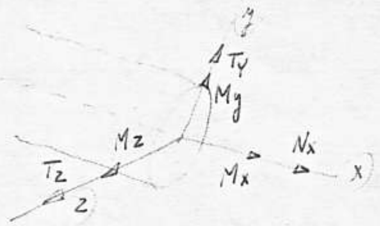
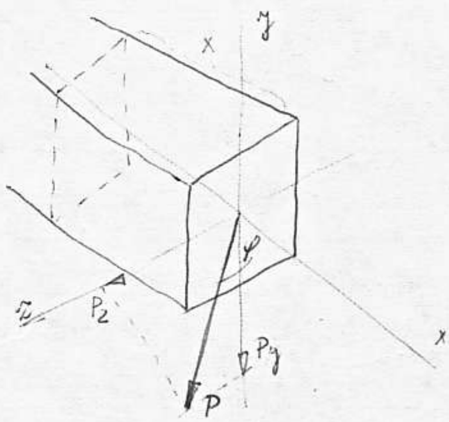
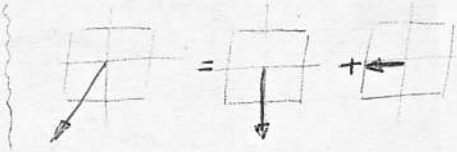
19. 11. 1969 (54)



Priestorový, (šikmý) ohyb.



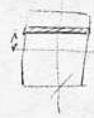
$$P \begin{cases} P_y = P\cos\varphi \\ P_z = P\sin\varphi \end{cases}$$



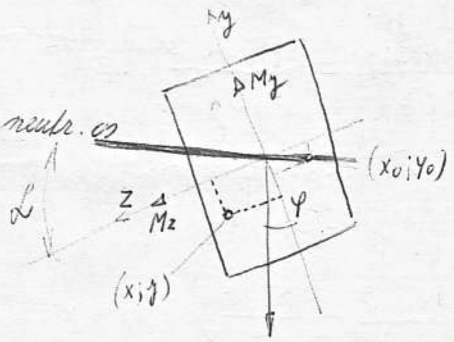
$$M_y = P_z . x = P . x . \sin\varphi = M \sin\varphi$$

$$M_z = P_y . x = P . x . \cos\varphi = M \cos\varphi$$

$$\sigma = \frac{M . y}{J_s}$$



$$\sigma = - \frac{M_z . y}{J_z} - \frac{M_y . z}{J_y}$$



$$\sigma = - M \left(\frac{y . \cos\varphi}{J_z} + \frac{z \sin\varphi}{J_y} \right)$$

Neutrálna os : $\sigma = 0$

$$\sigma = - M \left(\frac{y_0 \cos\varphi}{J_z} + \frac{z_0 \sin\varphi}{J_y} \right) = 0$$

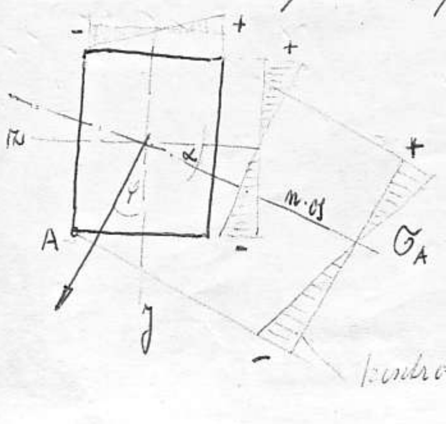
$$\frac{y_0 \cos\varphi}{J_z} + \frac{z_0 \sin\varphi}{J_y} = 0$$ ~ rovnica neutr. osy.

$$\frac{y_0}{z_0} = - \frac{\sin\varphi}{\cos\varphi} \cdot \frac{J_z}{J_y}$$

$$\operatorname{tg} \alpha = - \operatorname{tg} \varphi \cdot \frac{J_z}{J_y}$$ obecne $\operatorname{tg}\alpha \neq \operatorname{tg}\varphi$

Ak profil je ⌀ ; ◻ $\operatorname{tg}\alpha = \operatorname{tg}\varphi$ ... $J_z = J_y$

Maximálne napätie (pre dimenz.)



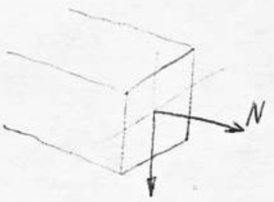
$$J_z = \frac{1}{12} b h^3 \qquad W_z = \frac{1}{6} b h^2$$

$$+ \quad J_y = \frac{1}{12} h b^3 \qquad W_y = \frac{1}{6} h b^2$$

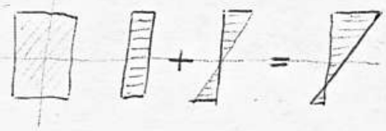
$$\sigma_A = - \left(\frac{M_z}{W_z} + \frac{M}{W_y} \right) = - \left(\frac{6 M \cos\varphi}{b h^2} + \frac{6 M \sin\varphi}{h b^2} \right)$$

kontrola rozmeru

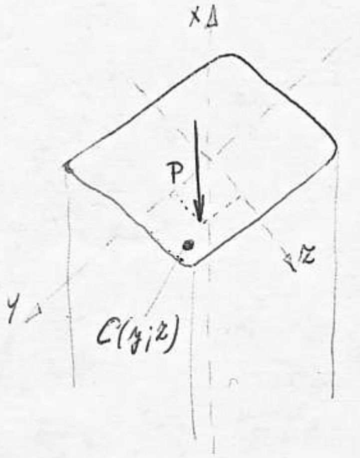
– výsledná def. = vekt. súčtu def.



$$\sigma_n = \frac{N}{F} \qquad \sigma_{oh} = \frac{M_{o}}{W_{o}}$$



Excentrický tlak (ťah)



$$\left.\begin{aligned} &P \\ &M_z = P \cdot y_P \\ &M_y = P \cdot z_P \end{aligned}\right\}$$ 3 zložky keď P posunieme do osy x

Nap. v obecnom bode C:

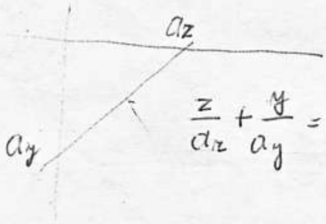
$$\sigma_C = -\frac{P}{F} - \frac{P \cdot y_P \cdot y}{J_z} - \frac{P \cdot z_P \cdot z}{J_y} = -P\left(\frac{1}{F} + \frac{y_P \cdot y}{J_z} + \frac{z_P \cdot z}{J_y}\right)$$

$$\left| \begin{aligned} J_z &= i_z^2 \cdot F \\ J_y &= i_y^2 \cdot F \end{aligned}\right.$$

$$\sigma_C = -\frac{P}{F}\left(1 + \frac{y_P \cdot y}{i_z^2} + \frac{z_P \cdot z}{i_y^2}\right)$$

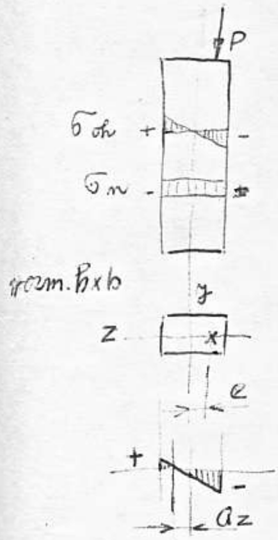
pre centr. os: $\sigma_C = 0 = 1 + \frac{y_P \cdot y}{i_z^2} + \frac{z_P \cdot z}{i_y^2}$

$$\frac{z}{\left(\frac{i_y^2}{z_P}\right)} + \frac{y}{\left(\frac{i_z^2}{y_P}\right)} = -1$$



$\frac{z}{a_z} + \frac{y}{a_y} = 1$ ~ úsekový tvar rovn. priamky

$$\left.\begin{aligned} a_z &= -\frac{i_y^2}{z_P} \\ a_y &= -\frac{i_z^2}{y_P} \end{aligned}\right\}$$ => poloha c.o. závisí od tvaru prierezu a od veľk. sily P.



$y_P = 0$
$z_P = e$
$a_y = -\infty$

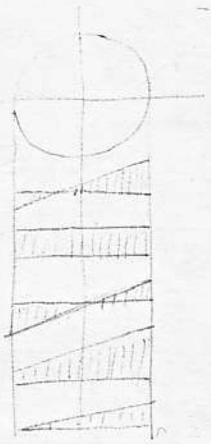
$$a_z = -\frac{i_y^2}{e}$$

$$i_z^2 = \frac{J_y}{F} = \frac{\frac{1}{12}bh^3}{bh} = \frac{h^2}{12}$$

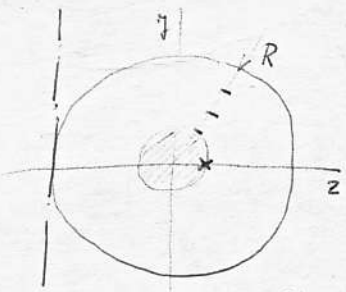
$$a_z = -\frac{h^2}{12e}$$

$$\sigma = -\frac{P}{F} - \frac{P e z}{J_y} \qquad \sigma_{max} = -\frac{Pb}{bh}\left(1+\frac{6e}{h}\right)$$

N.o. chceme dostat' mimo prierez:



$$z_P \equiv z_j = \frac{i_y^2}{a_z} \qquad y_j = -\frac{i_z^2}{a_y}$$



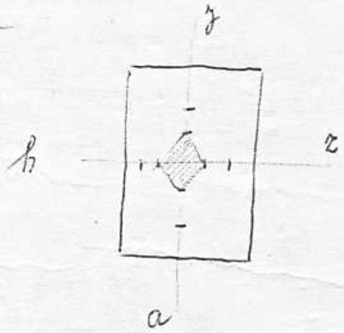
ak $a_z = R$ .... $i_y^2 = \frac{\pi R^4}{4}\cdot\frac{1}{\pi R^2} = \frac{R^2}{4}$

$$z_j = \frac{R^2}{4R} = \frac{R}{4}$$

Ak sila pôs. vo vnútr. kruhu (jadre), neutr. os je mimo kruhu (s polom R)

⇓ Nevzniká dvojaké (znamienko!) nap. v tyči.

N.o. ...



$$i_y^2 = \frac{a^2}{12}$$

$$i_z^2 = \frac{h^2}{12}$$

$$a_y = \frac{h}{2}$$

$$y_j = -\frac{h^2}{12}\cdot\frac{2}{h} = -\frac{h}{6} \qquad z_j = \frac{a}{6}$$

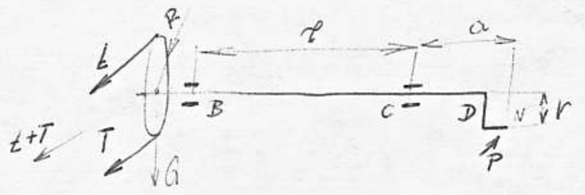
/ pri udávaní aj v tabuľkách.

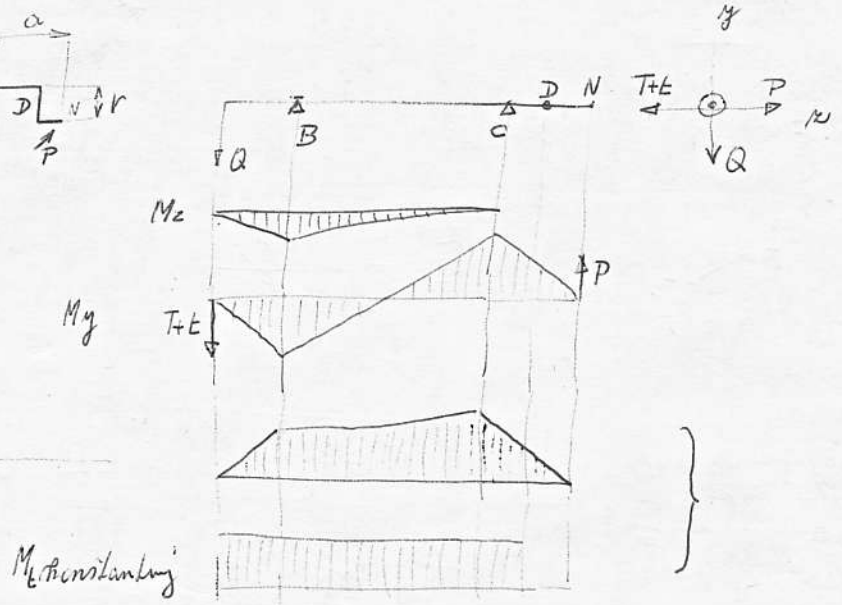
→

Ohyb a krut

výkon $N\ (k)$
otáčky $n\ (^1/_{min})$

$$M_k = 716 \frac{N}{n} \quad kpm$$

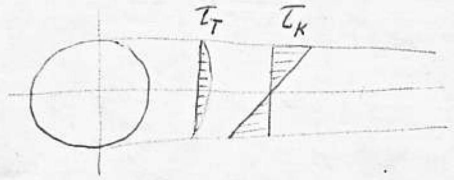




$$M_{výsl} = \sqrt{M_z^2 + M_y^2}$$

$$\sigma_o = \frac{M_o}{W} \qquad \tau_k = \frac{M_k}{W_p} \qquad \tau_T = \frac{T.S}{J.b}$$

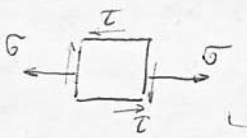
ohyb — krut — šmyk



τ_T vynecháme – [súčet: $\tau_T + \tau_k$!]

$$\underline{\underline{\sigma_{red} \leq \sigma_{dov}}}$$

I. $\sigma_1 \leq \sigma_{dov}$



$$\sigma_1 = \frac{\sigma_\alpha - \sigma_\beta}{2} \pm \frac{1}{2}\sqrt{(\sigma_\alpha - \sigma_\beta)^2 + 4\tau^2}$$

$$\sigma_1 = \frac{1}{2}\left(\sigma \pm \sqrt{\sigma^2 + 4\tau^2}\right) \leq \sigma_{dov}$$

II. Teor. najv. deformácie

$$\varepsilon_{max} \leq \varepsilon_{dov}$$

$$\sigma_{red} = \sigma_1 - \mu(\sigma_2 + \sigma_3) \qquad \sigma_{1,3} = \frac{1}{2}\left(\sigma + \sqrt{\sigma^2 + 4\tau^2}\right)$$

$$\sigma_{red} = 0{,}35\sigma + 0{,}65\sqrt{\sigma^2 + 4\tau^2}$$

III. Najväčš. šmyk. nap.

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} \qquad \sigma_{red} = \sqrt{\sigma^2 + 4\tau^2} \leq \sigma_{dov}$$

IV. Energetická t.

$$\sigma_{red} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_1\sigma_3 - \sigma_2\sigma_3} \qquad \text{ak } \sigma_2 = 0 : \sigma_{red} = \sqrt{\sigma_1^2 + \sigma_3^2 - \sigma_1\sigma_3}$$

$$\sigma_{red} = \sqrt{\sigma^2 + 3\tau^2} \leq \sigma_{dov}$$

I. – historický význam

II. – $\sigma_{zr.} = 0{,}35\sigma + 0{,}65\sqrt{\sigma^2 + 4\tau^2}$ kruh. $W = \frac{\pi d^3}{32}$ $W_p = \frac{\pi d^3}{16}$ / $W_p = 2W$

$$\sigma = \frac{M_o}{W} \qquad \tau = \frac{M_k}{W_p}$$

$$\sigma_{zr} = 0{,}35\frac{M_o}{W} + 0{,}65\sqrt{\frac{M_o^2}{W^2} + \frac{4M_k^2}{(2W)^2}} = 0{,}35\cdot\frac{M_o}{W} + 0{,}65\frac{\sqrt{M_o^2 + M_k^2}}{W}$$

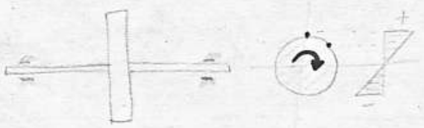
$$\sigma_{zr} = \frac{0{,}35\cdot M_o + 0{,}65\sqrt{M_o^2 + M_k^2}}{W} = \frac{M_{red.}}{W} \leq \sigma_{dov}$$

III. – $\sigma_{zr} = \sqrt{\sigma^2 + 4\tau^2}$

$$= \sqrt{\frac{M_o^2}{W^2} + \frac{4M_k^2}{4W^2}} = \frac{\sqrt{M_o^2 + M_k^2}}{W} \leq \sigma_{dov}$$

IV. (energetická) $\sigma_{zr} = \sqrt{\sigma^2 + 3\tau^2} = \sqrt{\frac{M_o^2}{W^2} + \frac{3M_k^2}{4W^2}} = \frac{\sqrt{M_o^2 + 0{,}75M_k^2}}{W} \leq \sigma_{dov}$

Premenlivé namáhanie :

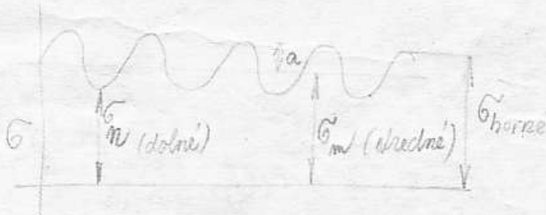


únava materiálu

striedavé – súmerné namáhanie

str. – nesúmerné nam.

~~k~~miznúce napätie

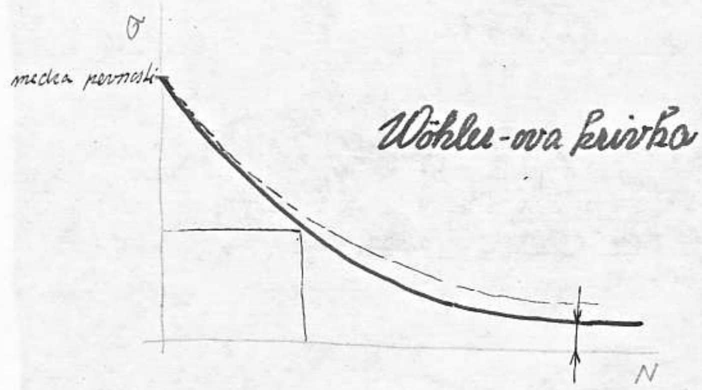


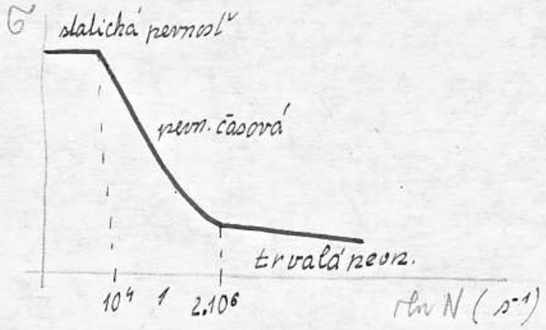
pulzujúce nap.

a – amplitúda

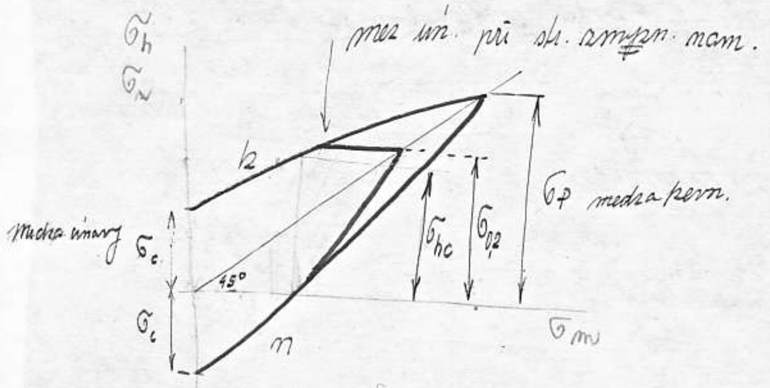
$$\sigma_m = \frac{\sigma_h + \sigma_n}{2} \qquad \sigma_a = \frac{\sigma_h - \sigma_n}{2}$$

Experimentálne určenie medze únavy.



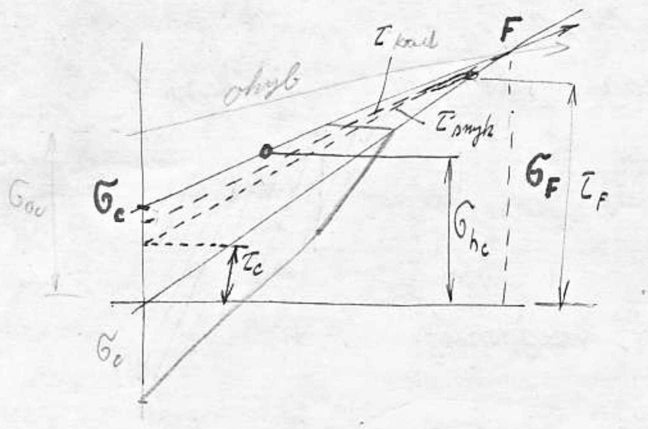


((obmedzená životnosť, ale ľahšia konšt.)



Smith-ov diagram.

Krivky sa nahradia priamkami



Medz. únavy záv. od druhu namáhania

σ_{oc} – medz. ún. v ohyb

σ_c – —"— v ťahu $= \nu_0$ ~ koef. veľkosti

τ_c – medz. ún v č. šmyku

τ_{02}

$$\frac{\tau_{02}}{\tau_c} = \nu_t$$

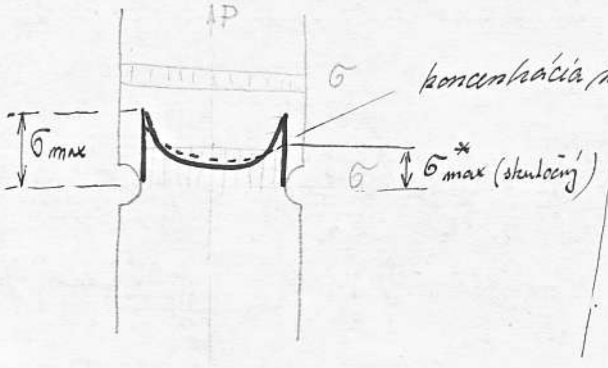
$\tau_F = 0{,}7\,\sigma_F$

$\tau_c = 0{,}7\,\sigma_c$

$\sigma_c = 0{,}32\,\sigma_P \pm 5\%$

→

Na m. ún. má vplyv tvar, vruby, ...



zväčša sa urč. experimentálne

$$\alpha = \frac{\sigma_{max}}{\sigma} > 1 \quad \text{koeficient tvaru}$$

(záv. aj od tvaru; rozm. vrubu)

σ_{max} závisí aj od materiálu.

$$\frac{\sigma^*_{max} - \sigma}{\sigma_{max} - \sigma} = \eta_c < 1 \quad \text{citlivosť}$$

Ak vyj: $\sigma_{max} = \alpha \cdot \sigma$: $\quad \sigma^*_{max} - \sigma = \eta_c (\alpha\sigma - \sigma)$

$$\sigma^*_{max} = \eta_c \alpha \sigma - \eta_c \sigma + \sigma = \sigma\left[1 + \eta_c(\alpha - 1)\right]$$

$$1 + \eta_c(\alpha - 1) = \beta \sim \text{koeficient vrubu}$$

η_c, α – sú tabuľkované. Približne: $\eta_c = \frac{\sigma_K}{\sigma_P}$

$$V_o = \frac{\sigma_{co}}{\sigma_c} \qquad V_t = \frac{\tau_{tc}}{\tau_c}$$

Mez.ún. je ovplyvnený aj akosťou povrchu.

$$\eta_p = \frac{\sigma_c'}{\sigma_c} < 1$$

[zaoblenie — zápich] ← nepatrí sem

[guličkovanie, nitridov., pochromovanie ..]

1., ťah – tlak

mez ún. v skut.: $\sigma_c^* = \sigma_c \cdot \frac{1}{\beta} \cdot \eta_p$ / β - tvar — môže byť rozdielny na súčiastke

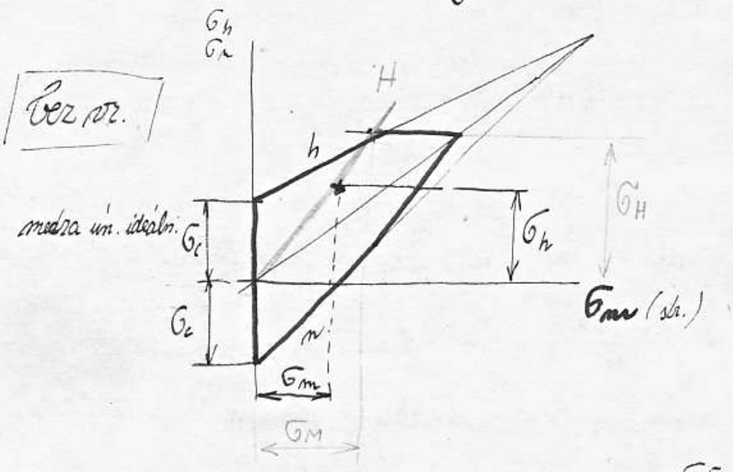
$\sigma \leqq \sigma_c^*$ + ešte pridáme bezpečnosť.

2., ohyb $\quad \sigma^*_{c,o} = \sigma_c \cdot \frac{1}{\beta_o} \eta_{p_o} \cdot V_o$

3., krut $\quad \tau^*_{c\,kor.} = \tau_c \frac{1}{\beta_t} \cdot \eta_{p_t} \cdot V_t$ / τ_c – mez ún. ideálu.

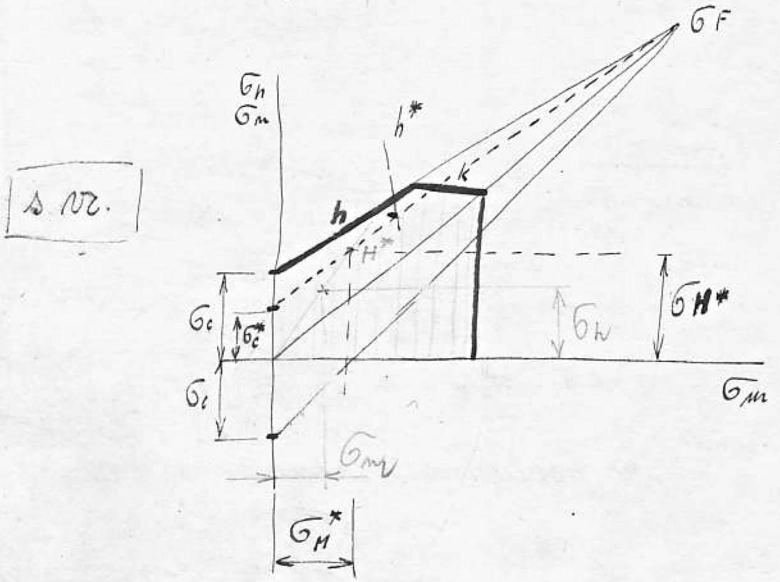
Namáhanie:

$K = \frac{\sigma_k^*}{\sigma}$ $K = 1{,}5 - 2{,}5$



keď bod je vonku, bezpečn. < 1.

$$K = \frac{\sigma_H}{\sigma_h} = \frac{\sigma_M}{\sigma_m}$$ bez vrubu

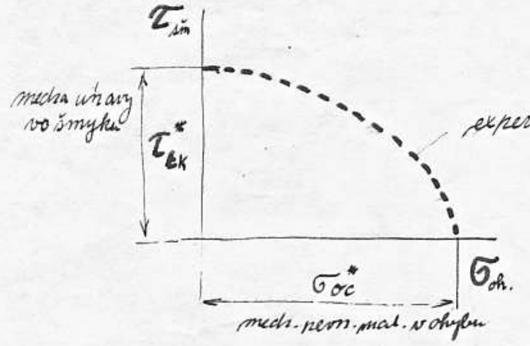


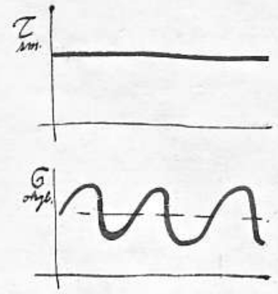
$$\sigma_c^* = \sigma_c \frac{1}{\beta} \cdot \eta_p$$

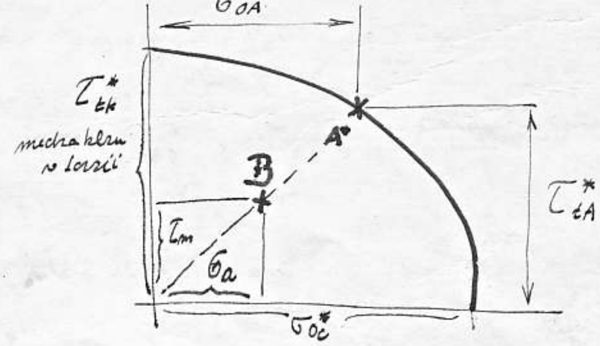
$$K = \frac{\sigma_H^*}{\sigma}$$

5. III. 1970.

Zložité namáh. vypočítame $\sigma_{ZROVN.}$







$$\left(\frac{\sigma_{oA}^*}{\sigma_{oc}^*}\right)^2 + \left(\frac{\tau_{tA}^*}{\tau_{tk}}\right)^2 = 1$$

aby súč. nedeform., bod musí byť vo vnútri ako napr. B.

Bezpečnosť: $k = \frac{\sigma_{oA}^*}{\sigma_a} = \frac{\tau_{tA}^*}{\tau_m}$

$K_{oh.} = \frac{\sigma_{oc}^*}{\sigma_a}$ — čistý ohyb.

$K_t = \frac{\tau_{tk}^*}{\tau_m}$ — čistý šmyk.

$K_\sigma \cdot \sigma_a = \sigma_{oc}^*$, $K_\tau \cdot \tau_m = \tau_{tk}^*$

$$\left(\frac{k\,\sigma_a}{\sigma_{oc}^*}\right)^2+\left(\frac{k\,\tau_m}{\tau_{tk}^*}\right)^2=1$$

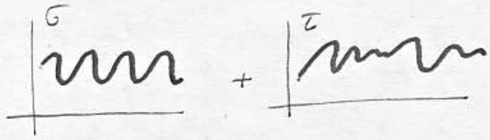
$$\left(\frac{k\,\sigma_a}{k_o\sigma_a}\right)^2+\left(\frac{k\,\tau_m}{k_t\,\tau_m}\right)^2=1$$

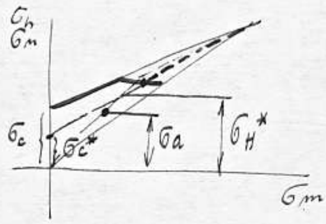
$$\frac{1}{k_o^2}+\frac{1}{k_t^2}=\frac{1}{k^2}$$

$$k=\frac{k_o \cdot k_t}{\sqrt{k_o^2+k_t^2}}$$ ~ celková bezpečn.

k_o – bezpečnosť v ohybe
k_t – v torzii
k – bezp. pri zlož. namáh.

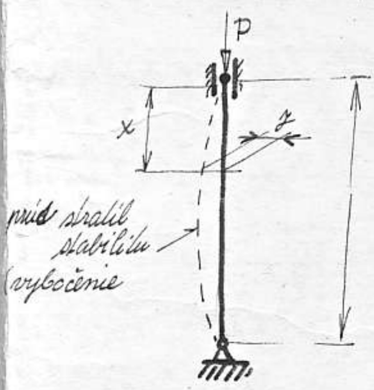
Pri predpokl., že čiara je elipsa. (súhlasí s exper. pokusmi)





$K_o = \dfrac{\sigma_H^*}{\sigma_a}$

VZPER



Kritická sila (prút ešte udrží stabilitu)

$y'' = -\dfrac{M_{(x)}}{EJ}$ / $M_{(x)} = P.y$

$y'' = -\dfrac{P\,y}{EJ}$

$y'' + \dfrac{P.y}{EJ} = 0$ / ozn. $k^2 = \dfrac{P}{EJ}$

$r^2 = -k^2$
$r = \pm ik$

$y'' + k^2 y = 0$ dif. rovn. [y - je funkc. x]

$\Downarrow$ [(pri $k^2>0$!)]

$y = A \sin kx + B \cos kx$ / $A; B$ sú integračné konštanty.

$x=0 \;\dots\; y=0$ $\longrightarrow$ $B=0$ $\dots$ $y = A \sin kx$

$x=l \;\dots\; y=0$

$\dots\dots\; 0 = A \sin kl \quad \dots\; kl = 0; \pi; 2\pi; \dots$

$kl = \pi$

$$k^2 = \frac{\pi^2}{l^2} = \frac{P}{EJ}$$

$$\boxed{P_{krit} = \frac{\pi^2 EJ}{l^2}}$$

$y = A \sin \dfrac{\pi}{l} x$

$x = \frac{l}{2}$ $\quad y' = 0$, $\quad y = y_0$ $\Big\}$ $y = y_0 \sin \frac{\pi x}{L}$

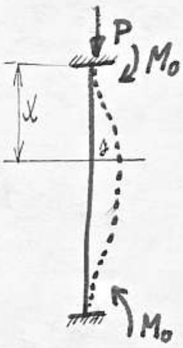
Sila P... Eulerova s.: P_E

Kritická sila závisí aj od spôsobu uchytenia tyče (väzba).

$$\sigma_{krit} = \frac{P_k}{F} = \frac{\pi^2 EJ}{l^2 F} = \frac{\pi^2 E . i^2 F}{l^2 F} = \frac{\pi^2 E i^2}{l^2}$$

$\frac{l}{i} = \lambda$ - štíhlostný pomer.

$$\sigma_k = \frac{\pi^2 E}{\lambda^2}$$



$$M_{(x)} = -M_0 + P.y$$

$$y'' = \frac{-M_{(x)}}{EJ} = \frac{M_0 - P.y}{E.J}$$

$\frac{M_0}{EJ} \Big/ \frac{P}{EJ}$ k^2

$$y'' + \frac{P}{EJ} . y - \frac{M_0}{EJ} = 0$$ ozn. $k^2 = \frac{P}{EJ}$

$$y'' + k^2 y - \frac{M_0}{EJ} = 0 \implies y = A \sin kx + B \cos kx + \frac{M_0}{P}$$

$x = 0 \ldots y = 0$ $\quad 0 = A \sin 0 + B \cos 0 + \frac{M_0}{P} \rightarrow B = -\frac{M_0}{P}$

$x = 0$ $\quad y' = 0$ $\quad y' = kA \cos kx - kB \sin kx = 0 \rightarrow A = 0$

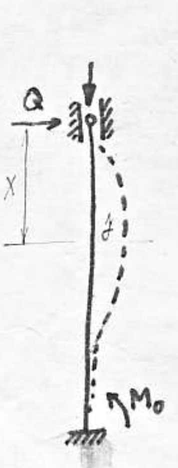
$$y = \frac{-M_0}{P} \cos kx + \frac{M_0}{P}$$

$x = l \ldots y = 0$ $\quad -\frac{M_0}{P} \cos kl + \frac{M_0}{P} = 0$ $\quad \cos kl = \frac{M_0 P}{M_0 P} = 1$

$kl = 0;\ 2\pi;\ 4\pi$

$$k^2 = \frac{4\pi^2}{l^2} = \frac{P}{EJ} \longrightarrow$$

$$P_k = \frac{4\pi^2 EJ}{l^2}$$ [vetknutie 4krát zvýšilo kritickú silu]



$$M_{(x)} = P.y - Qx$$

$$y'' = -\frac{M_{(x)}}{EJ} = -\frac{P.y + Qx}{EJ}$$ $\Big/$ $k^2 = \frac{P}{EJ}$

$$y'' + k^2 y - \frac{Qx}{EJ} = 0 \rightarrow y = A \sin kx + B \cos kx + \frac{Q.x}{P}$$

$x = 0$ $\quad y = 0$ $\quad \rightarrow B = 0$

$y = A \sin kx + \frac{Q.x}{P}$

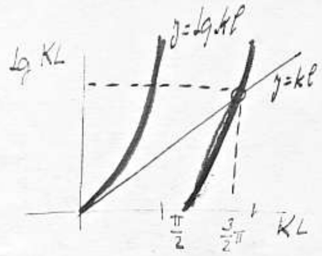
$x = l$ $\quad y = 0$ $\quad \rightarrow A \sin kl + \frac{Q.L}{P} = 0$ $\qquad A = -\frac{Q.L}{P \sin kL}$

$$Y = \frac{Q \cdot x}{P} - \frac{Q \cdot L \cdot \sin kx}{P \cdot \sin KL} \qquad x = L \quad y' = 0$$

$$\frac{Q}{P} - \frac{KQL}{P \sin KL} \cdot \cos k L = 0$$

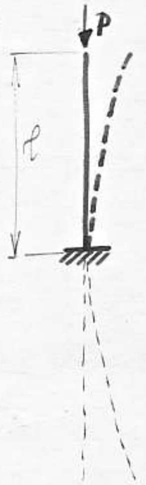
$$\frac{Q}{P} = \frac{QKL}{P} \cot g\, kl \longrightarrow tg\, KL = KL \Rightarrow kl = 4{,}493$$

$(3{,}141 + 1{,}351)$



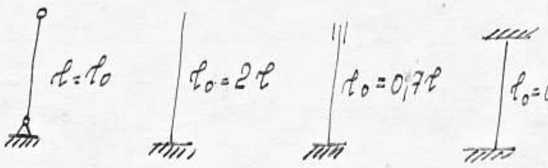
$$K^2 L^2 = 4{,}493^2 \rightarrow K^2 = \frac{20{,}16}{L^2} = \frac{P}{EJ}$$

$$P = \frac{20{,}16\, EJ}{L^2} \doteq \boxed{\frac{\pi^2 EJ}{(0{,}7\, l)^2}}$$



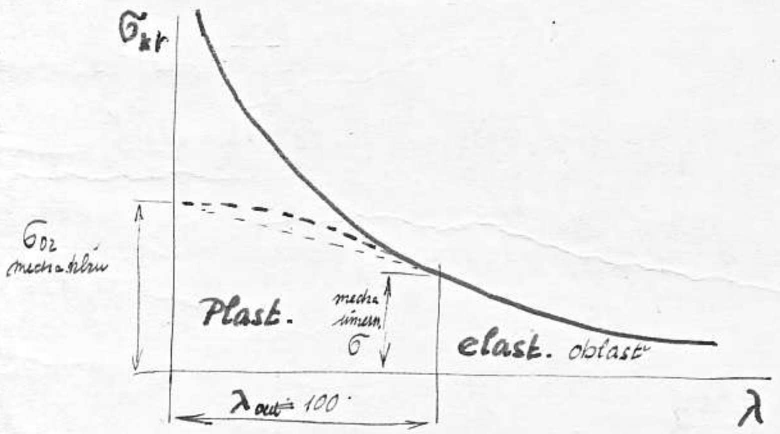
$$P_k = \frac{\pi^2 EJ}{(2l)^2} = \frac{\pi^2 EJ}{4l^2}$$

$$P_{kr} = \frac{\pi^2 EJ}{l_0^2}$$



$$\lambda_0 = \frac{l_0}{i}$$

$$\left[\sigma_k = \frac{\pi^2 E}{\lambda^2} \right]$$



vzorec platí, kým prút je namáhaný pod medz. úmernosti.

$$\sigma_{kr} \leq \sigma_{úm.}$$

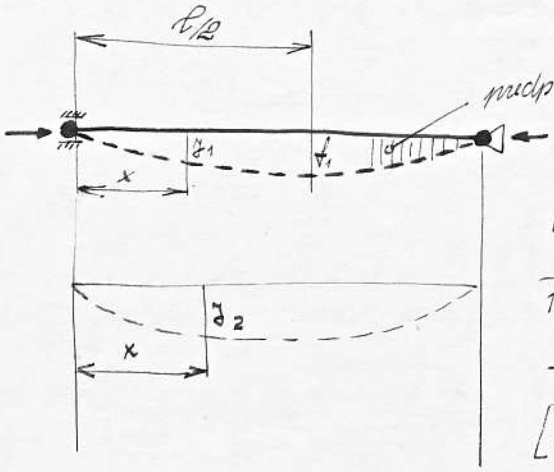
$$\lambda_0 = \pi \sqrt{\frac{E}{\sigma_ú}}$$

pre oceľ: $\lambda \doteq 100$

Pri dimenz. $\frac{P \cdot c}{F} \leq \sigma_{dov.}$ $\qquad c = f \cdot (\lambda)$

rovnica platí už pre celú oblasť grafu. (je v tabuľkách)

$$\sigma = \boxed{\frac{P \cdot c}{F}} \leq \sigma_{dov.}$$



predp. fikt. zaťaženie $\Rightarrow M_f$

ohybová č. udávajúca momentovú čiaru.

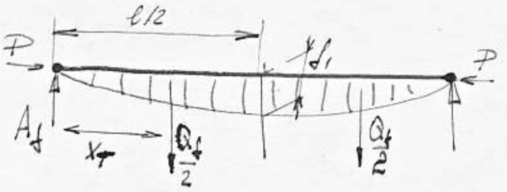
$$M_{(x)} = P \cdot y_1 = P_k \cdot y_2$$

$$P_{krit} = P \frac{y_1}{y_2}$$

[keď y_1/y_2 = konšt. správne zvol. čiary]

$$\left(\frac{y_1}{y_2}\right)_{min} < P_k < \left(\frac{y_1}{y_2}\right)_{min} \quad c = f(\lambda)$$

Pr.



zvol. oh. č.: $y_1 = f_1 \sin\frac{\pi}{L} x$

zvol $P = 1$

$$y = \frac{M_f}{E.J}$$

$$\frac{Q_f}{2} = f_1 \int_0^{L/2} \sin\frac{\pi x}{L} dx = -\frac{f_1 L}{\pi}$$

$$A_f = \frac{Q_f}{2} = \frac{f L}{\pi}$$

$$X_T = \frac{S_A}{\frac{Q_f}{2}} \quad X_T = \frac{L}{\pi}$$

$$f_2 = \frac{M_f}{EJ} = \dots$$

$$S_A = \int_0^{L/2} x.y dx = \int_0^{L/2} x_0 \sin\frac{\pi}{L} x . dx = \left[-\frac{xL}{\pi} \cos\frac{\pi x}{L}\right] - \int 1. \left[-\frac{L}{\pi} \cos\frac{\pi x}{L}\right] dx$$

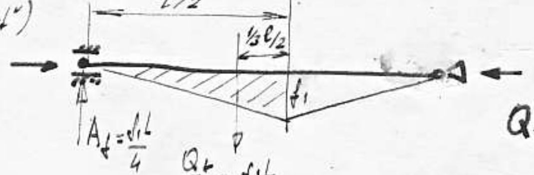
$$M_{f\,max} = X_T \cdot \frac{Q_f}{2} = \frac{L}{\pi} \cdot \frac{f_1 L}{\pi} = \frac{f_1 L^2}{\pi^2}$$

[je vlastne stat. mom!]

$$f_2 = \frac{f_1 L^2}{EJ.\pi^2}$$

$$P_k = P \frac{f_1}{f_2} = 1 \cdot \frac{f_1 EJ.\pi^2}{f_1 L^2} = \frac{\pi^2 EJ}{L^2} \quad !$$

zvol si Δ ohyb. čiaru
(taká č. nemôže byť)



$$Q_f = \frac{f_1 L}{2}$$

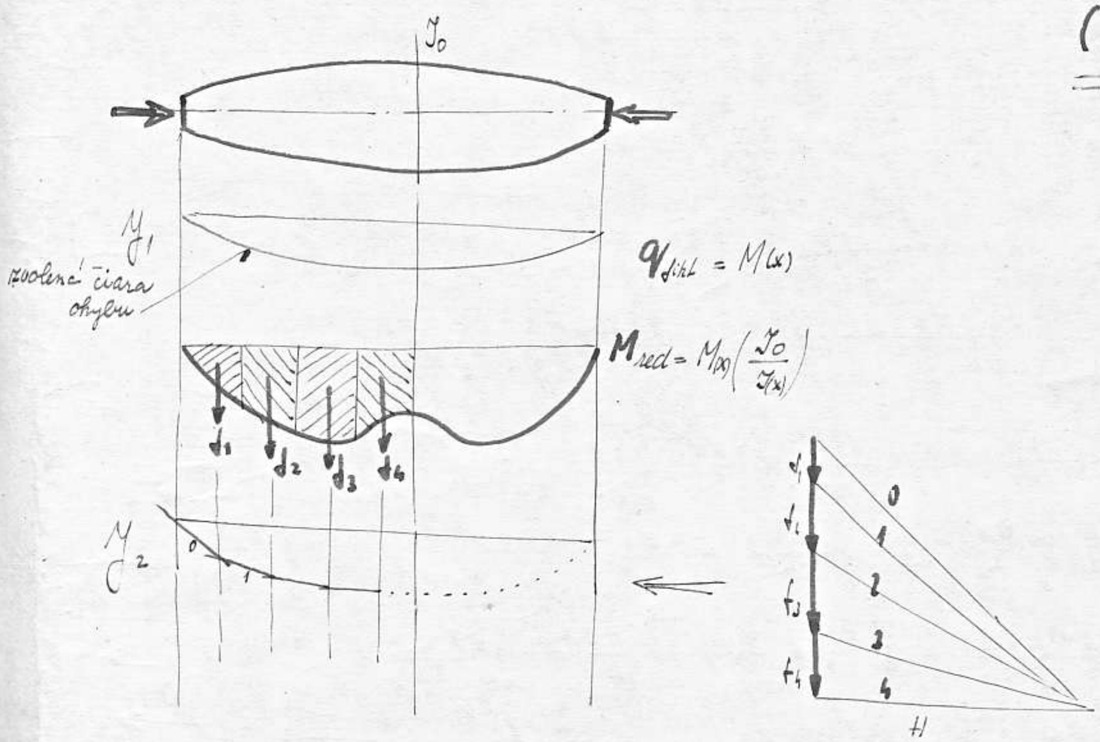
$$M_{f\,max} = A_f \cdot \frac{L}{2} - \frac{Q_f}{2} \cdot \frac{L}{6} = \frac{f_1 L^2}{24} - \frac{f_1 L^2}{24} = f_1 L^2 \left(\frac{1}{8} - \frac{1}{24}\right) = \frac{f_1 L^2}{12}$$

$$f_2 = \frac{M_{f\,max}}{EJ} = \frac{f_1 L^2}{12 EJ}$$

$$P_k = \frac{f_1}{f_2} = \frac{f_1 . 12 EJ}{f_1 L^2} = \frac{12 EJ}{L^2}$$

Grafická metóda má výhodu, keď nosník nemá konštantný priemer.

(Vianello)

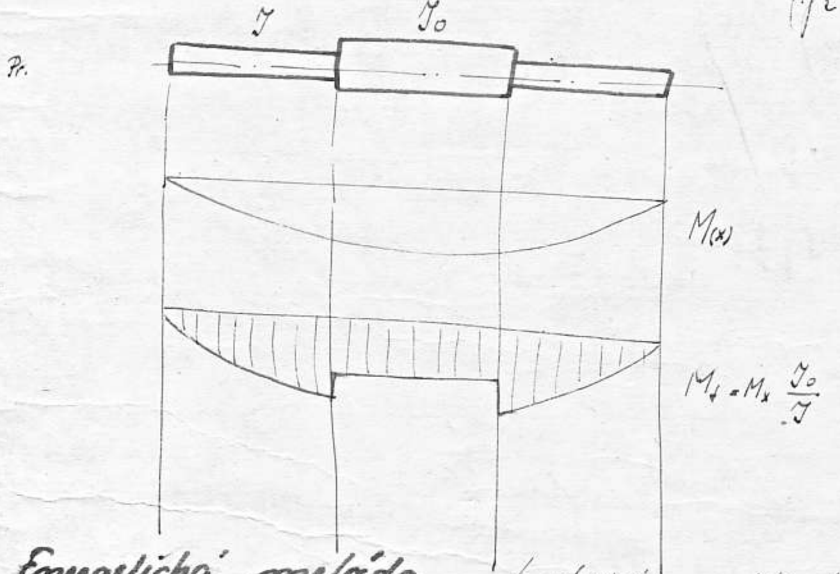


$$P_k = \frac{y_1}{y_2} \qquad y_2 = m_y \cdot \eta_2$$

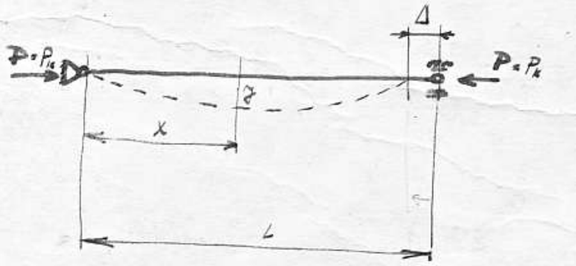
meritko: $m_y = \dfrac{m_\eta \cdot m_f \cdot H}{EJ_0}$

$$\left(y_2 = \frac{\eta \cdot H \cdot m_y \cdot m_f}{EJ_0}\right)$$

Pr.



Energetická metóda. (vhodná pre riešenie zložitých)



$A = U$ (vnútorná práca = energ.)

$$A = P_k \Delta \qquad U = \frac{1}{2}\int_0^L \frac{M_{(x)}^2}{EJ}\,dx$$

$$y'' = -\frac{M_{(x)}}{EJ} \Rightarrow M_{(x)}^2 = y''^2 (EJ)^2$$

$$U = \frac{1}{2}\int_0^L \frac{y''^2 (EJ)^2}{EJ}\,dx = \frac{1}{2}\int_0^L y''^2 EJ\,dx$$

$$\Delta = \tilde{L} - L \qquad \tilde{L} = \int_0^L \sqrt{1+y'^2}\,dx \qquad \Big|\ \sqrt{1+y'^2} \doteq 1 + \frac{1}{2}y'^2$$

$$\tilde{L} = \int_0^L dx + \frac{1}{2}\int_0^L y'^2\,dx = L + \frac{1}{2}\int_0^L y'^2\,dx$$

$$\Delta = \frac{1}{2}\int_0^L y'^2\,dx$$

$P_k = \frac{A}{\Delta}$ $\qquad P_k\cdot\Delta = U$

$$P_k\cdot\frac{1}{2}\int_0^L y'^2\,dx = \frac{1}{2}\int_0^L y''^2 EJ\,dx$$

$$\boxed{P_k = \frac{\int_0^L y''^2 EJ\,dx}{\int_0^L y'^2\,dx}}$$

Pr.

zvol: $y = f\sin\frac{\pi x}{L}$

$y' = \frac{\pi}{L} f\cos\frac{\pi x}{L} \qquad y'^2 = \frac{\pi^2 f^2}{L^2}\cos^2\frac{\pi x}{L}$

$y'' = -\frac{\pi}{L} f\cdot\frac{\pi}{L}\sin\frac{\pi x}{L} \qquad y''^2 = \frac{\pi^4}{L^4} f^2\sin^2\frac{\pi x}{L}$

$\cos^2\varphi = \frac{1+\cos 2\varphi}{2}$

$\sin^2\varphi = \frac{1-\cos 2\varphi}{2}$

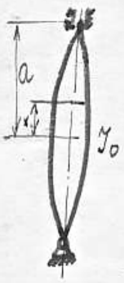
$U = \frac{1}{2}\int_0^L y''^2 EJ\,dx = \frac{1}{2}\frac{\pi^4}{L^4} f^2 EJ\int_0^L \sin^2\frac{\pi x}{L} = \dots = \frac{EJ\pi^4}{4L^3} f^2$

$\Delta = \frac{1}{2}\int_0^L y'^2\,dx = \frac{1}{2}\frac{\pi^2}{L^2} f^2\int_0^L \cos^2\frac{\pi x}{L} = \dots = \frac{\pi^2 f^2}{4L}$

$P_k = \frac{EJ\pi^4 f}{4L^3}\cdot\frac{4L}{\pi^2 f} = \frac{\pi^2 EJ}{L^2}$

$$\left[P_k = EJ\,\frac{\int y''^2\,dx}{\int y'^2\,dx}\right]$$

Pr. 8



$J_x = J_0\left(1-\frac{x^2}{a^2}\right)$ ~ zvol. zmena momentu zotrv.

$y'' = ?\,\frac{M}{EJ}$

$y' = \int_0^x \frac{M}{EJ}\,dx$

$$P_k = \frac{\int_0^a \frac{M^2}{EJ}\,dx}{\int_0^a y'^2\,dx}$$

priebeh M zvol. $M = \left(1-\frac{x^2}{a^2}\right)$

$\frac{M}{J} = \frac{a^2-x^2}{a^2}\cdot\frac{a^2}{(a^2-x^2)EJ_0} = \frac{1}{EJ_0}$ $\qquad y' = \int_0^x \frac{dx}{EJ_0} = \frac{x}{EJ_0}$

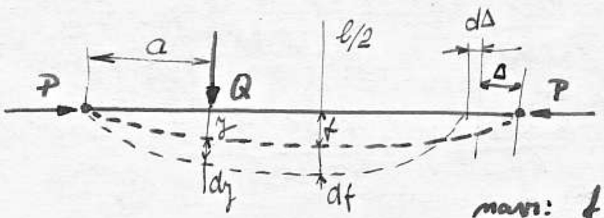
$\int y'^2\,dx = \int\frac{x^2}{E^2J_0^2}\,dx = \frac{a^3}{3E^2J_0^2}$

$\int\frac{M^2}{EJ}\,dx = \frac{1}{EJ_0}\int_0^a\left(1-\frac{x^2}{a^2}\right)dx = \frac{1}{EJ_0}\cdot\frac{2a}{3}$

$P_k = \frac{2a\cdot 3E^2J_0^2}{3EJ_0\cdot a^3} = \frac{2EJ_0}{a^2} = \frac{8EJ}{L^2}$

V praxi sa vyskytuje : excentrický tlak, nepriamy prút } =>

Vzper a ohyb



nelineárne zvýšenie ohybu !

navr.: $f \cdot \sin \frac{\pi x}{L} = y$ ohybovú krivku.

$$U = \frac{EJ\pi^4}{4L^3} f^2$$

$$\Delta = \frac{\pi^2 f^2}{4L} = \frac{\pi^2 f^2}{4L}$$

$$dU = \frac{2EJ\pi^4}{4L^3} f\,df$$

$$dy = df \cdot \sin \frac{\pi x}{L}$$

pre $x = a$: $dy = df \sin \frac{\pi a}{L}$

$$d\Delta = \frac{\pi^2 f df}{2L}$$

$$U = \frac{1}{2}\int \frac{M^2 dx}{EJ} = \frac{1}{2}\int \frac{(y'' EJ)^2}{EJ} dx$$

$$y'' = -f\frac{\pi^2}{L^2} \sin \frac{\pi x}{L}$$

$$U = \frac{1}{2} \frac{f^2 \pi^4}{L^4} EJ \int_0^L \underbrace{\sin^2 \frac{\pi x}{L}}_{\frac{1-\cos 2\frac{\pi x}{L}}{2}} dx$$

$$U = \frac{f^2\pi^4 EJ}{4L^4}\left[x + \frac{L}{2\pi} \sin \frac{2\pi x}{L}\right]_0^L$$

$$U = \frac{f^2\pi^4 EJ}{4L^4} \cdot L$$

$dA = P . d\Delta$

$$Q \cdot dy + P \cdot d\Delta = \overbrace{\frac{\pi^4 EJ}{2L^3} f\,df}^{dU}$$

$$f\frac{\pi^4 EJ}{2L^3} df = Q \cdot df \cdot \sin \frac{\pi a}{L} + P \cdot \frac{\pi^2 df}{2L} f$$

$$f\left(\frac{\pi^4 EJ}{2L^3} - \frac{P\pi^2}{2L}\right) = Q \sin \frac{\pi a}{L}$$

$$f = \frac{2QL \sin \frac{\pi a}{L}}{\pi^2\left(\frac{\pi^2 EJ}{L^2} - P\right)}$$

$$\frac{\pi^2 EJ}{L^2} = P_{kr}$$

$$f = \frac{2QL \sin \frac{\pi a}{L}}{\pi^2 (P_{kr} - P)}$$

pred. s P_k :

$$f = \frac{2QL \sin \frac{\pi a}{L} \cdot \frac{L^2}{\pi^2 EJ}}{\pi^2\left(1 - \frac{P}{P_k}\right)} = \frac{\boxed{\frac{2QL^3}{\pi^4 EJ}} \sin \frac{\pi a}{L}}{\left(1 - \frac{P}{P_k}\right)}$$

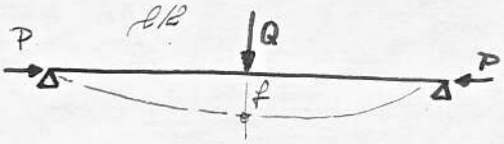
$$\frac{2QL^3}{\pi^4 EJ} \doteq \underbrace{\frac{QL^3}{48EJ}}_{} = f_0$$

priehyb nosn. bez osových síl. (na zač. látky)

ozn. $\frac{P}{P_{kr}} = \mathcal{L}$

$$f = \frac{f_0 \sin \frac{\pi a}{L}}{1 - \mathcal{L}} \qquad f = \frac{f_0}{1 - \mathcal{L}}$$

f_0 – priehyb bez osov. síl.



$$M_{max} = \frac{QL}{4} + P.f. \qquad f = \frac{f_0}{1-\alpha} \qquad \alpha = \frac{P}{P_K}$$

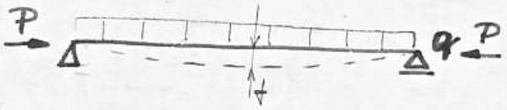
$$f = \frac{QL^3}{48EJ(1-\alpha)}$$

$$= \frac{QL}{4} + P\frac{QL^3}{48EJ} \cdot \frac{1}{1-\alpha} = \frac{QL}{4}\left(1 + \frac{PL^2}{1{,}21.\underbrace{EJ\pi^2}_{P_k}} \cdot \frac{1}{1-\alpha}\right) \qquad 48 \doteq 1{,}21.\pi^2$$

$$\frac{QL}{4}\left[1 + \frac{0{,}822.\alpha}{1-\alpha}\right] \doteq \underbrace{\frac{0{,}91}{1-\alpha} \cdot \frac{QL}{4}}$$

$$\sigma = \frac{M}{W} - \frac{P}{F} \leq \sigma_{0,2}$$

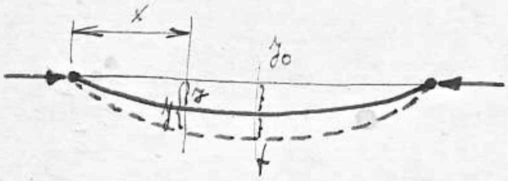
[Keď nap. je > než mez sklzu, Hookov z. neplatí]



$$f = \frac{f_0}{1-\alpha} \qquad f_0 = \frac{5}{384}\frac{qL^4}{EJ}$$

$P = 0 \ldots$ $\quad M_{max} = \frac{q\ell^2}{8}$

$P = P \ldots$ $\quad M_{max.} = \frac{q\ell^2}{8} \cdot \frac{1}{1-\alpha} + ?$



$$y_1 = y_0 \sin\frac{\pi x}{L} \quad \text{– Gezzalöz.}$$

$$y = (y_0 + f)\sin\frac{\pi x}{L}$$

zmena pot. energ.

$$U = \frac{\pi^4 EJ}{4L^3} f^2 \qquad \Delta = \frac{\pi^2}{4L}(f + y_0)^2 - \frac{\pi^2}{4L}y_0^2 = \frac{\pi^2}{4L}(f^2 + 2fy_0)$$

$$dU = \frac{\pi^4 EJ}{2L^3} f\,df$$

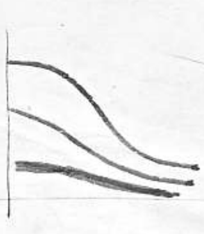
$$dU = dA$$

$$P.d\Delta = P\frac{\pi^2}{2L}(f + y_0)df$$

$$\frac{\pi^4 EJ}{2L^3} f = \frac{\pi^2 P}{2L}(f + y_0) \longrightarrow f = \frac{Py_0}{\frac{EJ\pi^2}{L^2} - P} = \frac{y \cdot \frac{P}{P_k}}{1 - \frac{P}{P_k}} = \frac{y_0\alpha}{1-\alpha}$$

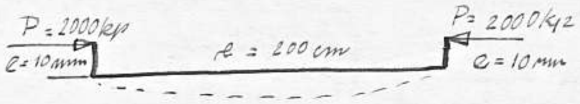
$$y_{max.} = \frac{y_0\alpha}{1-\alpha} + y_0 = \frac{y_0}{1-\alpha}$$

$$M_{max} = y_m P = \frac{Py_0}{1-\alpha} \longrightarrow \sigma = -\frac{P}{F} = -\frac{M}{W} \leq \sigma_{0,2}$$



Nap. sa zväčší ak prút je už na začiatku zohnutý!!

Def. rastie nelineárne s zaťaž.



tiahka ϕ 50/3 C 52 $\sigma_{02} = 3600$ kp/cm^2 kr.

$E = 2{,}1.10^6$ kp/cm^2

$F = 4{,}43$ cm^2

$J = 12{,}28$ cm^4

$i = 1{,}67$ cm $\qquad W = 4{,}91$

$\lambda = \frac{200}{1{,}67} = 120$ štíhl. pom.

$\alpha = \frac{P}{P_{kr}}$

$P_{kr} = \frac{\pi^2 EJ}{l^2} = \frac{\pi^2\, 2{,}1.10^6\, 12{,}28}{4.10^4}$

$P_{kr} = 6350$ kp

$$M_{max} = P.e \cdot \frac{1{,}1}{1-\frac{P}{P_{kr}}} = \frac{2000 \cdot 1 \cdot 1{,}1}{1-\frac{2000}{6350}}$$

$$M_{max} = 3220 \text{ kpcm}$$

$$\sigma_{max} = -\frac{P}{F} - \frac{M}{W} = -\frac{2000}{4{,}43} - \frac{3220}{4{,}91} = -1107 \text{ kp/cm}^2$$

$$k = \frac{\sigma_{02}}{\sigma} = \frac{3600}{1107} = 3{,}26$$

→

$P^\circ = 2 \times 2000 = 4000$ kp

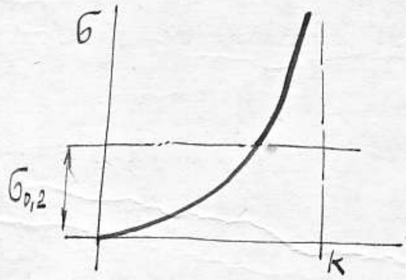
$$M_{max} = \frac{1{,}1 \cdot 4000}{1-\frac{4000}{6350}} = \ldots = 11\,900 \text{ kpcm}$$

$$\sigma_{max} = -\frac{4000}{4{,}43} - \frac{11\,900}{4{,}91} = -3585 \text{ kp/cm}^2 \;?$$

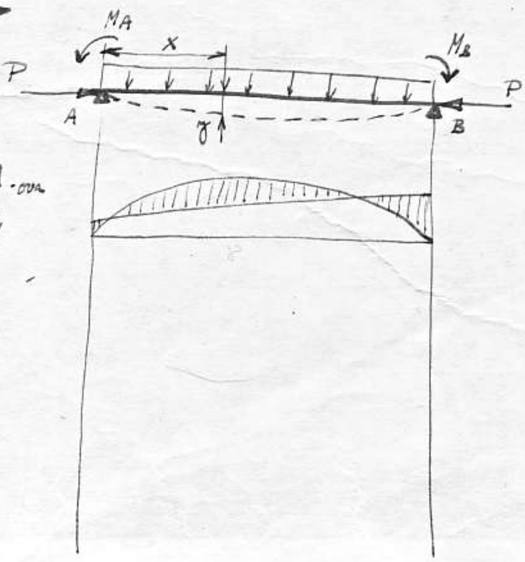
$P^\circ = 2{,}07 \times 2000 = 4140$ kp

$$M_{max} = \frac{1{,}1 \cdot 4140}{1-\frac{4140}{6350}} = \ldots = 13000 \text{ kp}$$

$$\sigma_{max} = -\frac{4140}{4{,}43} - \frac{13\,000}{4{,}91} = -3585 \text{ kp/cm}^2 \;? \qquad k = \frac{3600}{3585} \doteq 1{,}0$$



Čencev-Hoфward-ova metóda grafická



$$A = \frac{M_A - M_B + \frac{ql^2}{2}}{l}$$

moment, od zaťaž., od spojit., od P

$$M_{(x)} = -M_A - \frac{M_B - M_A}{l} \cdot x + \frac{ql}{2}x - \frac{qx^2}{2} + P.y$$

$$\frac{dM}{dx} = -\frac{M_B - M_A}{l} + \frac{ql}{2} - qx + P\frac{dy}{dx}$$

$$\frac{d^2M}{dx^2} = -q + P\frac{d^2y}{dx^2} \qquad \Big| \quad \frac{\partial^2 y}{dx^2} = -\frac{M_{(x)}}{EJ}$$

$$\frac{d^2M_{(x)}}{dx^2} = -q - \frac{P}{EJ} \cdot M_{(x)} \quad / \text{upravujeme} \quad / k^2 = \frac{P}{EJ}$$

$$\frac{d^2M_{(x)}}{dx^2} + k^2 M_{(x)} + q = 0$$ – hľadáme moment

$$M_{(x)} = A \sin kx + B \cos kx - \frac{q}{k^2}$$

→

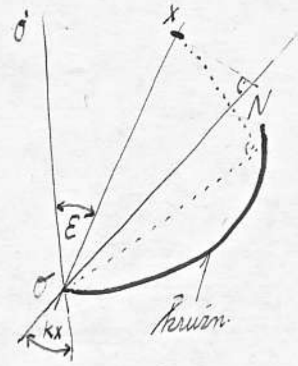
$M_{(x)} = A\sin kx + B\cos kx - \frac{q}{k^2}$ | $C = \sqrt{A^2+B^2}$ $\sin\varepsilon = \frac{A}{\sqrt{A^2+B^2}}$ $\cos\varepsilon = \frac{B}{\sqrt{A^2+B^2}}$ | 

$A = C\sin\varepsilon$ $B = C\cos\varepsilon$

$M_{(x)} = C\sin kx\cdot\sin\varepsilon + C\cos kx\cdot\cos\varepsilon - \frac{q}{k^2} = C(\sin kx\sin\varepsilon + \cos kx\cos\varepsilon) - \frac{q}{k^2}$

$M_{(x)} = C\cos(kx-\varepsilon) - \frac{q}{k^2}$ riešenie dif. rovn.

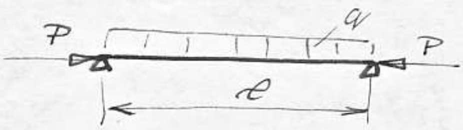
$M_{(x)} = M'_{(x)} + M''_{(x)}$ | $M'_{(x)} = C\cos(kx-\varepsilon)$ $M''_{(x)} = -\frac{q}{k^2}$ | $k = \sqrt{\frac{P}{EJ}}$



predp.: C, ε známe.

$\overline{OX} = C$

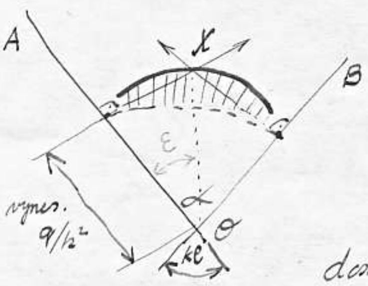
$ON = C\cos(kx-\varepsilon)$



$M = C\cos(kx-\varepsilon) - \frac{q}{k^2}$ ohybový mom.

$x = 0 \dots M_{(x)} = 0$

$x = L \dots M_{(x)} = 0$



$C\cos\varepsilon = \frac{q}{k^2}$

$C\cos(kl-\varepsilon) = \frac{q}{k^2}$

dostaneme $OX = C$ $AOX = \varepsilon$. | XO je priemer kružnice

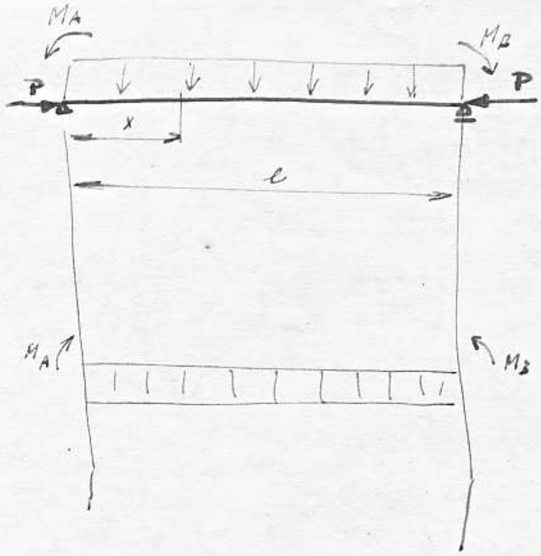
$k = \sqrt{\frac{P\pi^2 l^2}{EJ\pi^2 l^2}} = \frac{\pi}{l}\sqrt{\frac{P}{P_{kr}}} = \frac{\pi}{l}\sqrt{\alpha}$

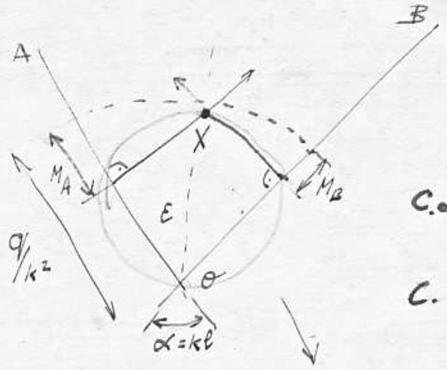
$\hat{\alpha} = kl = \pi\cdot\sqrt{\alpha}$

$\alpha^\circ = 180^\circ\sqrt{\alpha}$

1.)

2.)



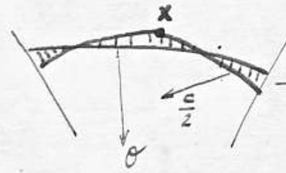


$x = 0 \quad M_{(x)} = -M_A$

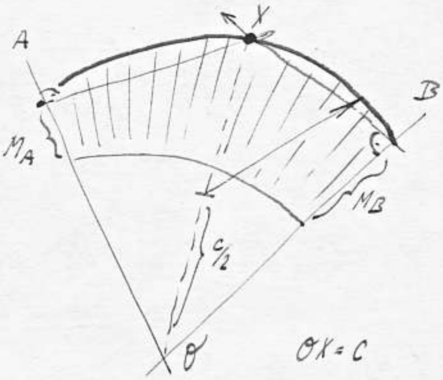
$x = l \quad M_{(x)} = -M_B$

$$C \cos \varepsilon = \frac{q}{k^2} - M_A$$

$$C \cdot \cos(kl - \varepsilon) = \frac{q}{k^2} - M_B$$



2.)



$OX = C$

$$C \cos \varepsilon = M_A + \frac{q}{k^2}$$

$$C \cos(kl - \varepsilon) = M_B + \frac{q}{k^2}$$

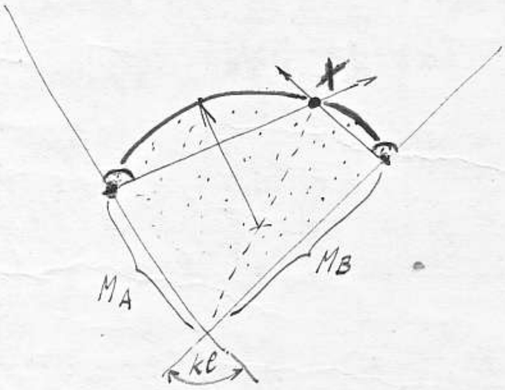
$\Longleftrightarrow$

$$M_{(x)} = C \cos(kx - \varepsilon) - 0$$

$x = 0 \quad M_{(x)} = M_A$

$x = l \quad M_{(x)} = M_B$

$M_A = C \cos \varepsilon \qquad M_B = C \cos(kl - \varepsilon)$

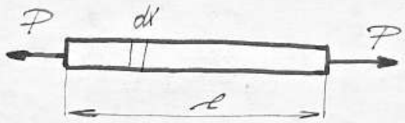


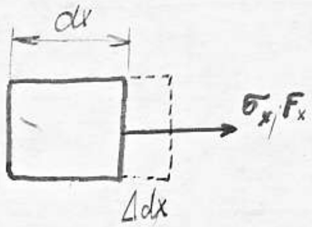
$$k = \frac{\pi}{l} \sqrt{\frac{P}{P_{kr.}}}$$

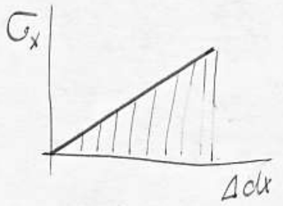
Potenciálna energia a jej súvislie

$A = U$

čistý ťah:







$$dA = \frac{1}{2}\sigma_x \cdot F_{(x)} \cdot \Delta dx$$

$$dA = \frac{1}{2}\sigma_x \cdot F_x \cdot \varepsilon \cdot dx$$

$$dA = \frac{1}{2}\sigma_x^2 F_x \cdot dx / E$$

$\frac{\Delta dx}{dx} = \varepsilon$ rel. predl.

$\varepsilon = \frac{\sigma_x}{E}$ Hookov z.

$\sigma_x = \frac{P_{(x)}}{F_{(x)}}$

$$dA = \frac{1}{2}\frac{P_x^2}{E \cdot F_x} dx \qquad A = \int \frac{1}{2} \cdot \frac{P_x^2}{E \cdot F_x} \cdot dx = U$$

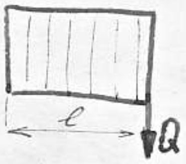
ak F = konšt.
P = konšt.

$$U = \frac{1}{2}\frac{P^2 \cdot l}{E \cdot F}$$ / EF – tuhosť v ť.

objemová en. : $u = \frac{U}{V} = \frac{1}{2}\frac{P^2}{EF^2} = \frac{1}{2}\cdot\frac{\sigma^2}{E}$ / $\sigma = \varepsilon E$

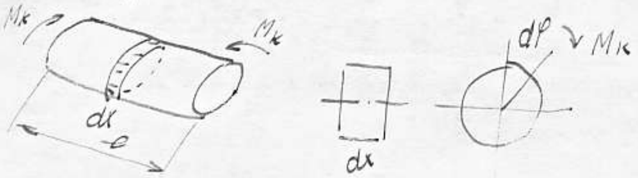
$$u = \frac{1}{2}\varepsilon \cdot \sigma$$

šmyk



$$U = \frac{1}{2}\frac{Q^2 l}{G \cdot F}$$

krut.



$$dU = dA = \frac{1}{2} M_{k(x)}\, d\varphi$$

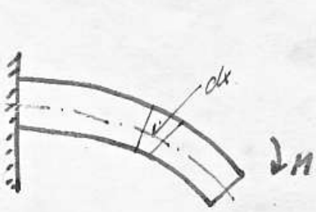
$$\vartheta = \frac{d\varphi}{dx} = \frac{M_k}{G \cdot J_p}$$

/ G modul pruž. vo šmyku
J_p – pol. moment

$$dA = \frac{1}{2}\frac{M_{kx}^2}{G \cdot J_p} dx$$

$$A = \frac{1}{2}\int_0^l \frac{M_{kx}^2}{G \cdot J_p} dx = \frac{1}{2} \cdot \frac{M_k^2 \cdot l}{G \cdot J_p} = U$$

ohyb.



$$\rho \cdot d\varphi = ds \doteq dx$$

$$d\varphi = \frac{dx}{\rho}$$

$\frac{1}{\rho} = \frac{M(x)}{EJ}$

$$dA = \frac{1}{2} M_{(x)} \cdot d\varphi$$

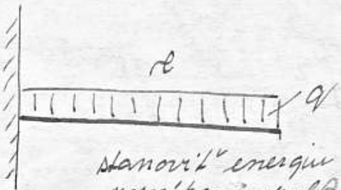
$$dA = \frac{1}{2} \cdot \cancel{d\varphi} \cdot \frac{M_{(x)}^2}{EJ} dx = \frac{1}{2}\frac{M_{(x)}^2}{EJ} dx$$

$$U = \frac{1}{2}\int_0^l \frac{M_x^2\, dx}{EJ}$$

$$U=\frac{1}{2}\int\frac{P^2}{EF}dx+\frac{1}{2}\int\frac{Q^2}{GF}dx+\frac{1}{2}\int\frac{M_x^2}{GJ_p}dx+\frac{1}{2}\int\frac{M_z^2}{EJ}dx$$ – celková p. energia

int od 0 po l

Pr

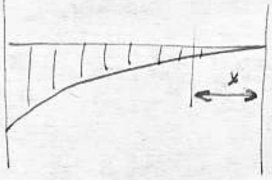


stanoviť energiu nosníka v ohybe

$$U=\frac{1}{2}\int_0^l\frac{M_z^2}{EJ}dx$$

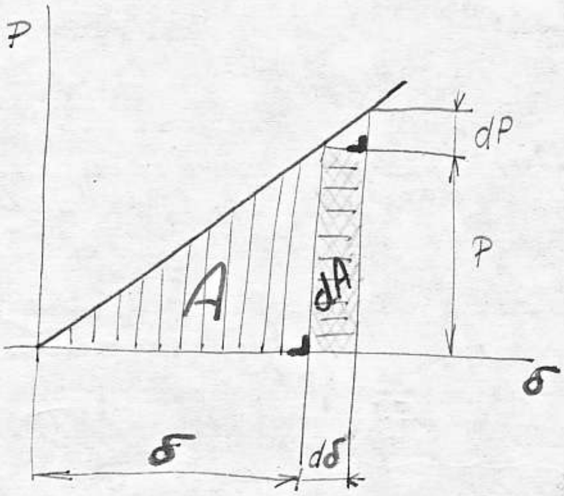
$$M_{(x)}=-q\cdot x\cdot\frac{x}{2}=-q\cdot\frac{x^2}{2}$$

„M"



$$U=\frac{1}{2}\int_0^l\frac{1}{EJ}\cdot\frac{q^2x^4}{4}dx=\frac{1}{4}\cdot\frac{1}{EJ}\cdot\frac{q^2}{4}\int_0^l x^4dx$$

$M(x)^2$



$$dA=d\delta\cdot P$$

$$\frac{dA}{d\delta}=P$$

z podobnosti Δ-ov: $\dfrac{dP}{d\delta}=\dfrac{P}{\delta}=\dfrac{\frac{dA}{d\delta}}{\delta}$

$$\delta\cdot dP=dA$$

$$\boxed{\delta=\frac{dA}{dP}}$$ I. Castigliana veta

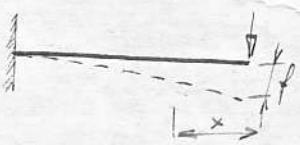
$$\frac{\partial U}{\partial P}=\delta\qquad\frac{\partial U}{\partial M}=\varphi$$

Pr:  $U=\frac{1}{2}\frac{P^2L}{EF}\rightarrow\Delta=\frac{\partial U}{\partial P}=\frac{PL}{EF}$

1970.
26. III.

$$U=f(P_1;P_2;\ldots M\ldots)$$

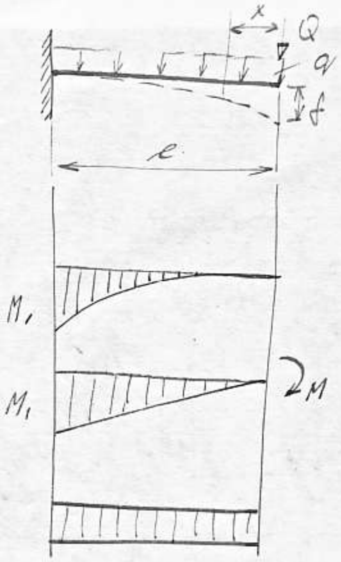
$$y=\frac{\partial U}{\partial P_1}\qquad\varphi=\frac{\partial U}{\partial M}$$ … dá sa určiť deformácia na urč. mieste. Niekedy stačí vo „význačných" miestach.



$$f=\frac{\partial U}{\partial P}\quad U=\frac{1}{2}\int\frac{M_{(x)}^2}{EJ}dx\qquad M_{(x)}=P\cdot x$$

$$U=\frac{1}{2}\int\frac{M_{(x)}^2}{EJ}dx=\frac{1}{2}\frac{P^2l^3}{3EJ}$$

$$f=\frac{\partial}{\partial P}\left(\frac{1}{2}\frac{P^2l^3}{3EJ}\right)=\frac{Pl^3}{3EJ}$$ – deformácia v mieste pôs. sily.



$$f = \frac{\partial U}{\partial Q} \qquad M_{(x)} = \frac{qx^2}{2} + Q(x)$$

$$f = \frac{\partial}{\partial Q}\,\frac{1}{2}\int \frac{M_{(x)}^2}{EJ}\,dx = \frac{1}{2}\int \frac{2M_{(x)}}{EJ}\cdot\frac{\partial M_{(x)}}{\partial Q}\,dx$$

$$\frac{\partial M_{(x)}}{\partial Q} = \frac{\partial}{\partial Q}\left(\frac{qx^2}{2} + Qx\right) = x = M_1$$ moment od jednotkovej sily klení pô. v mieste def.

$$f = \frac{2}{2EJ}\int_0^{\ell}\left(\frac{qx^2}{2} + Qx\right)x\,dx = \frac{1}{EJ}\int_0^{\ell}\frac{qx^3}{2}\,dx = \frac{q\ell^4}{8EJ}$$

$$M_{(x)} = \frac{qx^2}{2} + M_Q$$

$$\varphi = \frac{\partial U}{\partial M_Q} = \frac{\partial}{\partial M_Q}\,\frac{1}{2}\int\frac{M_x^2}{EJ}\,dx = \frac{2}{2EJ}\int M_{(x)}\,\frac{\partial M_{(x)}}{\partial M_Q}\,dx$$

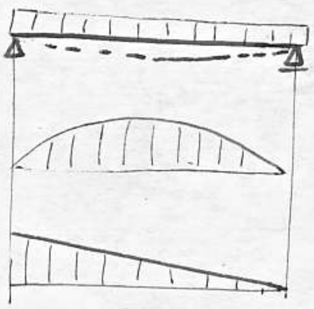
$$\frac{\partial M_{(x)}}{\partial M_Q} = 1 = M_1$$

$$\varphi = \frac{1}{EJ}\int\left(\frac{qx^2}{2} + \underbrace{M_Q}_{\emptyset}\right).1.dx = \frac{1}{EJ}\int\frac{qx^2}{2}\,dx = \frac{1}{EJ}\cdot\frac{q\ell^3}{6}$$

$$y = \int\frac{M_{0x}\cdot M_1}{EJ}\,dx$$

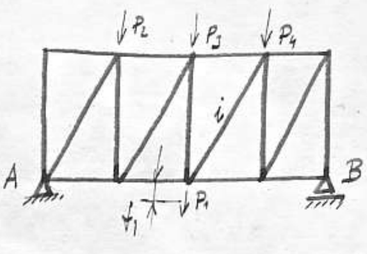
M_{0x} – skut. ohybový mom.
M_1 – mom. od jednotk. sily, zavedieme tam, kde hľadáme deformáciu.

$$\varphi = \int\frac{M_{0x}\cdot M_1}{EJ}\,dx$$



Mohr – Maxwell

Výpočet deform. – postup.



Energ. def.: $U_i = \frac{S_i^2\,\ell_i}{2F_i\,E}$ | práca celého súst. je Σ (sily od konšt.)

$$U = \sum_0^n \frac{S_i^2\,\ell_i}{2F_i\,E}$$

$$f_1 = \frac{\partial U}{\partial P_1} = \sum\frac{S_i\,\ell_i}{EF}\cdot\frac{\partial S_i}{\partial P_1}$$

$$S_i = k_{i1} P_1 + k_{i2} P_2 + \dots$$

$$\frac{\partial S_i}{\partial P_1} = k_{i1}$$ ~ osová sila, keby $P_1 = 1$ $\Rightarrow$ možno nap.: $k_{i1} = S_1$

osová sila vyvodená s silou P_1

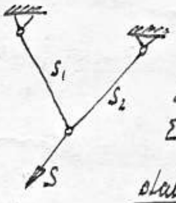
$$f_1 = \Sigma \frac{S_i \cdot S_{1i}}{EF} \cdot l_i$$

S_i – sila od vš. síl, ktoré tam pôs.
S_{1i} – od jednotk. zaťaž. v mieste kde kl. def.

$\longrightarrow$ S_i S_{1i}

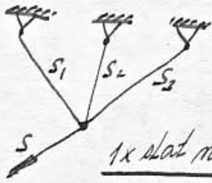
č. prútu	L_i	F	$\frac{L_i}{F_i E_i}$	S_i	S_{1i}	$S_i \cdot S_{1i}$	$\frac{L_i}{E_i F_i} S_i S_{1i}$
1							
2							
3							
					$\Sigma = f_1$		

Staticky neurčité úlohy

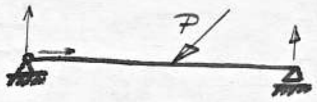


$\Sigma x = 0$
$\Sigma y = 0$

stat. urč.



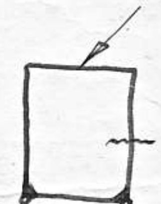
1x stat neurč



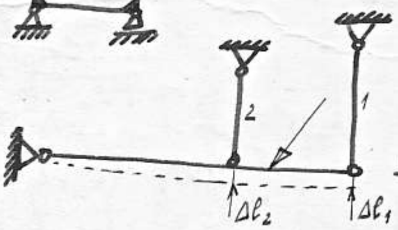
$\Sigma x = 0$
$\Sigma y = 0$
$\Sigma M = 0$

stat urč

1x stat neurč.



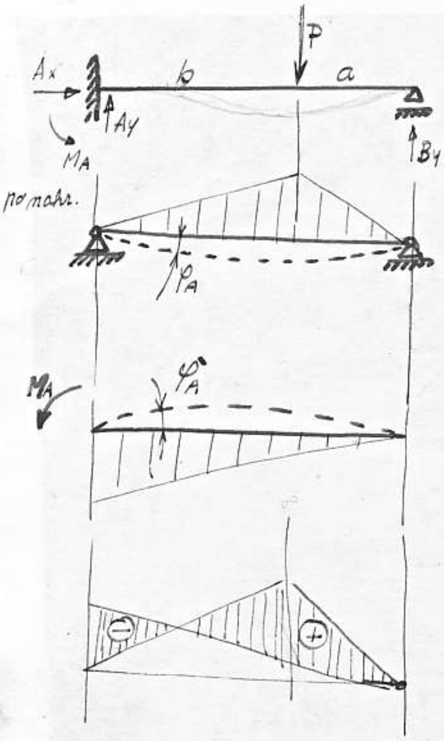
Vnútorne 3x neurč.



stat. neurč

$$\Delta l_1 = \frac{S_1 \cdot l_1}{E_1 F_1}$$

$\rightarrow$



$\Sigma Y = 0$ 1.)

$\Sigma M = 0$ 2.)

$X = 0$

} stat. rovn.

ale φ_A musí byť 0

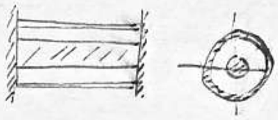
⇓

pridáme moment (ešte nezn. veľk.)

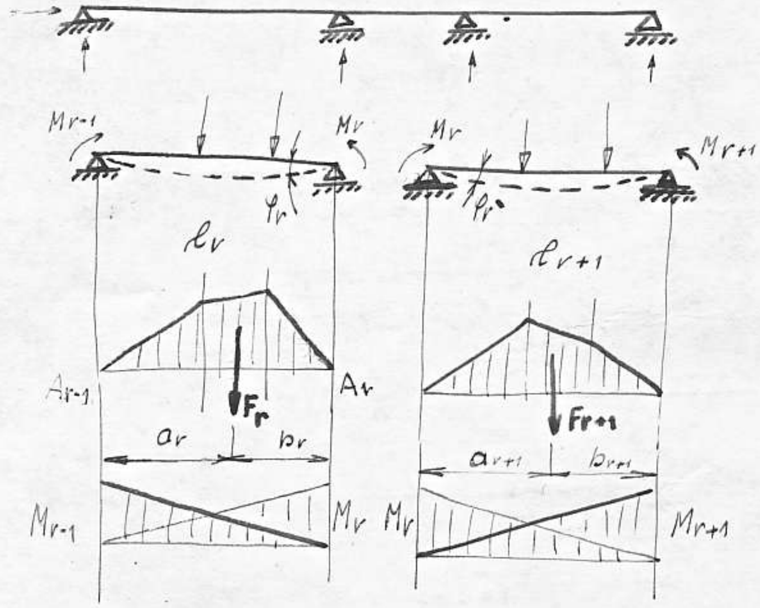
⇓

$\varphi_A = \varphi_A'$ 3.) rovnica z pružn. (deform.)

— výsledný moment.



$\varphi_1 = \varphi_2$



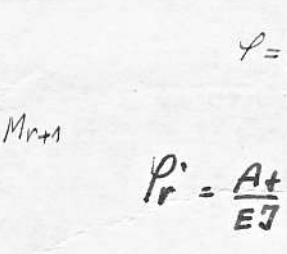
2x st. neurč. → nap. 2 def. podm.

$\varphi_r = \varphi_r'$

uhol nat. vyjadr. pom. mom. plochy

$\varphi = \frac{T_t}{EJ}$

$\varphi_r' = \frac{A_t}{EJ} = \frac{A_r . F_r}{l_r . EJ}$



Pos. sila od M_{r-1}

$\varphi_r'' = \frac{M_{r-1} . l_r}{6EJ}$

$\varphi_r''' = \frac{M_r . l_r}{3EJ}$ pos. s. („reakcia")

$\varphi_r = \frac{F_r . a_r}{l_r . EJ} + \frac{M_r . l_r}{3EJ} + \frac{M_{r-1} . l_r}{6EJ}$

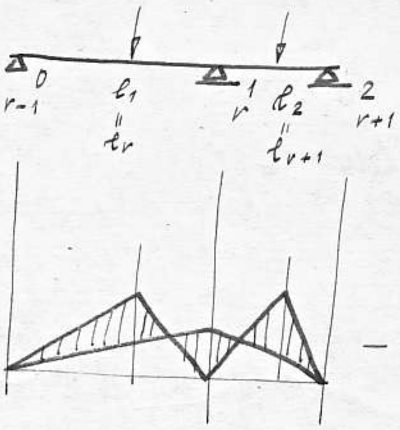
$\varphi_r' = \frac{F_{r+1} . b_{r+1}}{l_{r+1} . EJ} + \frac{M_r . l_{r+1}}{3EJ} + \frac{M_{r+1} . l_{r+1}}{6EJ}$

$$\frac{M_{r+1}\cdot l_{r+1}}{6EJ} + \frac{M_r\cdot l_{r+1}}{3EJ} + \frac{b_{r+1}\cdot F_{r+1}}{l_{r+1}\cdot EJ} + \frac{M_{r-1}\cdot l_r}{6EJ} + \frac{M_r\cdot l_r}{3EJ} + \frac{F_r\cdot a_r}{l_r\cdot EJ} = 0 \qquad /\cdot 6EJ$$

$$M_{(r-1)}\cdot l_r + 2M_r(l_r+l_{r+1}) + M_{r+1}\cdot l_{r+1} + \frac{6F_r a_r}{l_r} + \frac{6F_{r+1}\cdot b_{r+1}}{l_{r+1}} = 0$$

Clapeyron-ova rovnica

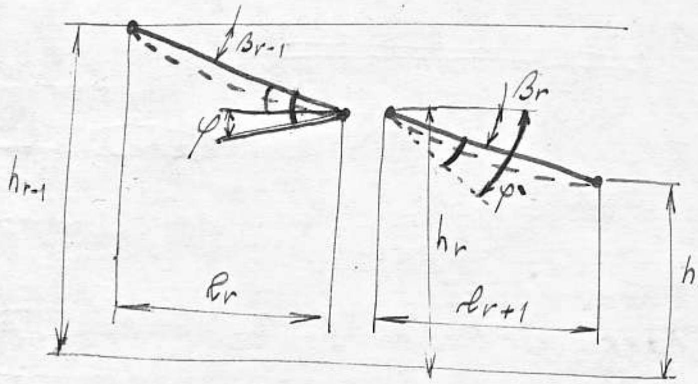
Musíme nap. toľko rovníc, koľko nezn. momentov máme.



$$\underbrace{M_0}_{0}\cdot l_1 + 2M_1(l_1+l_2) + \underbrace{M_2}_{0}\cdot l_2 + \frac{6F_1a_1}{l_1} + \frac{6F_2b_2}{l_2} = 0$$

– výsl. moment.

Nesledná výška podpor: (vznik momentu)



$\varphi + \beta_{r-1}$

$\varphi' - \beta_r$

po def. musí byť spoločn. tangenta. uhol vyjadr. pom. moment. plôch.

$$\varphi = \frac{M_r l_r}{3EJ} + \frac{M_{r-1}\cdot l_r}{6EJ} + \frac{F_r\cdot a_r}{l_r\cdot EJ} - \beta_{r-1}$$

$$\varphi' = \frac{M_{r+1}\cdot l_{r+1}}{6EJ} + \frac{M_r\cdot l_{r+1}}{3EJ} + \frac{F_{r+1}\cdot b_{r+1}}{l_{r+1}\cdot EJ} + \beta_r$$

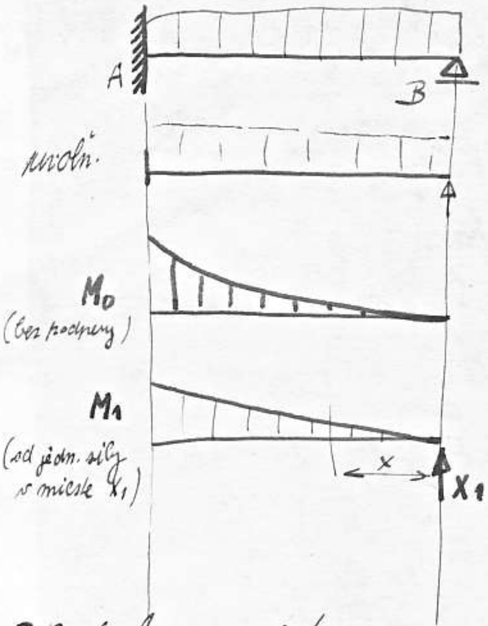
$$M_{(r-1)}\cdot l_r + 2M_r(l_r+l_{r+1}) + M_{r+1}\cdot l_{r+1} + \frac{6F_r\cdot a_r}{l_r} + \frac{6F_{r+1}\cdot b_{r+1}}{l_{r+1}} + 6EJ(\beta_r - \beta_{r-1}) = 0$$

$$\beta_r \doteq \frac{h_r - h_{r+1}}{l_{r+1}}$$

$$\beta_{r-1} \doteq \frac{h_{r-1} - h_r}{l_r}$$

$tg\alpha \doteq \alpha$ pre malé uhly

2. IV. 1970.



$\delta_B = \frac{\partial U}{\partial X_1} = 0$ – def. pri podpere.

$$U = \frac{1}{2EJ}\int_0^l (M_0+M_1)^2\,dx \qquad / \; M_1 = X_1 \cdot x$$

$$\frac{\partial U}{\partial X_1} = \frac{1}{EJ}\int (M_0+M_1)\,\frac{\partial}{\partial X_1}(M_0+M_1)\,dx$$

$/ \; \frac{\partial}{\partial X_1}(M_0 + X_1 \cdot x) = X$ ~ moment od jednotk. sily.

$$= \frac{1}{EJ}\int_0^l (M_0+M_1)\cdot X\,dx = 0$$

$$U = f(q;\, X_1)$$

2. Castiglian-ova veta.

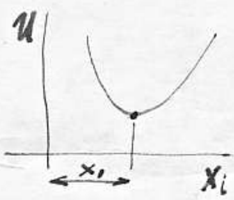
princíp minima. $\frac{\partial U}{\partial X_1} = 0$ – Keď potenc. energia je minimálna, hodnota X_1 je správna. [

$\longrightarrow$

$X_1 \quad X_2 \quad X_3 \quad \dots$

$$\left.\begin{array}{l} \frac{\partial U}{\partial X_1} = 0 \\ \frac{\partial U}{\partial X_2} = 0 \\ \dots \end{array}\right\}$$

dostaneme sústavu rovníc. Použ. princíp minima.



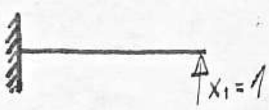
$\longrightarrow$

Nech poz.: $M_0 \quad T_0 \quad S_0 \quad M_K$

Za odstránené väzby zavedieme $X_1 = 1 \quad X_2 = 1 \dots$

v mieste x podobne

$$\left.\begin{array}{l} M_{(x)} = M_0 + X_1 M_1 + X_2 M_2 + \dots X_n M_n \\ T_{(x)} = T_0 + X_1 T_1 + X_2 T_2 + \dots \\ S_{(x)} = S_0 + S_1 X_1 + X_2 S_2 + \dots \\ M_{k(x)} = M_k + X_1 M_{k1} + \dots \end{array}\right\} \text{malé } X$$



Def. práca celého systému: $U = \frac{1}{2}\int \frac{M_x^2}{EJ}\,dx + \frac{1}{2}\int \frac{T_x^2}{GF} + \frac{1}{2}\int \frac{S_x}{EF}\,dx + \frac{1}{2}\int \frac{M_k^2}{GJ_p}\,dx$

$$\frac{\partial U}{\partial X_1} = \int \frac{M_x}{EJ}\cdot\frac{\partial M_x}{\partial X_1}\,dx + \int \frac{T}{GF}\,\frac{\partial T_x}{\partial X_1}\,dx + \int \frac{S_x}{EF}\,\frac{\partial S_x}{\partial X_1}\,dx + \int \frac{M_{kx}}{GJ_p}\cdot\frac{\partial M_{kx}}{\partial X_1} = 0$$

$\Leftarrow$ neznáme veličiny vypoč.

$$\frac{\partial U}{\partial X_2} = \dots\dots$$

$\dots\dots$

Napr.:

$$\frac{\partial M_x}{\partial X_1} = \frac{\partial}{\partial X_1}(M_0 + X_1 M_1 + \dots X_n M_n) = M_1$$

$$\frac{\partial T_x}{\partial X_1} = \frac{\partial}{\partial X_1}(T_0 + X_1 T_1 + X_2 T_2 \dots) = T_1$$

$$X_1\left[\int\frac{M_1^2}{EJ}dx+\int\frac{T_1^2}{GF}dx+\int\frac{S_1^2}{EF}dx+\int\frac{M_{k1}^2}{GJ_p}dx\right]+$$

$$+X_2\left[\int\frac{M_1\cdot M_2}{EJ}dx+\int\frac{T_1T_2}{GF}dx+\int\frac{S_1S_2}{EF}dx+\int\frac{M_{k1}M_{k2}}{GJ_p}dx\right]+$$

$$+X_3\left[\;\cdots\;\right]+\cdots+\int\frac{M_0M_1}{EJ}dx+\int\frac{T_0T_1}{GF}dx+\int\frac{S_0S_1}{EF}dx+\int\frac{M_{k0}M_{k1}}{GJ_p}dx\;.$$

moment jednotk.

$$a_{11}=\int\frac{M_1^2}{EJ}dx+\int\frac{T_1^2}{GF}dx+\int\frac{S_1^2}{EF}dx+\int\frac{M_k^2}{GJ_p}dx$$

$$a_{12}=\int\frac{M_1M_2}{EJ}dx+\int\frac{T_1T_2}{GF}dx+\int\frac{S_1S_2}{EF}dx+\int\frac{M_{k1}M_{k2}}{GJ_p}dx$$

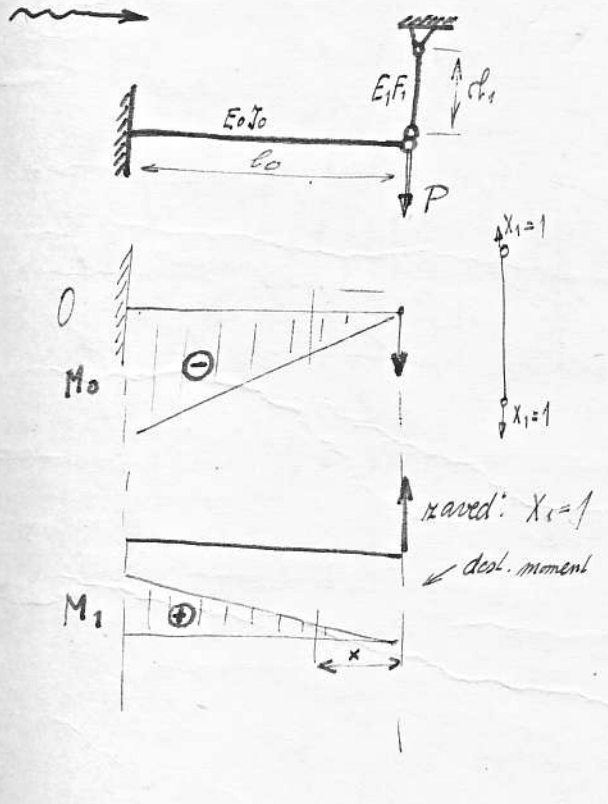
$$\cdots$$

$$a_{10}=\int\frac{M_0M_1}{EJ}dx+\int\frac{T_0T_1}{GF}dx+\int\frac{S_0S_1}{EF}dx+\int\frac{M_{k0}M_{k1}}{GJ_p}dx$$

Odvodenie kanon. rovníc.

V skrátenej forme:

$$\left.\begin{aligned}&a_{11}\cdot X_1+a_{12}X_2+\cdots+a_{1m}X_n+a_{10}=\emptyset\\&a_{21}X_1+a_{22}X_2\cdots\cdots+a_{2m}X_m+a_{20}=\emptyset\\&\vdots\\&a_{m1}X_1+a_{m2}X_2\cdots\cdots a_{mm}X_n+a_{m0}=\emptyset\end{aligned}\right\}$$ sústava rovníc.



$$a_{11}\cdot X_1+a_{10}=\emptyset$$

$$a_{11}=\int_0^{l_0}\frac{M_1^2}{E_0J_0}dx+\int_0^{l_1}\frac{S_1^2}{E_1F_1}dx$$

$M_1=X$ lebo $X_1=1$

$S_1=1$

$$=\int_0^{l_0}\frac{x^2}{E_0J_0}dx+\int_0^{l_1}\frac{dx}{E_1F_1}=\frac{l_0^3}{3E_0J_0}+\frac{l_1}{E_1F_1}$$

$$a_{10}=-\int_0^{l}\frac{M_0M_1}{E_0J_0}dx+\underbrace{\int_0^{l_1}\frac{S_1S_0}{E_1F_1}dx}_{S_0=\emptyset}$$

$M_0=P.x$

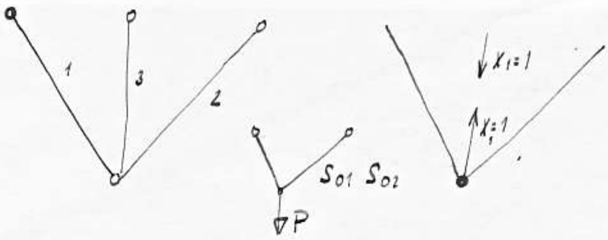
$$=-\int_0^{l}\frac{P.x.X}{E_0J_0}dx=-\frac{Pl_0^3}{3E_0J_0}$$

$$X_1=-\frac{a_{10}}{a_{11}}=\frac{\quad}{\quad}=+\frac{\quad}{\quad}=\cdots$$

$$M_x=M_0-X_1M_1=\underline{\underline{P.x-X_1x}}$$

$$S=S_0+X_1S_1=\underline{X_1}$$

Časť sily P prenáša nosník „l_0“, časť „l_1“.



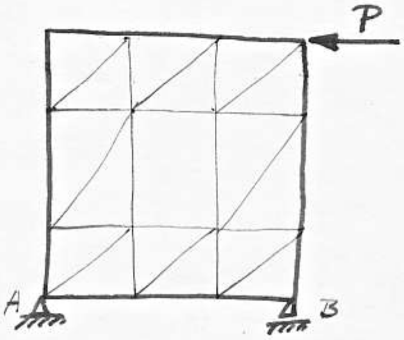
S_{11} S_{12} S_{13}

[v 3. prútu — ozn.]

$$a_{11} = \frac{S_{11}^2}{E_1F_1}\cdot l_1 + \frac{S_{12}^2\cdot l_2}{E_2F_2} + \frac{S_{13}^2}{E_3F_3} l_3$$

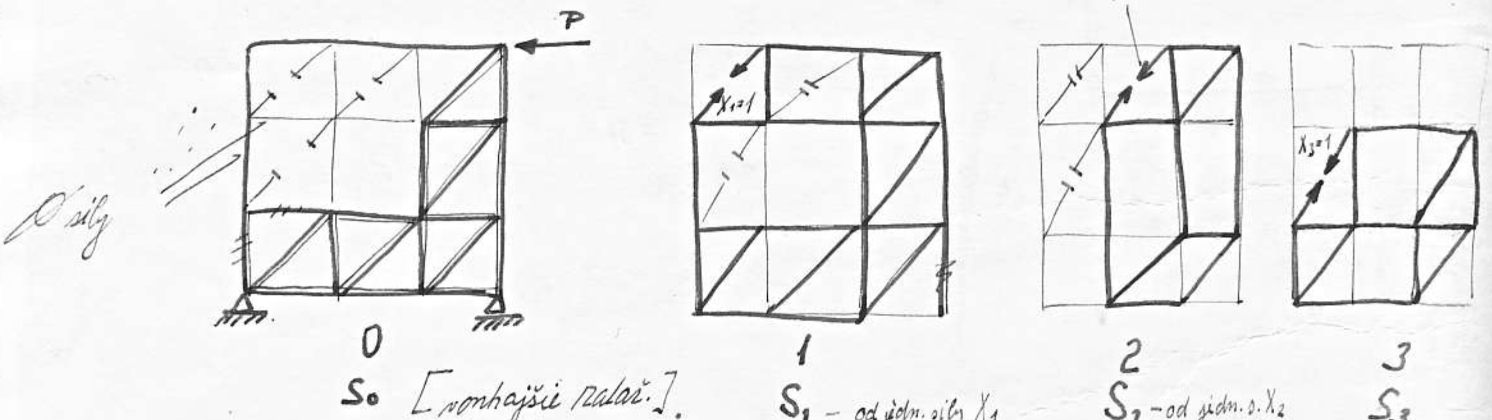
$$a_{10} = \frac{S_{11}S_{01}}{E_1F_1} l_1 + \frac{S_{12}\cdot S_{02}}{E_2F_2}\cdot l_2 \qquad a_{11}X_1 + a_{10} = \emptyset$$

Riešenie prútového systému . . .



3x stat. neurč.

Odstránime „st. neurčitosť":



0 — S_0 [vonkajšie zaťaž.].

1 — S_1 - od jedn. sily X_1

2 — S_2 - od jedn. s. X_2

3 — S_3

$$S = S_0 + X_1S_1 + X_2S_2 + X_3S_3$$

$$a_{11}X_1 + a_{12}X_2 + a_{13}X_3 + a_{10} = \emptyset$$
$$a_{21}X_1 + a_{22}X_2 + a_{23}X_3 + a_{20} = \emptyset$$
$$a_{31}X_1 + a_{32}X_2 + a_{33}X_3 + a_{30} = \emptyset$$

$$a_{ik} = \int \frac{S_iS_k}{EF} ds$$ pre konšt. prierez,... $$a_{ik} = \sum \frac{S_{ik}}{EF}\cdot l$$

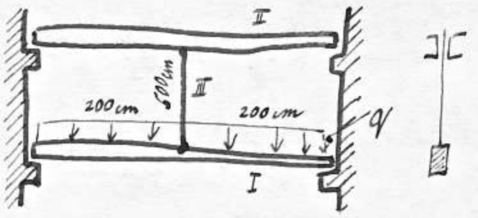
$$a_{k0} = \sum \frac{S_0\cdot S_k}{EF}\cdot l$$

} dos.

[Rozmerove nemusí vyjsť.]

9.IV. 1970.

Postup riešenia systému :

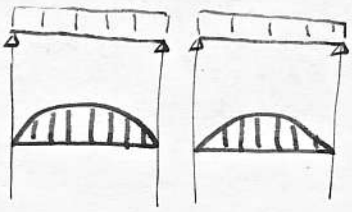


1× stat. neurč.

1.) $a \times a$ — $\sigma_{dov} = 100\,kp/cm^2$ $E = 10^5\,kp/cm^2$

2.)][— $\sigma_{dov} = 1500\,kp/cm^2$ $E = 2.10^6\,kp/cm^2$

3.) ○ — $\sigma_{dov} = 1500\,kp/cm^2$ $E = 2.10^6\,kp/cm^2$



$q = 10\,kp/cm$

Chceme dimenzovať (nosnosť, nosníka = ? " = ?)

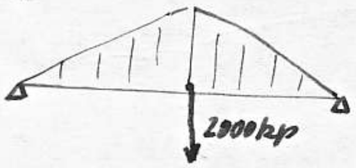
$$M_{max} = \frac{q l^2}{8} = \frac{10 \cdot 200^2}{8} = 50\,000\,kpcm$$

$$\sigma_{dov} = \frac{M}{W} \quad / \quad W = \frac{M}{\sigma_{dov}} = \frac{50\,000}{100} = 500\,cm^3$$

$$W = \frac{1}{6} a^3 = 500 \quad \rightarrow a = \sqrt[3]{6.500} = 14{,}4$$

zvol. $a = 15\,cm$

Zať. horný nosn :



$$M_{max} = \frac{PL}{4} = \frac{2000.400}{4} = 200\,000\,kpcm$$

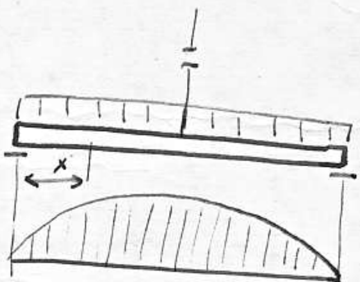
$$W = \frac{M}{\sigma_{dov}} = \frac{200\,000}{1500} = 134\,cm^3$$

/ hľadáme profil:

$I \times 2$ č 12 — z tab. $W = 60{,}7$ $2W = 121{,}4\,cm^3$

$$F = \frac{P}{\sigma_{dov}} = \frac{2000}{1500} = 1{,}33\,cm^2 \rightarrow d = 1{,}3\,cm$$

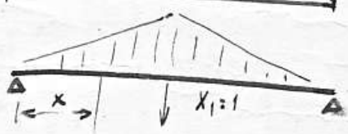
odstránime III :



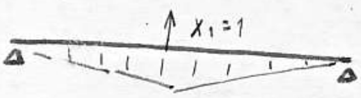
Mo

$$M_0 = \frac{ql}{2}x - \frac{qx^2}{2}$$

zaveď. jednotk. silu :



M₁



$M_1 = 0{,}5.x$ pre II $\quad M_1 = -0{,}5x$ pre I

$S_1 = 1$

→

$$a_{11} X_1 + a_{10} = 0$$

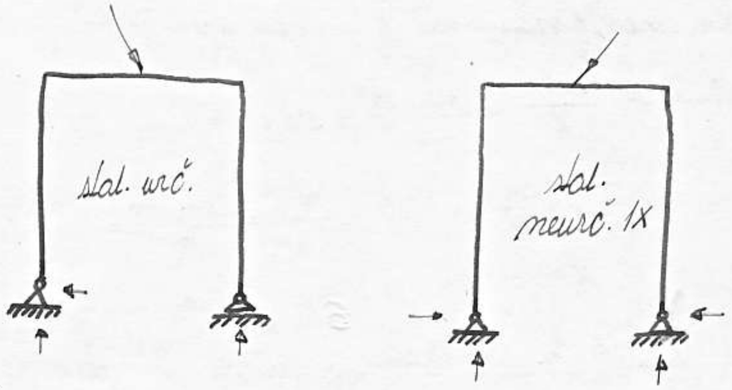
$$a_{11} = \int_0^l \frac{M_1^2 dx}{E_I J_I} + \int_0^l \frac{M_1^2 \cdot dx}{E_{II} J_{II}} + \int_0^h \frac{S_1^2 dx}{E_{III} \cdot F_{III}}$$

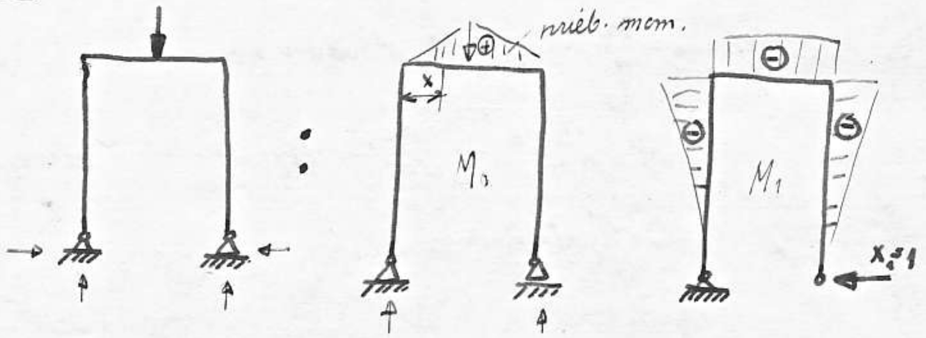
$$a_{10} = \int_0^l \frac{M_0 M_1}{E_I \cdot J_I} dx + \underbrace{\int_0^l \frac{M_0 M_1}{E_{II} J_{II}} dx}_{\emptyset} + \underbrace{\int_0^h \frac{S_1 S_0 \, dx}{E_{III} F_{III}}}$$

⋮

Ak vychádza nap. musíme zmeniť rozmery ⇒ znova opak. postup.

Táto úloha sa dá riešiť aj pom. deformácie





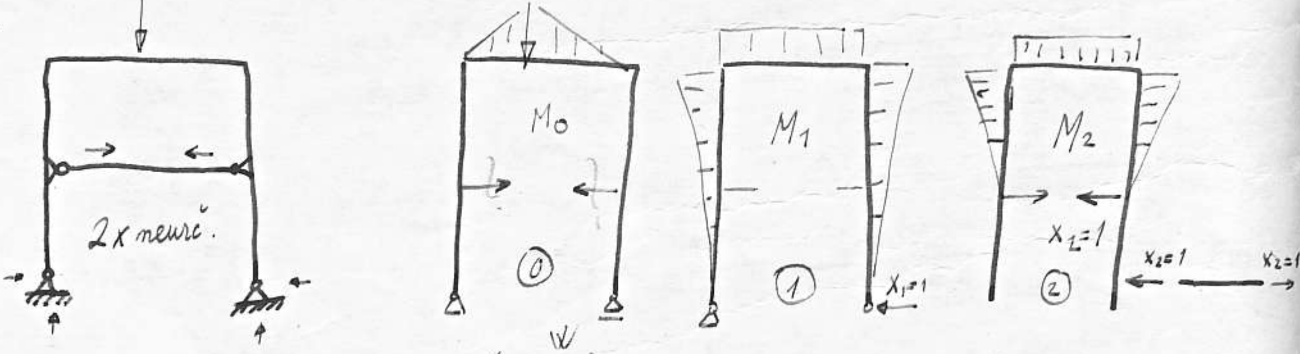
(násbu nahr. jednotk. silou)

$$a_{11} X_1 + a_{01} = 0$$

$$a_{11} = \int \frac{M_1^2 dx}{EJ}$$ po cel. rámu

$$a_{10} = \int \frac{M_1 M_0}{EJ} dx$$ po c. r.

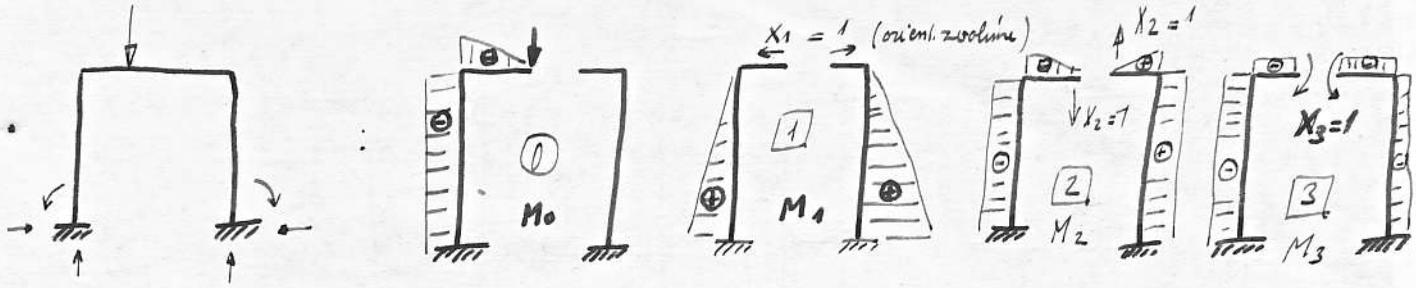
Ak rám má konšt. priemer j rovn. predel. s EJ.



$$a_{11} X_1 + a_{12} X_2 + a_{10} = \emptyset$$
$$a_{21} X_1 + a_{22} X_2 + a_{20} = \emptyset$$

$$a_{21} = a_{12}$$

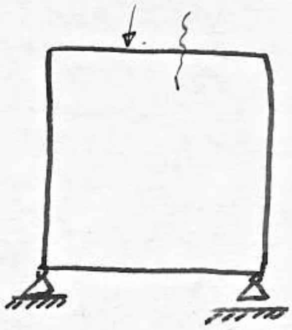
$$M = M_0 + X_1 M_1 + X_2 M_2$$



3X st. neurč.

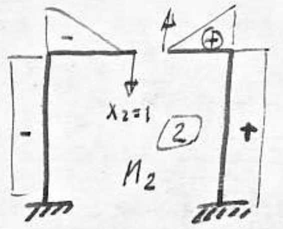
$$a_{11}X_1 + a_{12}X_2 + a_{13}X_3 + a_{10} = 0$$
$$a_{21}X_1 + a_{22}X_2 + a_{23}X_3 + a_{20} = 0$$
$$a_{31}X_1 + a_{32}X_2 + a_{33}X_3 + a_{30} = 0$$

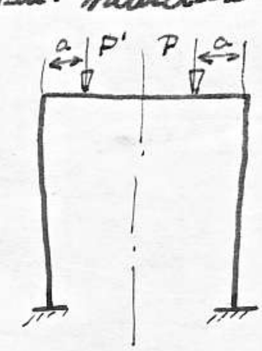
za $\underline{a}$ dosadíme



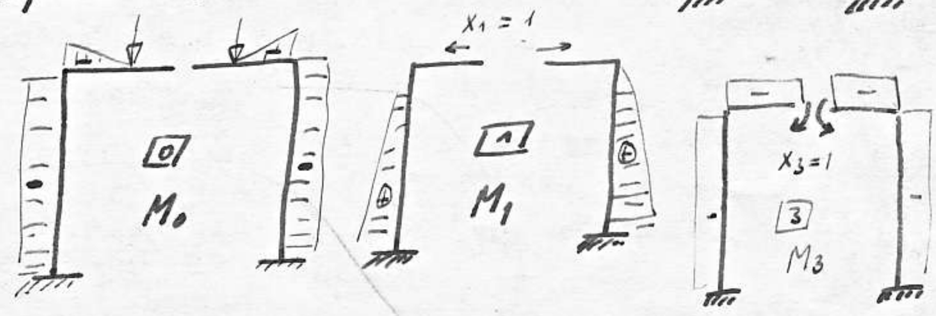
mnohonásobne neurč.!

Stat. neurčitosť niekedy môže byť znížená:





symetrický rám.



$$a_{12} = \int \frac{M_1 M_2}{EJ}\,dx = 0$$

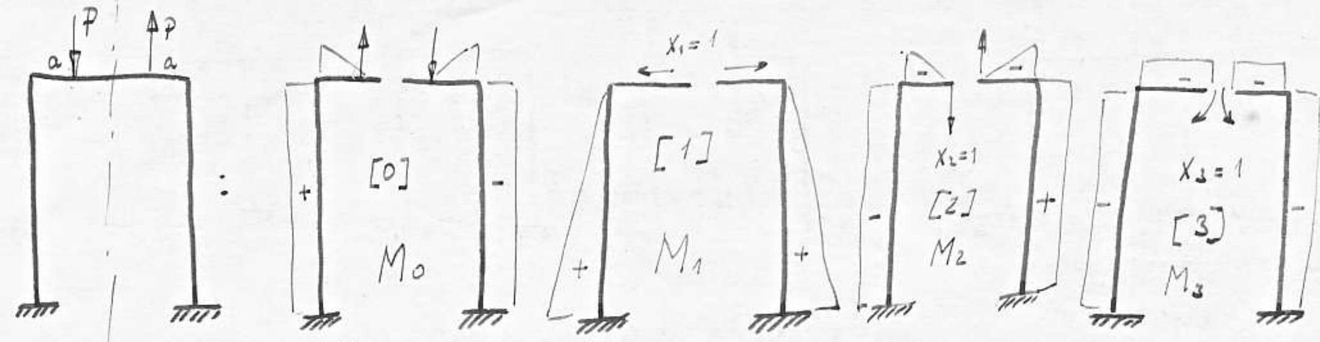
[súčet súčinov: M_1M_2 + na druhej str. −]

$$a_{21} = 0$$
$$a_{23} = \int \frac{M_2 M_3}{EJ}\,dx = 0$$
$$a_{32} = 0$$
$$a_{20} = \int \frac{M_0 M_2}{EJ}\,dx = 0$$

$$a_{11}X_1 + \cancel{a_{12}X_2} + a_{13}X_3 + a_{10} = 0$$
$$\cancel{a_{21}X_1} + a_{22}X_2 + \cancel{a_{23}X_3} + \cancel{a_{20}} = 0$$
$$a_{31}X_1 + \cancel{a_{32}X_2} + a_{33}X_3 + a_{30} = 0$$

$$a_{11}X_1 + a_{13}X_3 + a_{10} = 0$$
$$X_2\,a_{22} = 0 \rightarrow M_2 = 0$$
$$a_{31}X_1 + a_{33}X_3 + a_{30} = 0$$

zníž. stupeň!



$$a_{11} = \int\limits_{\text{ľavá strana}}^{L} \frac{M_1^2}{EJ}dx + \int\limits_{\text{pravá str.}} \frac{M_1^2}{EJ}dx \neq \varnothing$$

$$a_{12} = -\int\limits_{ľ} \frac{M_1M_2}{EJ}dx + \int\limits_{pr.} \frac{M_1M_2}{EJ}dx = a_{21} = \varnothing$$

$$a_{10} = +\int\limits_{ľav.\,str.} \frac{M_0M_1}{EJ}dx - \int\limits_{pr.\,str.} \frac{M_0M_1}{EJ}dx = \varnothing$$

$$a_{13} = -\int^{ľ} \frac{M_1M_3}{EJ}dx - \int^{pr.} \frac{M_1M_3}{EJ}dx \neq 0$$

$$a_{22} = +\int\limits_{ľ} \frac{M_2^2\,dx}{EJ} + \int\limits_{pr} \frac{M_2^2\,dx}{EJ} \neq 0$$

$$a_{23} = -\int \frac{M_2M_3}{EJ}dx - \int \frac{M_2M_3}{EJ}dx = 0$$

$$a_{33} = \int \frac{M_3^2\,dx}{EJ} \cdots \neq 0$$

$$a_{20} = -\int^{ľ} \frac{M_0M_2}{EJ}dx - \int^{pr.} \frac{M_0M_2}{EJ}dx \neq 0$$

$$a_{30} = \dots = \varnothing$$

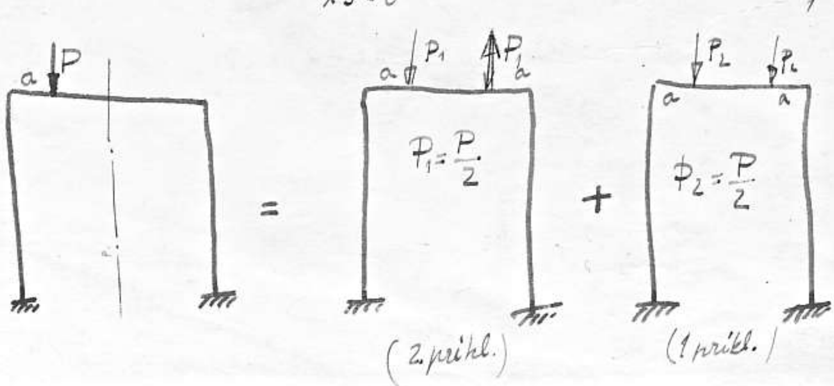
$a_{11} \neq 0$
$a_{12} = 0$
$a_{21} = 0$
$a_{10} = 0$
$a_{13} \neq 0$
$a_{31} \neq 0$
$a_{22} \neq 0$
$a_{23} = 0$
$a_{32} = 0$
$a_{33} \neq 0$
$a_{20} \neq 0$
$a_{30} = 0$

$$a_{11}X_1 + a_{12}X_2 + a_{13}X_3 + a_{10} = 0$$
$$a_{21}X_1 + a_{22}X_2 + a_{23}X_3 + a_{20} = 0$$
$$a_{31}X_1 + a_{32}X_2 + a_{33}X_3 + a_{30} = 0$$

po dos. zostane: $a_{22}X_2 + a_{20} = 0$

$x_1 = 0$
$x_3 = 0$

príkl. preš. na 1 stat. neurč.

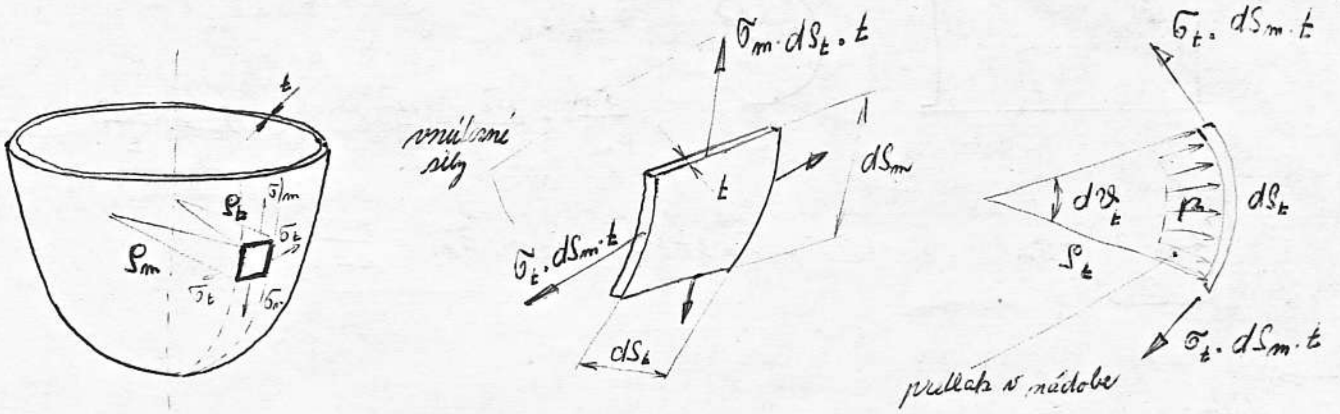


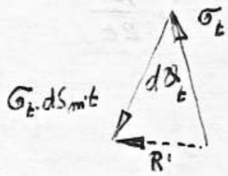
... superpoz. výsledkov

(2. príkl.) (1 príkl.)

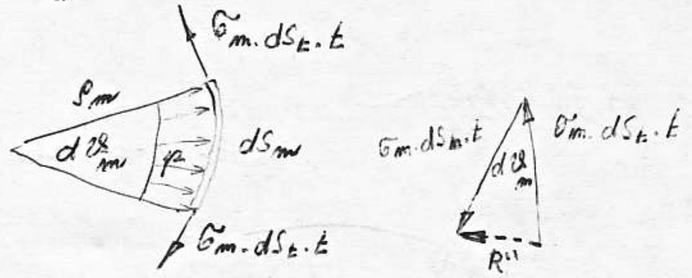
Tenkostenné nádoby.

Teleso vytv. rotáciou nejakej krivky





$$R' = \sigma_t \cdot dS_m \cdot t \cdot d\vartheta_t$$



$$R'' = \sigma_m \cdot dS_t \cdot t \cdot d\vartheta_m$$

$$\boxed{p \cdot dS_m \cdot dS_t = R' + R'' = \sigma_t\, dS_m \cdot t \cdot d\vartheta_t + \sigma_m \cdot dS_t \cdot t \cdot d\vartheta_m}$$

$$\rho_t \cdot d\vartheta_t = dS_t \quad \Downarrow \quad d\vartheta_t = \frac{dS_t}{\rho_t}$$

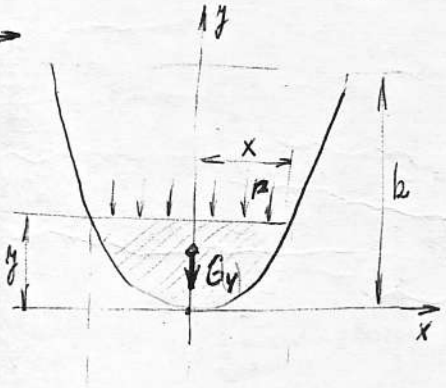
$$p \cdot dS_m \cdot dS_t = \sigma_t \cdot dS_m \cdot \frac{dS_t \cdot t}{\rho_t} + \sigma_m \cdot dS_t \cdot \frac{dS_m \cdot t}{\rho_m}$$

$$p = \frac{\sigma_t \cdot t}{\rho_t} + \frac{\sigma_m \cdot t}{\rho_m} \quad \longrightarrow \quad \boxed{\frac{\sigma_m}{\rho_m} + \frac{\sigma_t}{\rho_t} = \frac{p}{t}}$$

Laplace-ova rel.

rovnica s 2. neznámymi

potrebujeme ešte 1 rovnicu.



$$\sigma_{my} = \sigma_m \cos\vartheta$$

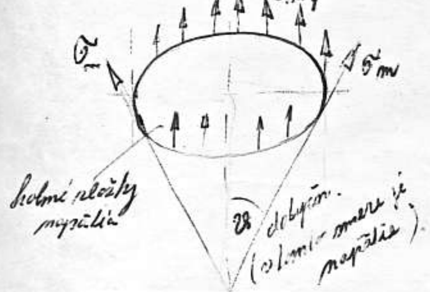
$$\underbrace{\underbrace{\sigma_m \cdot \cos\vartheta}_{\text{náp.}} \cdot \underbrace{2\pi x \cdot t}_{\text{plocha}}}_{\text{vnút. sila}} = \underbrace{\pi x^2 \cdot p + G_{(y)}}_{\text{vonk. sila}} \qquad / \text{nezn. } \sigma_m$$

$$/ \; p = \gamma \cdot (h - y)$$

$$\Downarrow$$

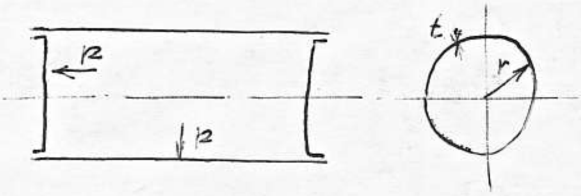
$$\boxed{\sigma_m = \frac{p \cdot x}{2t \cos\vartheta} + \frac{G_{(y)}}{2\pi x t \cdot \cos\vartheta}}$$

2. statická rovnica.



⇒

tenkostenný valec:

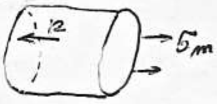


ρ_m (meridiánové) $= \infty$

$\rho_t = r$

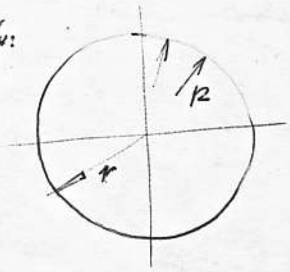
$$\frac{\sigma_m}{\rho_m} + \frac{\sigma_t}{r} = \frac{p}{t} \quad \longrightarrow \quad \underline{\underline{\sigma_t = \frac{p \cdot r}{t}}}$$

rozr. nádobu:



$$p r^2 \pi = \sigma_m 2 \pi r t \quad \longrightarrow \quad \underline{\underline{\sigma_m = \frac{pr}{2t}}}$$

Guľa:

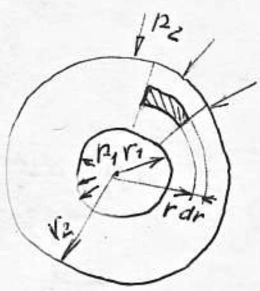


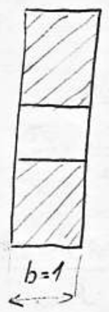
$\rho_m = \rho_t = r$

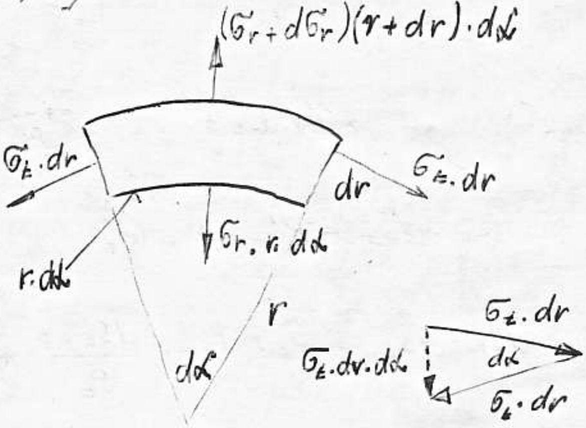
$\sigma_m = \sigma_t = \sigma$

$$\frac{\sigma}{r} + \frac{\sigma}{r} = \frac{p}{t} \quad \longrightarrow \quad \underline{\underline{\sigma = \frac{pr}{2t}}}$$

Hrubostenné nádoby







$$(\sigma_r + d\sigma_r) \cdot (r + dr) d\alpha - \sigma_r \cdot r \cdot d\alpha - \sigma_t \cdot dr \cdot d\alpha = 0$$

$$\cancel{\sigma_r \cdot r} + \sigma_r \cdot dr + d\sigma_r \cdot r + \underbrace{d\sigma_r \cdot dr}_{\sim 0} - \cancel{\sigma_r \cdot r} - \sigma_t \cdot dr = 0$$

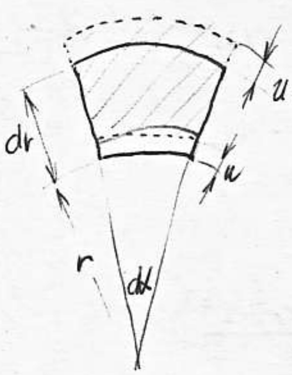
$$d\sigma_r \cdot r + \sigma_r \cdot dr - \sigma_t dr = 0 \quad \Big/ \cdot \frac{1}{dr}$$

$$\sigma_r - \sigma_t + r \cdot \frac{d\sigma_r}{dr} = 0 \quad \Big/ \; \sigma_r = f(r), \; \sigma_t = f(r)$$

neznáme σ_r a σ_t

potrebujeme ešte podmienku deformační.

$$\underline{\underline{\sigma_r - \sigma_t + r \cdot \frac{d\sigma_r}{dr} = 0}} \qquad (1)$$



$$\varepsilon_t = \frac{(r+u)d\alpha - r\,d\alpha}{r\cdot d\alpha} = \frac{u}{r}$$

$$\varepsilon_r = \frac{(dr+du)-dr}{dr} = \frac{du}{dr} = u'$$

$$\varepsilon_t = \frac{u}{r} = \frac{1}{E}(\sigma_t - \mu\cdot\sigma_r) \qquad (2)$$

$$\varepsilon_r = u' = \frac{1}{E}(\sigma_r - \mu\,\sigma_t) \qquad (3)$$

nezn. σ_t, σ_r, u

z (2) $\sigma_t = \frac{u}{r}\cdot E + \mu\,\sigma_r$

z (3) $\sigma_t = \frac{\sigma_r - u'E}{\mu}$

$$\frac{u}{r}E + \mu\sigma_r = \frac{\sigma_r - u'E}{\mu}$$

$$\mu E\frac{u}{r} + \mu^2\sigma_r = \sigma_r - u'E$$

$$\sigma_r(1-\mu^2) = E\left(u' + \mu\frac{u}{r}\right)$$

$$\sigma_r = \frac{E}{1-\mu^2}\left(u' + \mu\frac{u}{r}\right) \rightarrow \sigma_r' = \frac{\partial\sigma_r}{\partial r}$$

$$\sigma_t = \frac{u}{r}E + \frac{\mu E}{1-\mu^2}\left(u' + \mu\frac{u}{r}\right) = \frac{\frac{u}{r}E - \mu^2\frac{u}{r}E + \mu E u' + \mu^2 E\frac{u}{r}}{1-\mu^2}$$

$$\sigma_t = \frac{E}{1-\mu^2}\left(\frac{u}{r} + \mu u'\right)$$ — dos do (1) ✓

$$\frac{E}{1-\mu^2}\left(u' + \mu\frac{u}{r}\right) - \frac{E}{1-\mu^2}\left(\frac{u}{r} + \mu u'\right) + \frac{Er}{1-\mu^2}\left(u'' + \mu\frac{u'r - u\cdot 1}{r^2}\right) = 0$$

$$u' + \mu\frac{u}{r} - \frac{u}{r} - \mu u' + r\cdot u'' + \mu u' - \mu\frac{u}{r} = 0$$

$$r u'' + u' - \frac{u}{r} = 0$$

$u = r^m$

$u' = m r^{m-1}$

$u'' = m(m-1)\cdot r^{m-2}$

$$r\,m(m-1)\cdot r^{m-2} + m r^{m-1} - \frac{r^m}{r} = 0$$

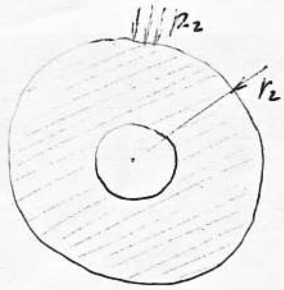
$$r^{m-1}\,[m(m-1) + m - 1] = 0$$

$$r^{m-1}\,(m^2 - m + m - 1) = 0 \longrightarrow r^{m-1}\cdot(m^2-1) = 0 \longrightarrow m = \pm 1$$

$$u = c_1 r + \frac{c_2}{r} = c_1 r^{+1} + c_2 r^{-1}$$

$$\sigma_t = \frac{E}{1-\mu^2}\left[\frac{c_1 r}{r} + \frac{c_2}{r^2} + \mu\left(c_1 - \frac{c_2}{r^2}\right)\right] = \frac{E}{1-\mu^2}\left[c_1 + \mu c_1\right] + \frac{c_2 E}{1-\mu^2}\left[\frac{1}{r^2} - \frac{\mu}{r^2}\right] = A + B\cdot\frac{1}{r^2}$$

$$\sigma_r = \frac{E}{1-\mu^2}\left\{\left[c_1 - \frac{c_2}{r^2}\right] + \mu\left[c_1 + \frac{c_2}{r^2}\right]\right\} = \frac{E}{1-\mu^2}(c_1 + \mu c_1) - \frac{E c_2}{1-\mu^2}\left(\frac{1}{r^2} - \frac{\mu}{r^2}\right) = A - B\cdot\frac{1}{r^2}$$



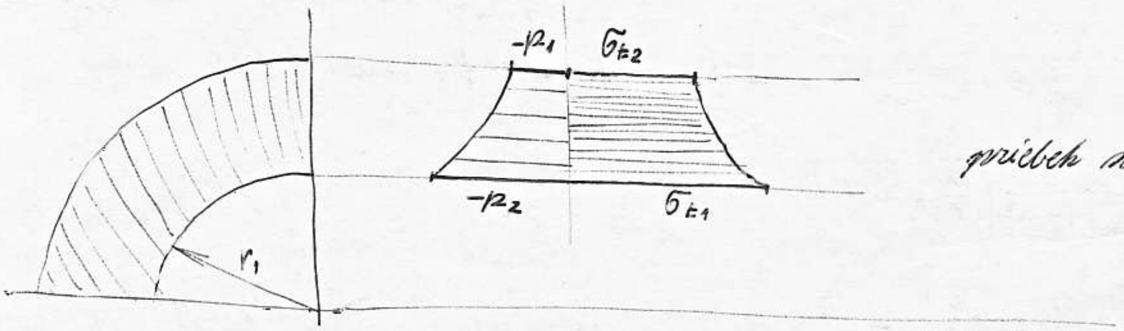
na okraji $(\sigma_r) = p$: $\left.\begin{aligned}(\sigma_r)_{r=r_1} &= -p_1\\ (\sigma_r)_{r=r_2} &= -p_2\end{aligned}\right\}$ dos.

$$\left.\begin{aligned}-p_1 &= A - B/r_1^2\\ -p_2 &= A - B/r_2^2\end{aligned}\right\} \longrightarrow \quad A = \frac{p_1 r_1^2 - p_2 r_2^2}{r_2^2 - r_1^2}, \quad B = (p_1 - p_2)\left(\frac{r_1^2 \cdot r_2^2}{r_2^2 - r_1^2}\right)$$

$$\sigma_r = \frac{p_1 r_1^2 - p_2 r_2^2}{r_2^2 - r_1^2} - (p_1 - p_2)\frac{r_1^2 \cdot r_2^2}{r_2^2 - r_1^2} \cdot \frac{1}{r^2}$$ radiálne nap.

$$\sigma_t = \frac{p_1 r_1^2 - p_2 r_2^2}{r_2^2 - r_1^2} + (p_1 - p_2)\frac{r_1^2 \cdot r_2^2}{r_2^2 - r_1^2} \cdot \frac{1}{r^2}$$ tangenciálne nap.

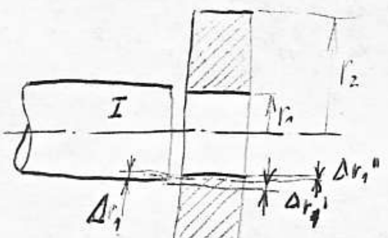
} v hrubost. nádobe.



priebeh napätia

$$\sigma_{red} = \sigma_1 - \sigma_3 \leq \sigma_D$$

$$\sigma_{t_1} - \sigma_{r_1} = \sigma_{t_1} + p_1 \leq \sigma_D$$



$$\Delta r_1 = (\Delta r_1') + (\Delta r_1'')$$

na čape: $\sigma_r = \sigma_t = -p_1$

$$\underset{\text{čap.}}{\varepsilon_t} = \frac{\Delta r_1'}{r_1} = \frac{1}{E}(\sigma_t - \mu\sigma_r) = -\frac{1}{E}(p_1 - \mu p_1) = -\frac{p_1}{E}(1-\mu)$$

Náboj: $p_2 = 0$

$\sigma_r = -p_1$

$$\sigma_{t_1} = \frac{p_1 r_1^2}{r_2^2 - r_1^2} + \frac{p_1 r_2}{r_2^2 - r_1^2} = \frac{p_1}{r_2^2 - r_1^2}(r_1^2 + r_2^2)$$

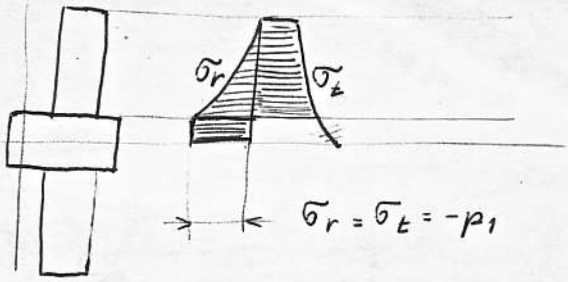
$$\varepsilon_t = \frac{\Delta r_1''}{r_1} = \frac{1}{E}(\sigma_{t_1} - \mu\sigma_{r_1}) = \frac{p_1}{E}\left(\frac{r_1^2 + r_2^2}{r_2^2 - r_1^2} + \mu\right)$$

$$\Delta r_1 = \frac{r_1 p_1}{E} \cdot \left(\frac{r_1^2 + r_2^2}{r_2^2 - r_1^2} + \mu + 1 - \mu\right) = \frac{r_1 \cdot p_1}{E}\left(\frac{r_2^2 + r_1^2}{r_2^2 - r_1^2} + 1\right) p_1$$

$$\sigma_t - \sigma_r \leq \sigma_D$$

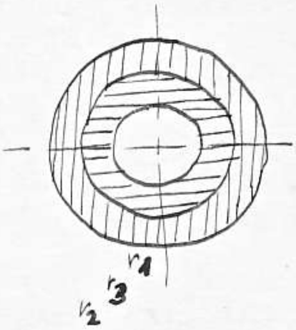
$$\sigma_{t_1} + p_1 \leq \sigma_D$$

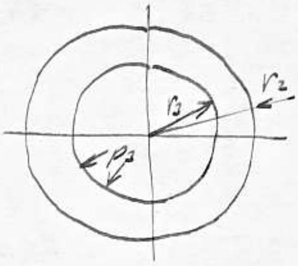
$$\frac{\sigma_D}{2}\left(1 - \frac{r_1^2}{r_2^2}\right) \geq p$$

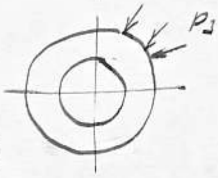


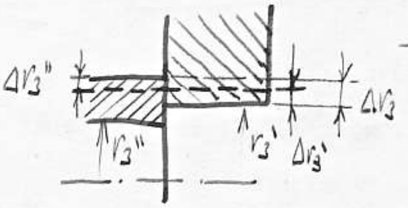
priebeh napätia

30. IV. 1970.









$$\Delta r_3 = |\Delta r_3''| + |\Delta r_3'|$$

$$\varepsilon_t' = \frac{1}{E}\left(\sigma_{t_3}' - \mu\, \sigma_{rad\,3}'\right) = \frac{p_3}{E}\left(\frac{r_1^2 + r_3^2}{r_2^2 - r_3^2} + \mu\right) = \frac{\Delta r_3'}{r_3}$$

$$\sigma_{t3}' = \frac{p_3 \cdot r_3^2}{r_2^2 - r_3^2} + \frac{p_3 r_2^2}{r_2^2 - r_3^2}$$

$$\sigma_{r3}' = \frac{p_3 r_3^2}{r_2^2 - r_3^2} - \frac{p_3 r_2^2}{r_2^2 - r_3^2} = -p_3$$

$$\Delta r_3' = \frac{r_3 p_3}{E}\left(\frac{r_2^2 + r_3^2}{r_2^2 - r_3^2} + \mu\right)$$

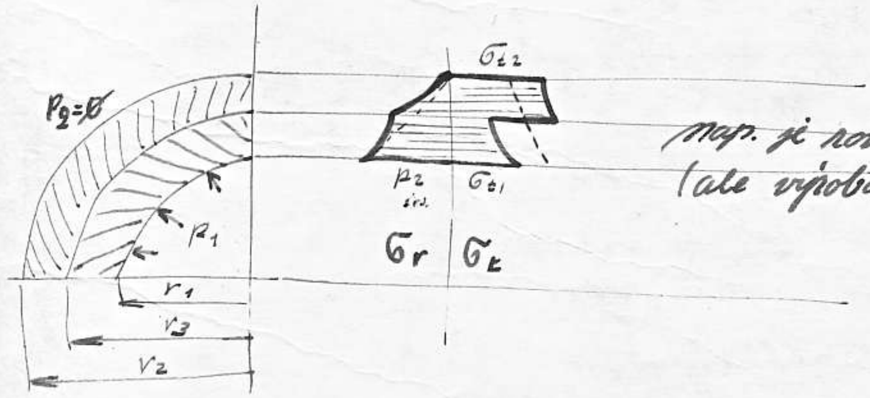
zmena polomeru vonk. valca

$$\Delta r_3'' = -\frac{p_3 \cdot r_3}{E}\left(\frac{r_3^2 + r_1^2}{r_3^2 - r_1^2} - \mu\right)$$

zmena polom. vnút. valca.

$$\Delta r = \Delta r_3' + \Delta r_3'' = \frac{p_3 r_3}{E}\left\{\frac{r_2^2 + r_3^2}{r_2^2 - r_3^2} + \mu\right\} + \left\{\frac{r_3^2 + r_1^2}{r_2^2 - r_1^2} - \mu\right\}\frac{p_3 r_3}{E}$$

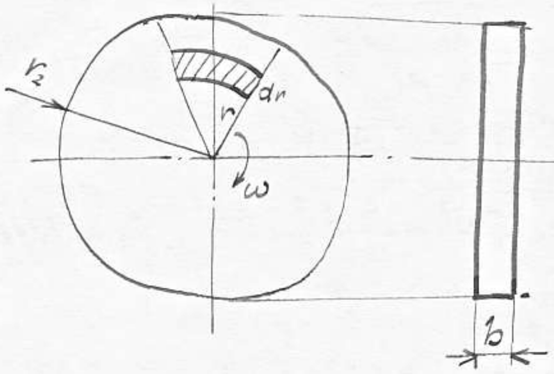
význam tohoto vzťahu:

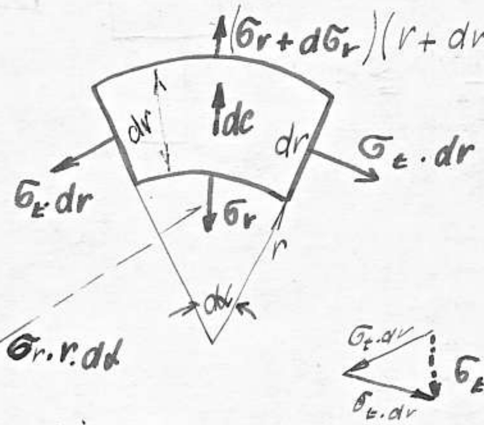


nap. je rovnomernejšie rozložené (ale výroba je drahšia)

Kotúče

Vzniká sila odstredivá, [illegible] aj vlastná váha





dc - centrifug. s.

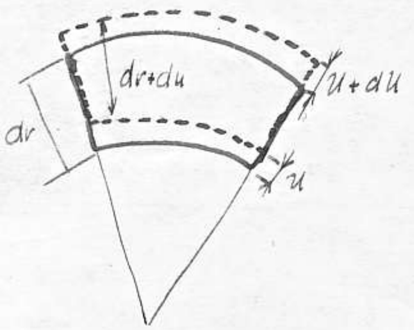
$$(\sigma_r + d\sigma_r)(r+dr) . d\alpha - \sigma_r . r . d\alpha + dc - \sigma_t . dr . d\alpha = 0$$

$$dc = r\omega^2 dm = \frac{r\omega^2 . \gamma . (r . d\alpha . dr)}{g} = \frac{\gamma \omega^2 r^2 d\alpha dr}{g}$$

$$(\sigma_r + d\sigma_r)(r+dr)d\alpha - \sigma_r . r . d\alpha + \frac{\omega^2 \gamma}{g} . r^2 d\alpha . dr - \sigma_t . dr . d\alpha = 0$$

$$\sigma_r - \sigma_t + r\frac{d\sigma_r}{dr} + \gamma . \frac{\omega^2 r^2}{g} = 0 \qquad \text{dif. r.} \qquad 1.)$$

Musíme rozl. dif. rovnicu, lebo horná d.r. obs. 2 nezn.



(z predch. látky)

$$\varepsilon_t = \frac{u}{r} = \frac{1}{E}(\sigma_t - \mu \sigma_r) \qquad 2.)$$

$$\varepsilon_r = \frac{du}{dr} = u' = \frac{1}{E}(\sigma_r - \mu \sigma_t) \qquad 3.)$$

Vyjadr. a dos. u.

ozn: $\frac{\omega^2 \gamma}{g} = A_0$

$\vdots$

$$r . u'' + u' - \frac{u}{r} + \frac{A_0}{E^*} . r^2 = 0 \qquad \text{– nezn. u.}$$

ozn: $E^* = \frac{E}{1-\mu^2}$

$\Downarrow u$ a potom dosadíme.

$$u_p = Kr^3$$

$$u_p' = 3kr^2$$

$$u_p'' = 6kr$$

$$6r^2K + 3Kr^2 - kr^2 + \frac{A_0}{E^*}r^2 = 0$$

$$6K + 3k - k + \frac{A_0}{E^*} = 0$$

$$K = -\frac{A_0}{8E^*}$$

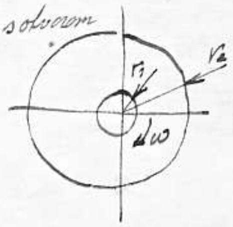
$$u = c_1 r + \frac{c_2}{r} - \frac{A_0}{8E^*}r^2 \qquad \text{všeob. ries.}$$

$$\sigma_t = A + \frac{B}{r^2} - \frac{1+3\mu}{8} A_0 r^2$$

$$\sigma_r = A - \frac{B}{r^2} - \frac{3+\mu}{8} A_0 r^2$$

$$A_0 = \frac{\omega^2 \gamma}{g}$$

1.) s otvorom



$$\left.\begin{aligned} \sigma_{(r_1)} &= 0 \\ \sigma_{(r_2)} &= 0 \end{aligned}\right\}$$ 2 rovn. o nezn. A; B

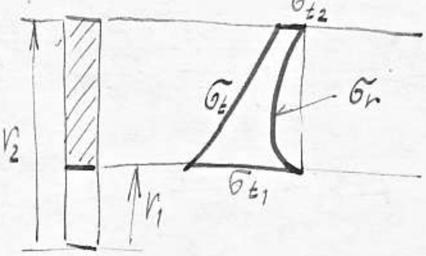
$\Downarrow$

$$A = \frac{3+\mu}{8} \cdot A_0 (r_1^2 + r_2^2)$$

$$B = \frac{3+\mu}{8} A_0 r_1^2 \cdot r_2^2$$

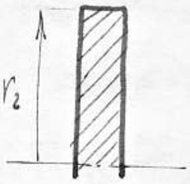
$$\sigma_r = \frac{3+\mu}{8g} \gamma \omega^2 \left(r_1^2 + r_2^2 - r^2 - \frac{r_1^2 r_2^2}{r^2}\right)$$

$$\sigma_t = \frac{\gamma \omega^2}{8g} \left[(3+\mu)\left(r_1^2 + r_2^2 + \frac{r_1^2 r_2^2}{r^2}\right) - (1+3\mu) r^2\right]$$



$$\sigma_{t_1} = \frac{\gamma \omega^2}{4g} \left[(1-\mu) r_1^2 + (3+\mu) r_2^2\right]$$

2.) bez otvoru



$r = 0 \quad \sigma_t = \sigma_r$

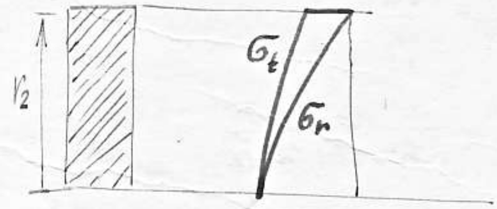
$r = r_2 \quad \sigma_r = 0$

$$B = 0$$

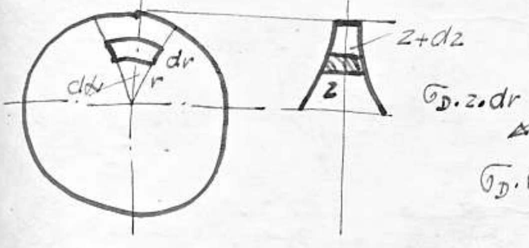
$$A = \frac{3+\mu}{8g} \underbrace{\gamma \omega^2}_{A_0} r_2^2$$

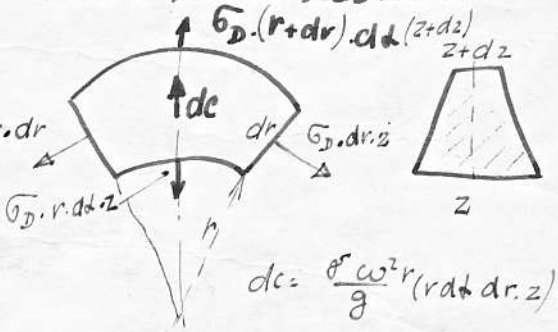
$$\sigma_t = \frac{3+\mu}{8} A_0 r_2^2 - \frac{1+3\mu}{8} A_0 r^2$$

$$\sigma_r = \frac{3+\mu}{8} A_0 r_2^2 - \frac{3+\mu}{8} A_0 r^2$$



Ideálny kotúč, ak napätia budú všade rovnaké:





$$\sigma_r = \sigma_t = \sigma_D$$

$$dc = \frac{\gamma \omega^2 r}{g} (r\, d\alpha\, dr . z)$$

neuvaž. vlastnú váhu.

$$\sigma_D \cdot (r+dr)\, d\alpha . (z+dz) + \frac{\gamma \omega^2 r^2}{g} z\, d\alpha\, dr = \sigma_D \cdot r \cdot d\alpha . z + \sigma_D \cdot z\, dr\, d\alpha$$

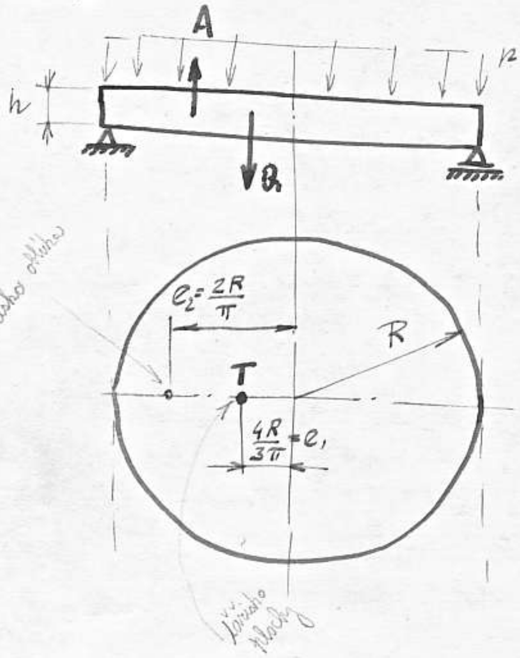
$$-\sigma_D \cdot r z + \sigma_D \cdot z \cdot dr + \sigma_D \cdot r \cdot dz + \frac{\gamma \omega^2 r^2}{g} z \cdot dr = \sigma_D \cdot r \cdot z + \sigma_D \cdot z \cdot dr$$

$$\sigma_D \cdot r \cdot dz + \frac{\gamma \omega^2 r^2}{g} z \cdot dr = 0 \qquad z = f(r)$$

$$\frac{dz}{z} = -\frac{\gamma \omega^2 r}{\sigma_D \cdot g} \cdot dr$$

$$\ln z = -\frac{\gamma \omega^2}{\sigma_D \cdot g} \cdot \frac{r^2}{2} + C \qquad \Big/ \frac{\gamma \omega^2}{2 \sigma_D \cdot g} = m$$

$$z = C \cdot e^{-m r^2}$$



Výpočet dosky:

$$e_2 = \frac{\int_0^{\frac{\pi}{2}} r \, d\varphi \cdot r \sin\varphi}{\frac{1}{4} \cdot 2\pi r} = \frac{2r}{\pi}\left[-\cos\varphi\right]_0^{\frac{\pi}{2}} = \frac{2r}{\pi}$$

$$e_1 = \frac{\int y \, dx \cdot \frac{y}{2}}{\frac{1}{4} F} = \frac{\int_0^{\pi/2} R^2 \sin^2 \varphi (-R \sin\varphi) \, d\varphi}{\frac{1}{4}\pi R^2} =$$

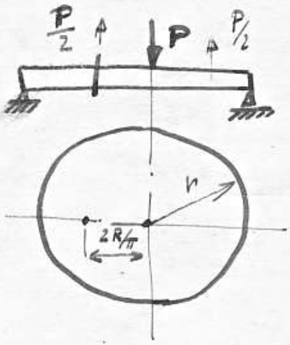
$$Q = \frac{\pi R^2 p}{2} = A$$

$$M_{max} = e_2 A - e_1 Q = Q\left(\frac{2R}{\pi} - \frac{4R}{3\pi}\right) = Q \frac{2}{3} \frac{R}{\pi} = \frac{pR^3}{3}$$

$$\sigma = \frac{M}{W} \qquad \Big/ W = \frac{1}{6} 2Rh^2 = \frac{1}{3} Rh^2$$

$$\sigma = \frac{\frac{pR^3}{3}}{\frac{1}{3} Rh^2} = \frac{pR^2}{h^2}$$

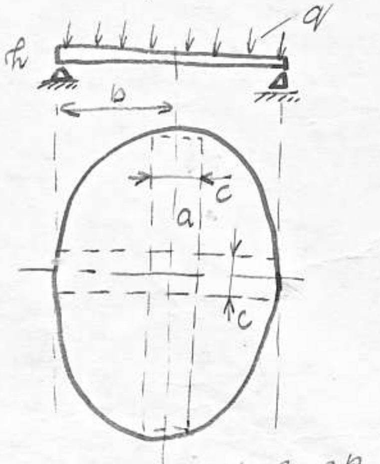
Pozr. presné ries.



$$M_{max} = \frac{P}{2} \cdot \frac{2R}{\pi} = \frac{PR}{\pi}$$

$$\sigma_{max} = \frac{M}{W} = \frac{PR}{\pi} \cdot \frac{3}{Rh^2} = \frac{3P}{\pi h^2} \implies \text{nezáv. nap. na mieste v doske}$$

$$\sigma = p \cdot \frac{b^2}{h^2}$$



$$q = c \cdot p$$

$$M_{max} = \frac{1}{8} q (2b)^2 = \frac{1}{2} q b^2 = \frac{p c b^2}{2 \cdot 1.1} \qquad \Big/ W = \frac{ch^2}{6}$$

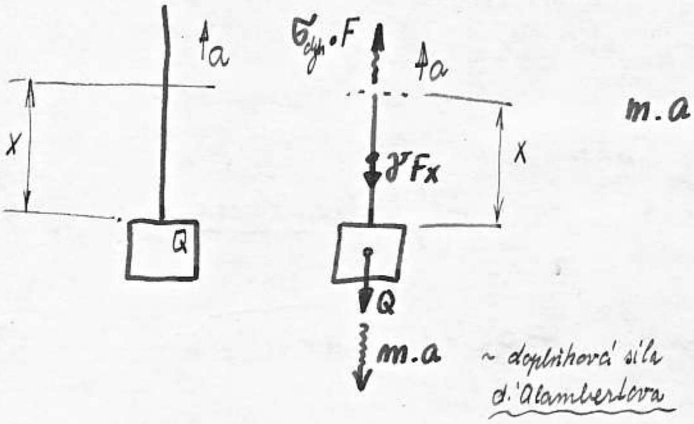
$$\sigma_{max} = \frac{M_m}{W} = \frac{p c b^2}{2} \cdot \frac{6}{h^2 c} = \frac{3 p b^2}{h^2}$$

$$\sigma = \alpha \cdot p \cdot \frac{b^2}{h^2} \qquad \Big/ \begin{array}{l} \alpha \text{ pre kruh} = 1 \\ \alpha \text{ pre elipsu} = 3 \end{array} \qquad \begin{array}{l} \frac{b}{a} = 1 \\ \frac{b}{a} = 3 \end{array}$$

$$\alpha = 3 - 2\frac{b}{a}$$

$$\sigma = \frac{3a - 2b}{a} \cdot p \cdot \frac{b^2}{h^2}$$

Dynamické namáhanie



$$m.a = \frac{Q + \gamma F.x}{g}.a$$

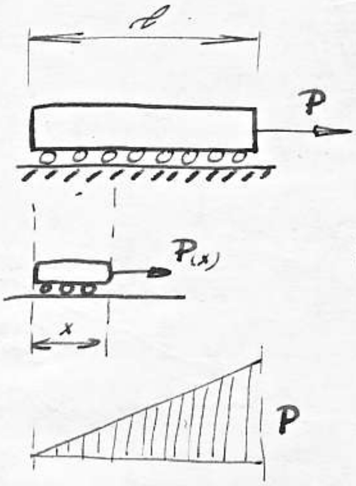
$$\sigma_{dyn}.F = Q + \gamma F.x + \frac{Q + \gamma F_x}{g}.a$$

$$\sigma_{dyn} = \frac{Q + \gamma F_x}{F} + \frac{Q + \gamma F.x}{gF}.a =$$

$$= \frac{Q + \gamma F_x}{F}\left[1 + \frac{a}{g}\right] = \sigma_{stat}\left[1 + \frac{a}{g}\right]$$

$$\sigma_d = k_d . \sigma_{st}$$

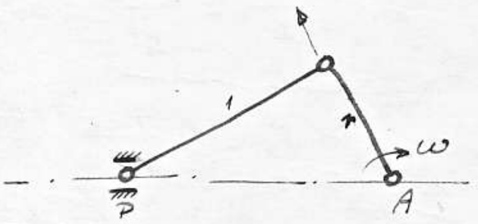
Bezváhový stav, ak $a = g$ a majú rovnaký smer.



$$P = m.a \qquad m = \frac{l F.\gamma}{g}$$

$$a = \frac{P}{m} = \frac{P.g}{l.F.\gamma}$$

$$P_{(x)} = a.m_{(x)} = \frac{Pg}{l.F.\gamma}.\frac{\gamma.x.F}{g} = \frac{Px}{l} = \frac{x}{l}.P$$

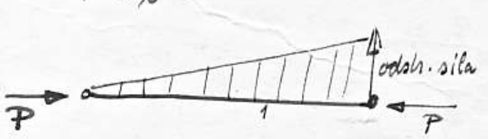


$$a_n = \omega^2 r$$

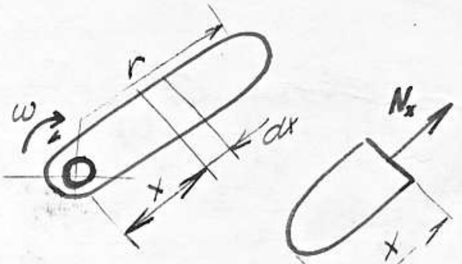
(uvaž 90°) [bude max. pos. zaťaž na „1"]

preveď na stat. príklad.

$|a_n = 0$



$$q_0 = \frac{F.\gamma.1}{g}.\omega^2 r$$



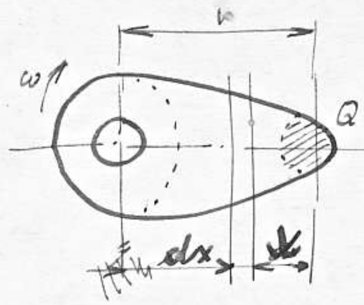
$$dq = a_n.dm = (\omega^2 x).\frac{F.\gamma.dx}{g}$$

$$N_x = \int_x^r dq = \int_x^r \frac{F.\gamma.\omega^2}{g}.x\,dx = \frac{\omega^2 F\gamma}{g}\left[\frac{r^2}{2} - \frac{x^2}{2}\right]$$

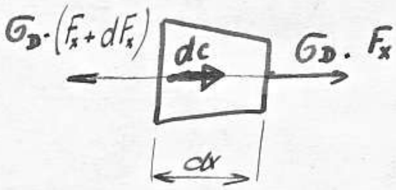
$$\left(N_x = \omega^2.\frac{F}{g}.\gamma[r-x]\frac{[r+x]}{2} \right)$$

pre $x = 0$ je $\sigma_{dyn} = \frac{\omega^2 \gamma}{2g}.r^2$ maximálne.

Hraď namiesto rovn. prírodi!



chceme aby $\sigma = \sigma_D$ všade.



$$dc = \frac{F_{(x)}\cdot \gamma \cdot dx}{g}\,\omega^2 (l-x)$$

$$\sigma_D \cdot F_{(x)} + \frac{F_{(x)}\cdot \gamma \cdot dx}{g}(l-x)\,\omega^2 - \sigma_D \cdot (F_{(x)} + dF_{(x)}) = 0$$

$$\frac{dF_{(x)}}{F_{(x)}} = \frac{\gamma \omega^2}{g \cdot \sigma_D}(l-x)\,dx \qquad d.r.$$

$$\ln F_{(x)} = \frac{\gamma \omega^2}{g \cdot \sigma_D}\left(lx - \frac{x^2}{2}\right) + C$$

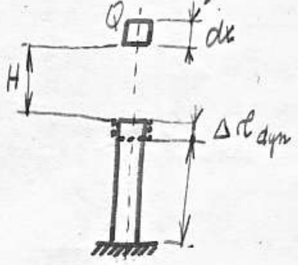
ak $x = 0$ } $F_{(x)} = F_{(0)}$

$$\ln \frac{F_{(x)}}{F_{(0)}} = \frac{\gamma \omega^2 x}{g \sigma_D}(2l - x)$$

$$F_{(x)} = F_{(0)} \cdot e^{\frac{\gamma \omega^2 x}{2 g \sigma_D}\cdot (2l-x)}$$

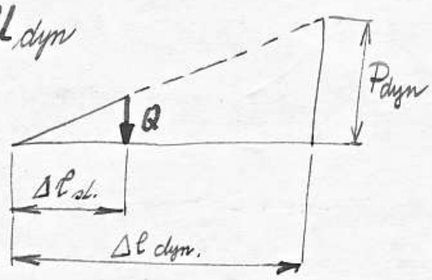
Rázy

Pohybová energia telesa sa premení pri ráze → zvýšené namáhanie telesa.



Predp. že energia telesa sa mení na def.

$T = U_{dyn}$



$$\frac{P_d}{Q} = \frac{\Delta l_d}{\Delta l_{st}} \rightarrow$$

$$P_d = Q\,\frac{\Delta l_{dyn}}{\Delta l_{st.}}$$

predl. ráz?

potenc. en. $T = Q\,(H + \Delta l_{dyn})$

$U = \frac{1}{2}\cdot P_{dyn} \cdot \Delta l_{dyn}$ prírl. energ. po deform.

$$\frac{1}{2} P_d \cdot \Delta l_d = Q(H + \Delta l_d)$$

$$\frac{1}{2}\cdot Q \cdot \frac{\Delta l_d}{\Delta l_{st}}\Delta l_d = Q\,(H + \Delta l_d) \qquad /\cdot 2\Delta l_{st}$$

$$\Delta l_{dyn}^2 = 2H \cdot \Delta l_{st} + 2\Delta l_{st}\cdot \Delta l_d.$$

$$\Delta l_d^2 - 2\Delta l_d \cdot \Delta l_{st} - 2H\Delta l_{st} = 0$$

$$\Delta l_d = \frac{2\Delta l_{st} \pm \sqrt{4\Delta l_{st}^2 + 8H\Delta l_{st}}}{2} = \Delta l_{st} \pm \sqrt{\Delta l_{st}^2 + 2H\Delta l_{st}}$$

$$\Delta l_{dyn} = \Delta l_{st} + \sqrt{\Delta l_{st}^2 + 2H\Delta l_{st}} = \Delta l_{st}\left(1 + \sqrt{1 + \frac{2H}{\Delta l_{st}}}\right) = \Delta l_{dyn}$$

dynamický koef.: $K_d = 1+\sqrt{1+\frac{2H}{\Delta l_{st}}}$ →

$$\Delta l_d = K_d \cdot \Delta l_{st}$$
$$\sigma_d = K_d \cdot \sigma_{st}$$

$$\frac{2H}{\Delta l_{st}} \cdot \frac{\frac{Q}{2}}{\frac{Q}{2}} = \frac{Q.H}{\frac{1}{2} Q.\Delta l_{st}}$$

preto $K_d = 1+\sqrt{1+\frac{T_0}{T_{st}}}$

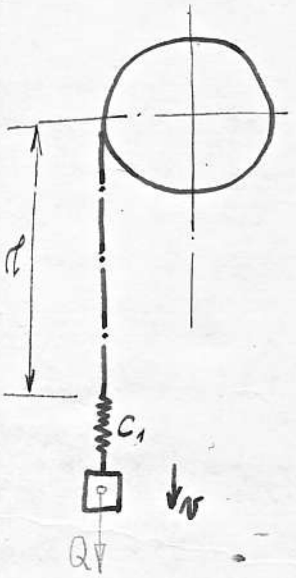
T_0 – energia (kin.) telesa

T_{st} – energ. telesa.

Keď $H=0$ $K_{dyn}=2$:

$H \gg \Delta l_{st}$: $K_d = 1+\sqrt{\frac{2H}{\Delta l_{st}}}$

$H \ggg \Delta l_{st}$: $K_d = \sqrt{\frac{2H}{\Delta l_{st}}}$

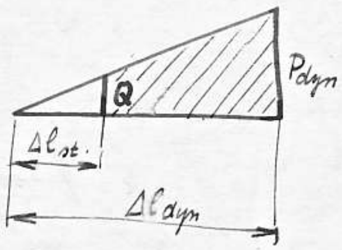


v [m/s] c_1 kp/cm, pruž. konšt. prík. lana.

c_2

Pri zabrzdení vzn. ráz.

$$T = U_d.$$



$$P_{dyn} = Q \cdot \frac{\Delta l_d}{\Delta l_{st}}$$

$$U_{dyn} = \frac{1}{2} P_d \Delta l_d - \frac{1}{2} Q \cdot \Delta l_{st} \qquad T = \frac{1}{2}\overset{m}{\frac{v^2}{g}\cdot Q} + Q(\Delta l_d - \Delta l_{st})$$

prírastky energ. v dôsl. dynam. namáh.: $\underbrace{\frac{1}{2}\frac{v^2}{g}\cdot Q + Q(\Delta l_d - \Delta l_{st})} = \underbrace{\frac{1}{2}\cdot\frac{\Delta l_d^2}{\Delta l_{st}} \cdot Q - \frac{1}{2} Q.\Delta l_{st}}$ $/ \cdot \frac{2}{Q}$

$$\frac{v^2}{g} + 2\Delta l_d - 2\Delta l_{st} = \frac{\Delta l_d^2}{\Delta l_{st}} - \Delta l_{st} \qquad / \cdot \Delta l_{stat.}$$

$$\Delta l_d^2 - 2\Delta l_d \Delta l_{st} + \Delta l_{st}^2 - \frac{v^2 \Delta l_{st}}{g} = 0$$

$$\Delta l_d = \Delta l_{st} \pm \sqrt{\frac{v^2 \Delta l_{st}}{g}} = \Delta l_{st}\left(1+\sqrt{\frac{v^2}{g\,\Delta l_{st}}}\right) = \Delta l_{st} \cdot K_{dyn}$$

$$\Delta l_{st} = \frac{QL}{EF} + \frac{Q}{c_1} = \frac{Ql}{EF}\left(1+\frac{EF}{l c_1}\right)$$

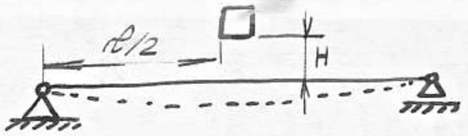
$$\sigma_{dyn} = k_{dyn} \cdot \sigma_{st}.$$

Pr.: $v = 1\,m/s$
$Q = 5000\,kp$
$F = 6\,cm^2$
$E = 17.10^6\,kp\,cm^{-2}$
$l = 200\,m$

$c_1 = 400\,kp/cm$

$\sigma_{st} = 835\,kp\,cm^{-2}$
$\sigma_{dyn} = 1560\,kp\,cm^{-2}$

Ak $c_1 = 0$ $\sigma_{dyn} = 3520\,kp\,cm^{-2}$



$T = U_d$

$K_d = 1+\sqrt{1+\frac{T_0}{U_{st}}}$ / $T_0 = H.Q$

$U_{st} = \frac{1}{2}\,\Delta l_{st}\,.\,Q$

$\Delta l_{st} = \frac{Ql^3}{48EJ}$ ✓

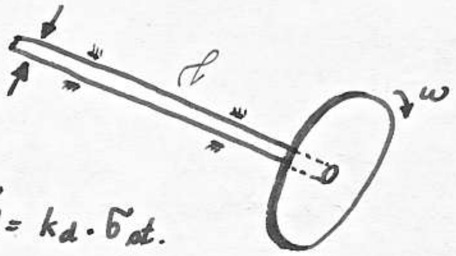
priečny raz?

$U_{st} = \frac{Q^2 l^3}{96EJ}$

$K_d = 1+\sqrt{1+\frac{96EJH}{Ql^3}}$

$\sigma_d = \sigma_{st}\cdot k_{dyn} = k_{dyn}\frac{QL}{4W}$

Vplyv hmoty sústavy!



$T = \frac{J_0}{2}\cdot\omega^2$

$J_0 = \frac{1}{8}D^2 m$ ~ mom. zotr. kolúča

$k_d = \sqrt{\frac{T_0}{U_{st}}}$

Energia pri zabrzdení deformuje hriadeľ.

$\sigma_d = k_d\cdot\sigma_{st}.$

$\tau_{st} = \frac{M_k}{W_p}$

$U_{st} = \frac{1}{2}\frac{M_k^2 L}{G J_p}$ ~ energia pri krute

$k = \sqrt{\frac{J_0\omega^2 G J_p}{M_k^2 L}}$

$\tau_d = \frac{M_k}{W_p}\cdot\sqrt{\frac{J_0\omega^2 G J_p}{M_k^2 L}} \rightarrow \tau_{dyn} = \sqrt{\frac{J_0\omega^2 G J_p}{L W_p^2}}$

Ráz pri krútení.

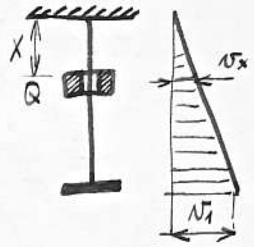
$\varphi_{dyn} = K.\varphi_{st}.$

$\varphi_{st} = \frac{M_k\cdot l}{G\cdot J_p}$

$\varphi_{dyn} = k_{dyn}\cdot\varphi_{st}.$

$\varphi_{dyn} = \sqrt{\frac{J_0\omega^2\, G.J_p}{M_k^2 L}}\cdot\frac{M_k\cdot l}{G J_p}$

$\varphi_{dyn} = \sqrt{\frac{\omega^2 L.J_0}{G.J_p}}$



$$dT_P = \frac{1}{2g}\gamma F.dx.\,v_x^2 \qquad v_x = \frac{v_1 . x}{l}$$

$$T = \frac{1}{2g}\int \gamma F\left(\frac{v_1 x}{l}\right)^2 dx = \frac{v_1^2}{2g}\cdot F\gamma\cdot\frac{l}{3} = \frac{v_1^2}{2g}\cdot G_{red.}$$

$$Qv = \left(Q + \frac{G_p}{3}\right)\cdot v_1 \quad \rightarrow \quad v_1 = \frac{v}{1+\frac{G_p}{3Q}}$$

$$T = \frac{v_1^2}{2g}\left(Q+\frac{G_p}{3}\right) + Q\,\Delta l_d = U_{dyn.}$$

$$\Delta l_{dyn}^2 - 2\Delta l_{st}\Delta l_{dyn} - \frac{v_1^2}{g}\cdot\Delta l_{st}\left(1+\frac{G_p}{Q}\right) = 0 = \frac{v_1^2}{2g}G_{red}$$

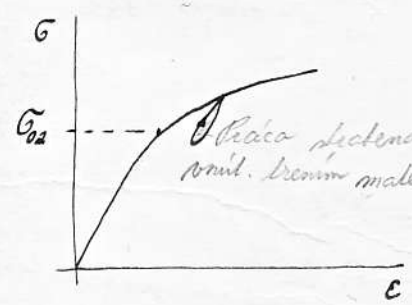
$$\Delta l_{dyn} = \Delta l_{st}\left(1+\sqrt{1+\frac{v_1^2}{g\Delta l_{st}\left(1+\frac{G}{3Q}\right)}}\right)$$

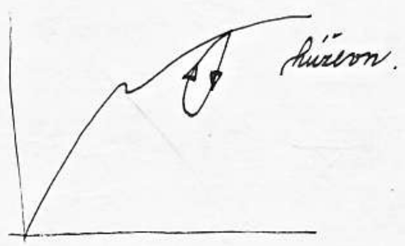
$$v_1^2 = \frac{v^2}{\left(1+\frac{G_p}{3Q}\right)^2} \nearrow$$

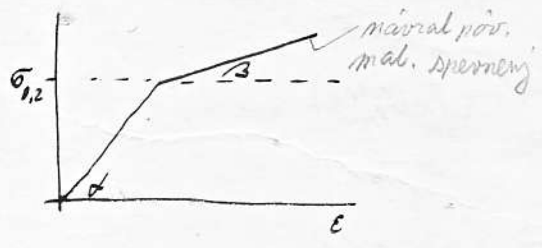
$$\underline{\underline{\Delta l_d}} = \Delta l_{st}\left[1+\sqrt{1+\frac{v^2}{g\cdot\Delta l_{st}}\cdot\frac{1}{\left(\frac{G_p}{3Q}+1\right)}}\right]$$

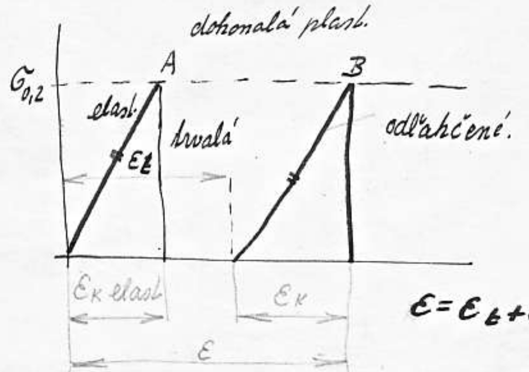
Základy plasticity

21. V. 1970.
(prep. predn.)









$$\underline{\varepsilon = \varepsilon_t + \varepsilon_K \rightarrow \varepsilon_t = \varepsilon - \varepsilon_K}$$

Podmienky plasticity: napätosť v každom bode iná. $F(\sigma_x\ \sigma_y\ \sigma_z\ \tau_x\ \tau_y\ \tau_z) = 0$

$\Phi(\sigma_1, \sigma_2\, \sigma_3) = 0$

1 osový stav napätosti: $\sigma_1 = \sigma_K$

viac-osový stav (nap. vo vš. smeroch) Saint - Venant $\sigma_1 > \sigma_2 > \sigma_3$

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \tau_{kr.} \qquad \tau_K = \frac{\sigma_K}{2}$$

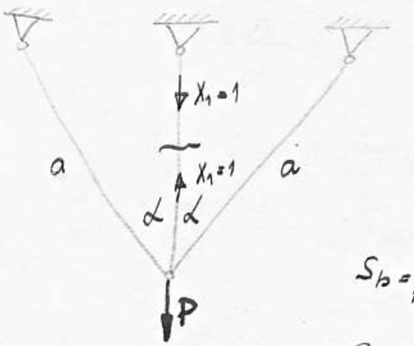
→

$\sigma_1 - \sigma_3 = \sigma_z$. *mater*

$$\sigma_{zr} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}$$

$\sigma_{zr} = \sigma_K$ ak dosiahne σ_{zr} — σ_K vzn. stav nap. — plastický

V praxi **Jednoosý stav nap.**



$a_{11} X_1 + a_{10} = \emptyset$ | $a_{11} = \sum \frac{S_i^2 l_i}{EF}$ $a_{10} = \sum \frac{S_1 S_0}{EF} . l_i$

od jedn. zaťaž.

$S_b = \frac{P}{1+2\cos^3\alpha}$ $S_a = \frac{P\cos^2\alpha}{1+2\cos^3\alpha}$ $\frac{S_a}{S_b} = \cos^2\alpha$ $S_a < S_b$

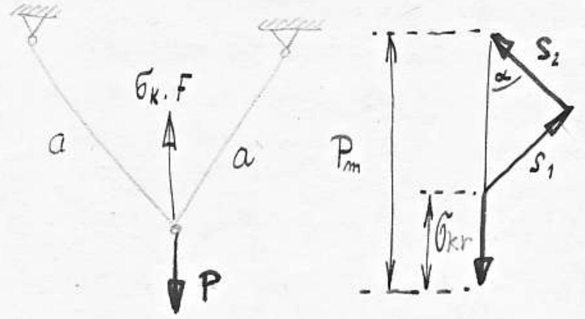
Pri dimenz.: $\sigma_b \leq \sigma_D$

$\sigma_{D_1} \leq \frac{\sigma_z . F}{k}(1+2\cos^3\alpha)$ / počítané podľa elast.

$S_b \leq \sigma_D . F = \frac{\sigma_z . F}{k}$ $\sigma_D = \frac{\sigma_z}{k}$

Sila - ktorá vyvodí medzu klzu. $P_{Klzu} = \sigma_z . F(1+2\cos^3\alpha)$ - zaťažujeme (a počítame podľa únosnosti celého systému) - ak dosiahn. v krajných σ_K

Ak stredný prút dosiahne σ_K, nemusíme počítať systém ako stat neurč., ale ako urč.



$P - F.\sigma_K = 2. S_a . \cos\alpha$

$P = F.\sigma_K + 2S(a)\cos\alpha$

$S(a) = F.\sigma_K$

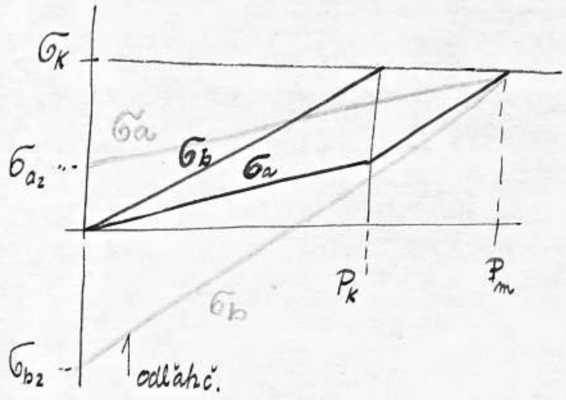
$P_m = F.\sigma_K(1+2\cos\alpha)$

$$P_{dov_2} = \frac{P_{medzné}}{2} = \frac{F.\sigma_K}{k}(1+2\cos\alpha)$$

pomer: $\frac{P_{dov\,2}}{P_{dov\,1}} = \frac{1+2\cos\alpha}{1+2\cos^3\alpha} > 1$

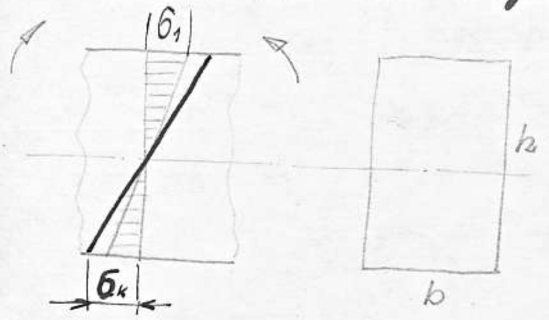
ak $\alpha = 30°$ $\frac{P_{dov_2}}{P_{dov_1}} = 1,18$

Po odľ: zostalo v systéme urč: zbytkové napätie:



Zákl. pojmy ; podmienky!

Namáhanie nosníka ohybom (nap. sa rozdelí nerovnomerne)



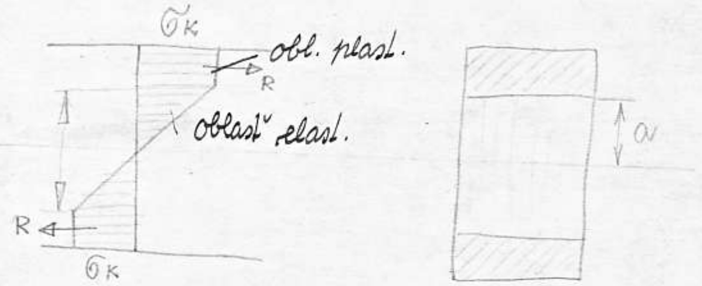
$\sigma_1 = \frac{M_o}{W}$ ak zaťažením predup. σ_K σ nebude rásť — len deformácia

$W = \frac{1}{6} bh^2$

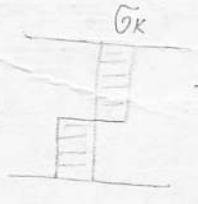
$R = b \cdot \frac{1}{2}(h-a)\sigma_K$?

? $M_{ep} = \frac{1}{6} bh^2 \sigma_K + R\frac{1}{2}(h+a)$ W

$\frac{1}{6} bh^2 \sigma_K + \frac{b}{4}(h^2 - a^2)\sigma_K$



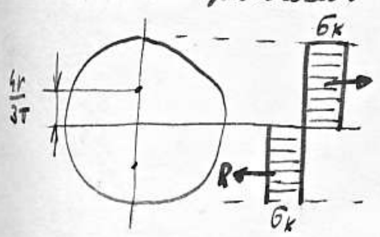
ak budeme ďalej zvyšovať, dost. horné medze $M_M = \frac{bh^2}{4}\sigma_K$ / $a=0$



— všade plastick. (nosnosť vyčerpaná)

$\frac{M_M}{M_K} = 1{,}5$

závisí od prierezu:



$W = \frac{\pi r^3}{4}$ $M_K = \frac{\pi r^3}{4}\sigma_K$

$R = \sigma_K \cdot \frac{1}{2}\pi r^2$

$M_M = R \cdot 2 \cdot \frac{4r}{3\pi}$

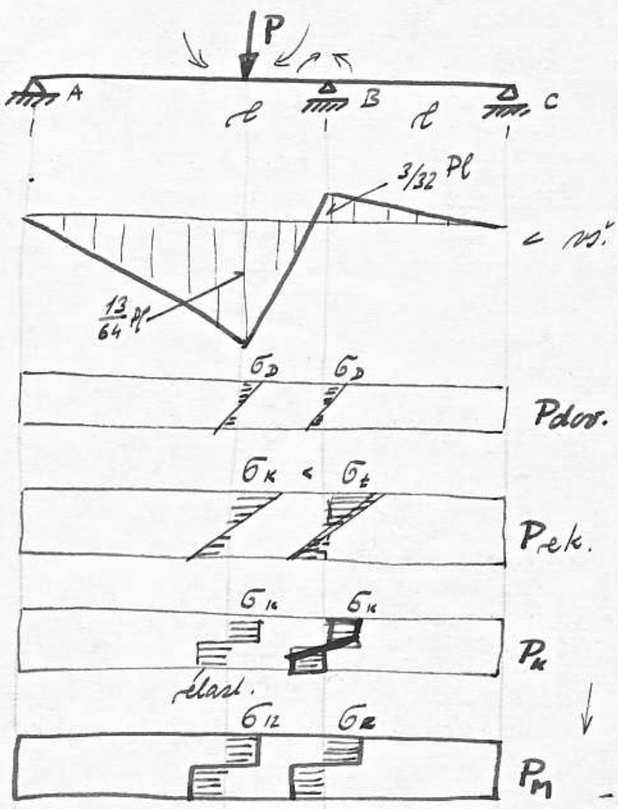
$M_M = \frac{8}{6} r^3 \sigma_K$

$\frac{M_M}{M_K} = \frac{16}{3\pi} = 1{,}7$

ohyb!

nosnosť asi 10 % čo nastane ak odľahč. zať. moment. — potom urč. vnút. napätie

*

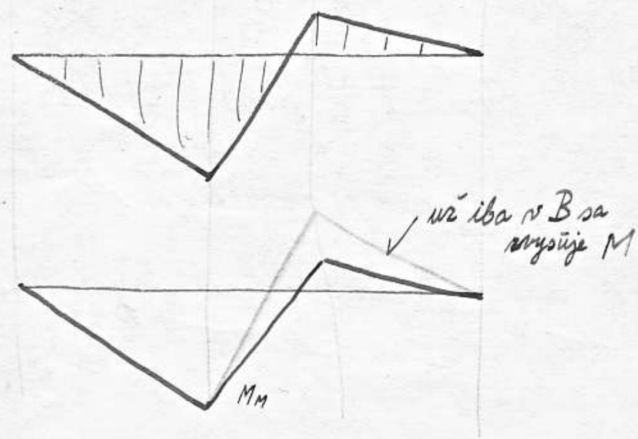


← vs. elast. namáhanie

↓ zvyšov. zaťaženia

ak ďalej zvyš. zaťaž.

— vyčerpaná vs. nosnosť nosníka.



$M_D = M_B = M_M = \sigma_K \cdot \bar{W}$

P_m sa vyvodí z priebehu moment.

$M_D = \frac{P_m \cdot l}{4} - \frac{M_M}{2} = M_M$ z obr. *

$P_M = \frac{6 M_M}{l} = \frac{6 \sigma_K \cdot \bar{W}}{l}$

$P_{M\,dov} = \frac{6 M_M}{l} = \frac{6 \sigma_K \cdot \bar{W}}{l} = \frac{P_m}{k}$

ak

M_{ek} — ak 1. vlákno je na medzi klzu.

$M_{ek} = \sigma_K \cdot W$

$\frac{13}{64} P_{ek} l = \sigma_K \cdot W \rightarrow P_{ek} = \frac{64}{13 l} \sigma_K \cdot W$ ~ sila, ktorá vyvodí stav napätosti σ_{ek}.

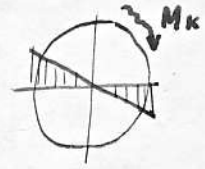
$P_{D_1} = \frac{P_{ek}}{k}$

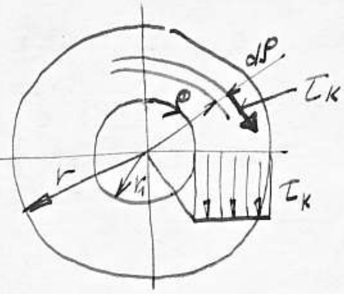
$\frac{P_M}{P_{ek}} = \frac{6 \sigma_K \bar{W}}{l} \cdot \frac{13 l}{64 \sigma_K W} = 1{,}21 \frac{\bar{W}}{W}$

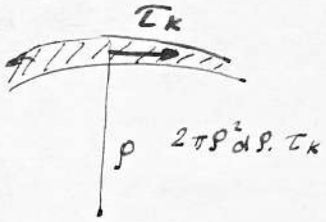
$\bar{W} = 1{,}5 W$ (elast. st.)

$1{,}21 \cdot 1{,}5 = 1{,}82 \Rightarrow$ medzná sila.

Pri kruhu:







pre elast.:

$M_K = W_K . \tau$ $\quad W_K = \frac{1}{2}\pi r^3$ $\boxed{\text{z pol. mom. rotr.}}$

$M_{eK} = W_K . \tau_K$

$M_{ep} = \frac{1}{2}\pi r_1^3 \tau_K + \int_{r_1}^{r} 2\pi\rho^2 d\rho . \tau_K =$

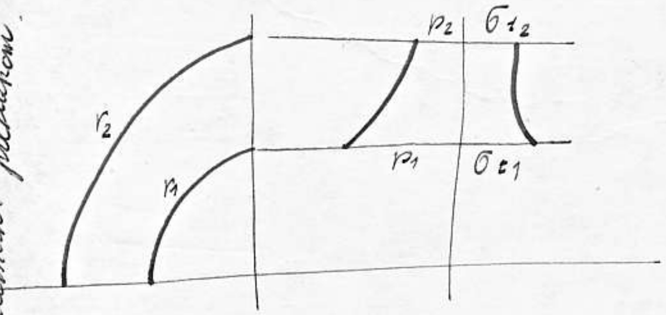
$= \frac{1}{2}\pi r_1^3 \tau_K + 2\pi\tau_K \int_{r_1}^{r} \rho^2 d\rho = \frac{1}{2}\pi r_1^3 \tau_K + \frac{2}{3}\pi\tau_K (r^3 - r_1^3).$

$M_M = M_{ep} = \frac{\pi \tau_K . r^3}{2.3}\left[4 - \left(\frac{r_1}{r}\right)^3\right]$

$M_{ep} = M_{eK} . \frac{1}{3}\left[4 - \left(\frac{r_1}{r}\right)^3\right]$

ak $r_1 = 0$ $\quad M_M = \frac{4}{3} M_{eK}.$

Hrubost. nádoby, namáh. pretlakom.



$\sigma_1 - \sigma_3 = \sigma_K$

$\sigma_1 = \sigma_{t_1} \qquad \sigma_3 = -p_1$

nastane potom plastický stav.

$\underbrace{\sigma_{t_1} + p_1}_{\sigma_K \text{ elast.}} = \sigma_K$ $\quad$ plast. stav v krajn. vlákne.

keď $p_1 = p_K$ nastane stav plasticity.

$\sigma_{t_1} = \frac{p_1 r_1^2 - p_2 r_2^2}{r_2^2 - r_1^2} + \frac{(p_1 - p_2) r_1^2}{r_2^2 - r_1^2}$

$r = r_1$:

$\sigma_{r_1} = \frac{p_1 r_1^2 - p_2 r_2^2}{r_2^2 - r_1^2} - \frac{(p_1 - p_2) r_1^2}{r_2^2 - r_1^2}$ $\qquad$ pri tom tlaku $p_1 = p_K$

$\sigma_{t_1} - \sigma_{r_1} = 2(p_1 - p_2)\frac{r_2^2}{r_2^2 - r_1^2} = \sigma_K$

$p_K - p_2 = \frac{\sigma_K}{2} . \frac{r_2^2 - r_1^2}{r_2^2} = \frac{\sigma_K}{2}\left[1 - \frac{r_1^2}{r_2^2}\right]$

Ak zvyšujeme pretlak p_K, hľadáme medzný pretlak, pri ktorom bude celá nom. v plast. oblasti.

$\sigma_t - \sigma_r = \sigma_K$

$\sigma_r - \sigma_t + \frac{d\sigma_r}{dr} . r = 0$

$\sigma_t - \sigma_r = \frac{d\sigma_r}{dr} . r$

$\frac{d\sigma_r}{dr} . r = \sigma_K$

$d\sigma_r = \sigma_K . \frac{dr}{r}$

$r = r_1 \quad \sigma_r = p_{1m} \sim$ medzný pretlak.

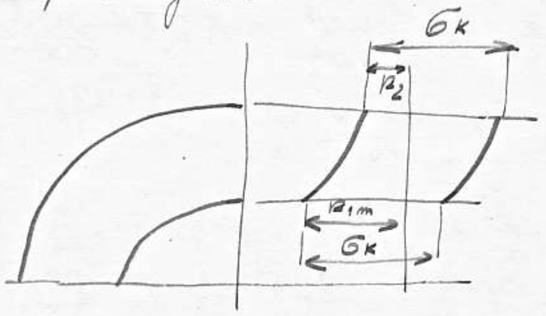
$$\int_{p_{1m}}^{\sigma_r} d\sigma_r = \sigma_k \int_{r_1}^{r} \frac{dr}{r}$$

$$\sigma_r - p_{1m} = \sigma_k \cdot \ln \frac{r}{r_1}$$

$$\sigma_t = p_{1m} + \sigma_k \cdot \ln \frac{r}{r_1}$$

r?

plastický stav



ak bude taký pretlak

$$p_{1M} = p_2 + \sigma_k \cdot \ln \frac{r_2}{r_1}$$

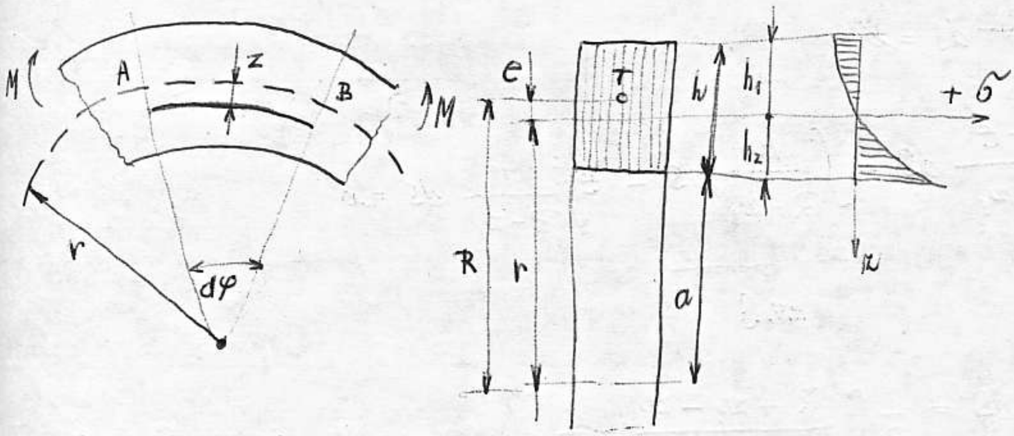
$p_2 = 0 \qquad r_2 = 2r_1$

$$p_{1M} = \sigma_k \cdot \ln \frac{r_2}{r_1}$$

$$p_{1K} = \frac{\sigma_k}{2} \left[1 - \left(\frac{r_1}{r_2} \right)^2 \right] \qquad \frac{p_{1M}}{p_{1t}} = 1{,}85.$$

Krivé prúty.

1) Rovinný ohyb:



Predp.: rovinné prierezy i po def. sú rovinné a ⊥ ku strednici.

Pomerné predĺženie vlákna „z":

$$\varepsilon = \frac{z \cdot \Delta d\varphi}{(r - z) \cdot d\varphi}$$

Napätie: $\sigma = E \cdot \varepsilon = E \, \frac{z}{r - z} \cdot \frac{\Delta d\varphi}{d\varphi}$ ⟶ hyperbolický priebeh.

Nepôs. norm. sila ⇒ $\int_S \sigma \, dS = E \cdot \frac{\Delta d\varphi}{d\varphi} \int_S \frac{z}{r - z} \, dS = 0$

Polomer krivosti neutr. plochy:

$$\int_S \frac{z}{r - z} \, dS = \int_S \frac{r - \rho}{\rho} \, dS = 0 \;\Rightarrow\; r \int_S \frac{dS}{\rho} - S = 0 \;\Rightarrow\; r = \frac{S}{\int_S \frac{dS}{\rho}}$$

Rovnováha namáh. sil. účinkov:

$$M = \int_S z \cdot \sigma \, dS = E \, \frac{\Delta d\varphi}{d\varphi} \int \frac{z^2}{r - z} \, dS$$

$$\int \frac{z^2 - rz + rz}{r - z} \, dS = -\int z \, dS + r \underbrace{\int \frac{z}{r - z} \, dS}_{0} = -S \cdot z_T = S \cdot e$$

e − vzd. ťažiska prierezu od neutr. osi.

$$M = E \, \frac{\Delta d\varphi}{d\varphi} \cdot S e \;\Rightarrow\; \frac{\Delta d\varphi}{d\varphi} = \frac{M}{ESe}$$

Dosad. za napätie:

$$\sigma = E \cdot \varepsilon = E \, \frac{z}{r - z} \cdot \frac{\Delta d\varphi}{d\varphi} = E \cdot \frac{z}{r - z} \cdot \frac{M}{ESe} = \frac{M}{eS} \cdot \frac{z}{r - z} = \sigma$$

Pri meraní vzdialenosti „od ťažiska":

$$\varkappa_1 = \varkappa + e$$
$$R = r + e$$

$$\int_S \sigma \cdot dS = E\frac{\Delta d\varphi}{d\varphi}\int \frac{\varkappa}{r-\varkappa}\, dS = 0$$

$$\int \frac{\varkappa_1 - e}{R-\varkappa_1}\, dS = \underbrace{\int \frac{\varkappa_1}{R-\varkappa_1}\, dS}_{m.S} - e\int \frac{1}{R-\varkappa_1}\, dS = 0$$

$$\int \frac{1}{R-\varkappa_1}\, dS = \frac{1}{R}\int_S \frac{R+\varkappa_1-\varkappa_1}{R-\varkappa_1}\, dS = \frac{1}{R}\left[\int_S dS + \int_S \frac{\varkappa_1}{R-\varkappa_1}\, dS\right] = \frac{S}{R}\left[1+m\right]$$

$$\int \frac{\varkappa_1 - e}{R-\varkappa_1} = m.S - e\,\frac{S}{R}(1+m) = 0$$

$$e = \frac{m}{1+m}R$$

$$M = \int_S \varkappa.\sigma.dS = E\frac{\Delta d\varphi}{d\varphi}\int \frac{\varkappa^2}{r-\varkappa}\, dS = E\frac{\Delta d\varphi}{d\varphi}\cdot Se \quad : \quad [\text{z predch. str.}]$$

$$\sigma = E.\varepsilon = E.\frac{\varkappa}{r-\varkappa}\cdot\frac{\Delta d\varphi}{d\varphi} = \frac{E\varkappa}{r-\varkappa}\,\frac{M}{ESe} = \frac{M}{e.S}\cdot\frac{\varkappa}{r-\varkappa}$$

$$\sigma = \frac{M}{\frac{mR}{1+m}\cdot S}\cdot\frac{\varkappa_1 - \frac{m}{1+m}R}{r-\left(\varkappa_1 - \frac{m}{1+m}R\right)} = \frac{M}{RS}\cdot\frac{(1+m)}{m}\cdot\frac{\varkappa_1 - \frac{m}{1+m}R}{R-\varkappa_1} = \frac{M}{RS}\,\frac{1+m}{m}\,\frac{\frac{(1+m)\varkappa_1 - mR}{1+m}}{R-\varkappa_1} =$$

$$= \frac{M}{RS}\left[\frac{\varkappa_1 - m(R-\varkappa_1)}{m.(R-\varkappa_1)}\right] = \frac{M}{mRS}\,\frac{\varkappa_1}{R-\varkappa_1} - \frac{M}{RS} = \frac{M}{RS}\left[\frac{1}{m}\cdot\frac{\varkappa_1}{R-\varkappa_1} - 1\right] = \sigma$$

$$\left.m.S = \int_S \frac{\varkappa_1}{R-\varkappa_1}\, dS = \int_S\left[\frac{\varkappa_1}{R}\cdot\left[1+\frac{\varkappa_1}{R}+\frac{\varkappa_1^2}{R^2}+\dots\right]\right] dS \quad\right\} \Rightarrow m$$

dá sa int. aj substitúciou

$S = b.h \qquad dS = b.d\varkappa$