = \frac{A_f}{EJ} = \frac{M_B . L}{6EJ}$$

$$\vartheta_B = \frac{T_{fB}}{EJ} = \frac{B_f}{EJ} = \frac{M_B . L}{3EJ}$$



$$F_1 = \frac{P.a.L}{2} \qquad F_2 = \frac{Pa^2}{2}$$

$$A_f = \frac{1}{3}F_1 \qquad B_f = \frac{2}{3}F_1$$

$$A_f = \frac{Pal}{6} \qquad B_f = \frac{Pal}{3}$$

$$M_{fc} = B_f\cdot a + F_2\cdot\frac{2}{3}a = \frac{Pa.a.l}{3} + \frac{Pa^2}{2}\cdot\frac{2}{3}a = \frac{Pa^2L}{3} + \frac{Pa^3}{3}$$

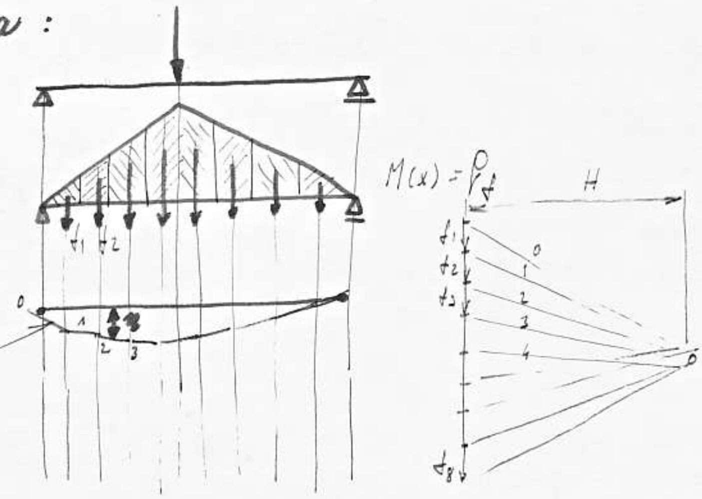
$$y_c = \frac{M_{fc}}{EJ} = \frac{Pa^2L}{3EJ} + \frac{Pa^3}{3EJ}$$

$\vartheta_c = \frac{T_{fc}}{EJ}$ | $T_{fc} = B_f + F_2 = \frac{Pal}{3} + \frac{Pa^2}{2}$

$$\vartheta_c = \frac{Pal}{3EJ} + \frac{Pa^2}{2EJ}$$

Grafická metóda :

(Mohrova m.) $q = \frac{M_f}{EJ}$



Ohybová čiara!

$M_{f(x)} = y \cdot EJ$

$M_{(x)} = H \cdot m$

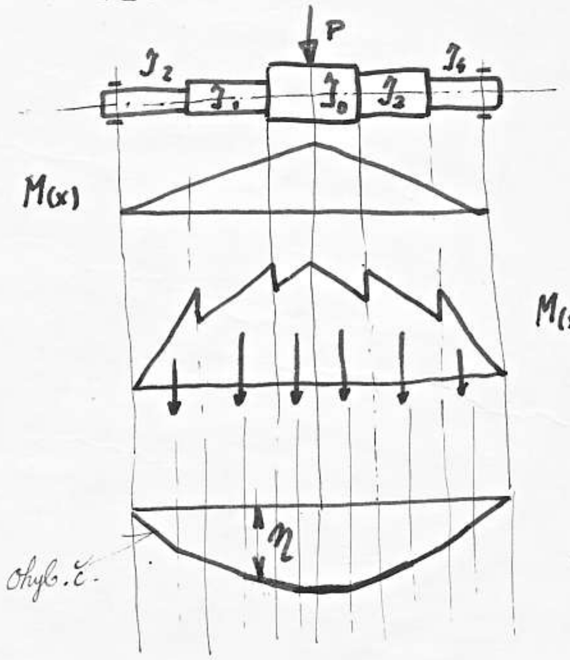
$M_{f(x)} = \eta_x \cdot H = y \cdot E \cdot J$

$y = \frac{\eta_x \cdot H}{E \cdot J}$

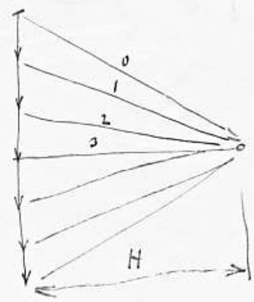
ozn.: $H = \frac{EJ}{K}$ potom $\eta_x \frac{EJ}{K} = y \cdot EJ$

$$y = \frac{\eta}{K}$$

$$m_y = \frac{m_d \cdot m_f \cdot H}{EJ} \quad \Big\} \quad y = m_y \cdot \eta$$



$M_{(x)} \cdot \frac{J_0}{J_i} = M_{presné} = q_f$



$y = m_y \cdot \eta$

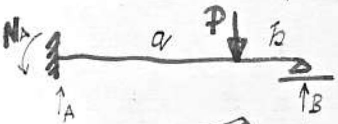
$m_y = \frac{m_d \cdot m_f \cdot H}{E \cdot J_0}$

Keď si zvol. malé úseky, lomená čiara prejde v spojitú (ohyb.) čiaru

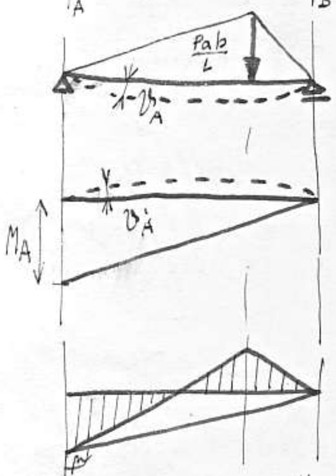
Staticky neurč. nosníky:

4 neurn. − 3 rovn.

Musíme nap. ešte deform. p

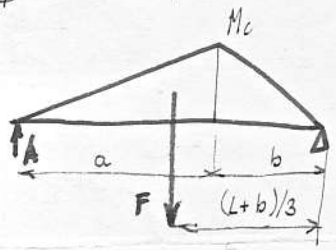


1x st. neurč. $\Sigma Y = 0$, $\Sigma M = 0$ | A B M_A



$$\vartheta_A - \vartheta_A' = 0$$

Výsl. mom je súčet 2 momentov. (2 hraných)

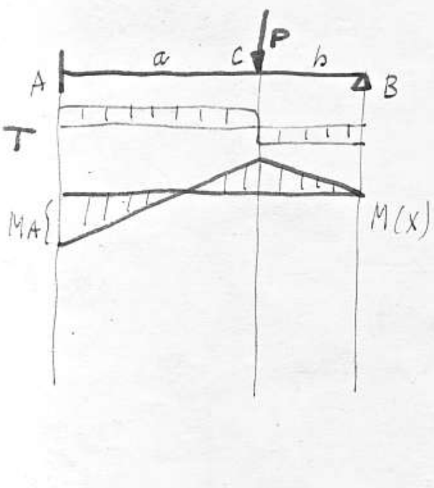


$$\vartheta_A = \frac{T_{vA}}{JE} = \frac{A_f}{EJ} \qquad F = \frac{M_c L}{2} = \frac{Pab}{2}$$

$$A_f \cdot L - \frac{(L+b)Pab}{3 \cdot 2} = 0 \quad \rightarrow \quad A_f = \frac{Pab(b+L)}{6L}$$

$$\underbrace{\vartheta_A' = \frac{A_f'}{EJ} = \frac{M_A \cdot L}{3EJ} \qquad \vartheta_A = \frac{Pab(b+L)}{6EJL}}$$

$$\frac{Pab(L+b)}{6EJL} = \frac{M_A \cdot L}{3EJ} \quad \rightarrow M_A = \frac{Pab(L+b)}{L^2 \cdot 2}$$



$$\Sigma M_A = 0 \qquad M_A + BL - Pa = 0 \quad \rightarrow B = \frac{Pa}{}$$

do za M_A: $$B = \frac{1}{L}\left(Pa - \frac{Pab(L+b)}{2L^2}\right)$$

$$A = P - B = \frac{1}{L}\left(\frac{Pab(L+b)}{2L^2} - Pa\right) + P \ \ldots$$