Štvrťvlnový prispôs. člen

$Z_{vst.} = e^{bl} \cdot Z'_{vst}$ | Z'_{vst} ~ vstupný odpor jednorodej sústavy bez strát, s vlnovým odporom w_{0l}.

$Z'_{vst} = \frac{w_{0l}^2}{w_{02}}$ | w_{0l} ~ vln. odpor meniča ; $l = \frac{\lambda}{4}$
| w_{02} ~ vln. odpor druhého prispôs. člena.

↓

pre $\frac{1}{4}$ vln. expon. prispôs. člen: $Z_{vst.} = \frac{w_{0l}^2}{w_{02}} e^{bl}$
| $Z_{vst} = w_{01}$ ~ vln. odp. 1. člena

$\Rightarrow w_{0l} e^{\frac{b}{2}l} = \sqrt{w_{01} \cdot w_{02}}$

Dáva väčšie možnosti, ako „rovnorodý" prispôs. člen.

Polvlnný prispôs. člen

pre exp.: $Z_{vst.} = e^{bl} \cdot Z_{zát.}$

$$\frac{w_{01}}{w_{02}} = e^{bl} \rightarrow bl = \ln \frac{w_{01}}{w_{02}}$$

Koncentrátory

Výkony: $P_{vst.} = P_{výst.}$

$$\dot{\xi}_{m1}^2 \frac{R_{vst}}{2} = \dot{\xi}_{m2}^2 \frac{R_{zát}}{2}$$

Pre exp. prispôs.: $\frac{R_{vst}}{R_{zát}} = \frac{S_1}{S_2}$

$$\Rightarrow \frac{\dot{\xi}_{m2}}{\dot{\xi}_{m1}} = \frac{\xi_{m2}}{\xi_{m1}} = \sqrt{\frac{S_1}{S_2}} = \frac{F_{m1}}{F_{m2}}$$

Intenzita (výkon/1cm²) na úzkom konci: $I_2 = \frac{S_1}{S_2} I_1$

Pre kruhové prierezy: pretransformovaný odpor žiarenia — ku začiatku koncentrátora:

$R'_{ž} = R_{ž} \left(\frac{d_1}{d_2}\right)^2$ ~ nie je viazané s vlnovým odporom.

$l = \frac{v}{2f}\sqrt{1 + \left(\frac{\ln \frac{d_1}{d_2}}{\pi}\right)^2}$ $b = \frac{2}{l} \ln \frac{d_1}{d_2}$ $\ln \frac{d_1}{d_2} < l \frac{2\pi}{v} f \Rightarrow$ krit. fr. | $v = \sqrt{\frac{E}{\rho}}$

má byť splnené: $d_1 < \frac{\lambda}{2}$

Uzol kmitov: $x_0 = \frac{l}{\pi} \operatorname{arcctg}\left(\frac{1}{\pi} \ln \frac{d_1}{d_2}\right)$ | x_0 – od širš. konca.

15. XII. 75. : Zmeny na dokumentácii ur. kmit. sústavy – Vráble. :

1. – Radiátor M10x1SH8 → SH6
– oprava vely nad rohovou pec.
2. – matica : výška na 33, správny materiál (14260.)
4 – str. elektróda ; priemer $\phi 38{,}2$ (3,9) ; M3 – 4.
5 – reflektor : hr. príruby 7 ; spr. materiál (14 331.)
6 – skrutka SH8 → SH6
7 – medzičlánok SH8 → SH6
8 – nástroj SH8 → SH6
12 – skrutka – $l = 20$ (z $l = 22$)

Pri $\frac{d_1}{d_2} \geq 2{,}0$ je nutné pri nahradzovaní koncentrátora kužeľovým telesom, použ. iné vzorce.

Obecne, pri $l = n \cdot \frac{\lambda}{2}$:

rez. dĺžka: $l = \frac{v}{2f}\sqrt{\frac{(n\pi)^2 + \left(\ln\frac{d_1}{d_2}\right)^2}{\pi^2}}$

miesto uzlov: $x_0 = \frac{1}{\sqrt{\left(\frac{2\pi}{\lambda}\right)^2 - b^2}} \operatorname{arctg} \frac{\sqrt{\left(\frac{2\pi}{\lambda}\right)^2 - b^2}}{b}$

keď d_1 je relatívne veľké, ($\rightarrow 0{,}3 \div 0{,}4\,\lambda$) : $l = \frac{v}{2f}(1-\delta)\sqrt{\frac{(n\pi)^2 + \left(\ln\frac{d_1}{d_2}\right)^2}{\pi^2}}$

$\delta = \frac{n^2\pi^2\sigma^2}{8}\left(\frac{d_1}{2l}\right)^2 \frac{\left(\frac{d_1}{d_2}\right)^2 - 1}{\left(\frac{d_1}{d_2}\right)^2 \ln\frac{d_1}{d_2}}$ / σ – Poissonovo číslo

---

Kónický koncentr. ; $d_{(x)} = d_1\left(1 - \frac{d_1 - d_2}{d_1 l}\, x\right)$

Rez. dĺžka : $l = \frac{v}{f}\frac{(\alpha l)}{\pi}$ | (αl) je koreň rovnice: $\operatorname{tg}\alpha l = \frac{\alpha l}{1 + (\alpha l)^2 \frac{d_1 d_2}{(d_1 - d_2)^2}}$ / $\alpha = \frac{\omega}{v}$

Koef. zosilnenia konc-a : $\left|\frac{\xi_{m2}}{\xi_{m1}}\right| = \left|\frac{d_1}{d_2}\left(\cos\alpha l - \frac{d_1 - d_2}{\alpha l d_2}\sin\alpha l\right)\right|$

Uzol rýchlosti $x_0 = \frac{1}{\alpha}\operatorname{arctg}\alpha l\,\frac{d_1}{d_1 - d_2}$

Katenoidálny koncentr. : $d_x = d_2 \operatorname{ch}\gamma(l - x)$ / $\gamma = \frac{1}{l}\operatorname{arch}\frac{d_1}{d_2}$

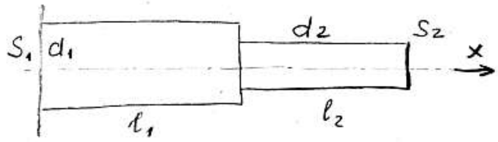
Rovnica frekvencií : $\alpha' l\,\operatorname{tg}\alpha' l = -\sqrt{1 - \left(\frac{d_2}{d_1}\right)^2}\operatorname{arch}\frac{d_1}{d_2}$ / $\alpha' = \sqrt{\alpha^2 - \gamma^2}$

Rez. dĺžka : $l = \frac{v}{2f}\sqrt{\frac{(\alpha' l)^2 + \left(\operatorname{arch} d_1/d_2\right)^2}{\pi^2}}$ / (αl) ⟵

Koef. zos-ia : $\left|\frac{\xi_{m2}}{\xi_{m1}}\right| = \left|\frac{d_1}{d_2 \cos\alpha' l}\right|$ ⟹ efektívnejšie transf. rýchl. a kmitov, než expon.

Uzol rýchlosti : $x_0 = \frac{1}{\alpha'}\operatorname{arctg}\left[\frac{\alpha'}{\gamma}\operatorname{cth}\gamma l\right]$.

Stupňovité koncentrátory.



$$\frac{tg\,\alpha l_1}{tg\,\alpha l_2} = \frac{S_2}{S_1}$$

$$\alpha l_1 = \frac{\omega}{v} l_1$$

$$l_2 = \frac{v}{\omega}\,\mathrm{arc\,tg}\left(\frac{S_1}{S_2}\, tg\,\frac{\omega}{v} l_1\right)$$

Koef. zos.: $\dfrac{\dot{\xi}_{m2}}{\dot{\xi}_{m1}} = \dfrac{S_1}{S_2}\,\dfrac{\sin\alpha l_1}{\sin\alpha l_2}$ | max. – pri $l_1 = l_2 = l$; | $l = \frac{v}{4f}$

Uzol kmitov – v prechode priemerov.

Opory: nesú (celkom, čiastočne) váhu kmit. sústavy.

Reaktívne: (hmota, pružnosť) – pozmenia rezonančné a kmitavé vlastnosti

Aktívne: – prakticky nezmení vlastnosti km. sústavy.

Stojaté – pracujú na tlačenie

Podvesné – na ťah.

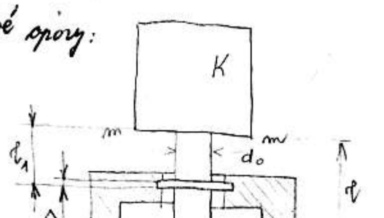
Uchytenia – spojenie opory s kmit. sústavou. – praktický význam majú aktívne. (môžu byť pružné)

Spojenia – kvalit. akust. kontakt, pružnosť. (Minim.-odraz, -straty.)

Pevné: pájkované, zvárané, skrutkované, klinované, ...

Kvapalinové:

Polvlnové opory:



M - doplňuje akust. dĺžku do $\frac{\lambda}{2}$.

Sústreď. hmota, keď: $l_2 \le (0{,}1 \div 0{,}125)\lambda$

$d \le 1{,}2\, d_0$

rez. dĺžka: $l = \frac{v}{\omega}\left(\mathrm{arctg}\,\frac{w_0}{X_M} + \frac{\pi}{2}\right)$

w – vln. odpor tyče

$X_M = \omega M$ ~ inerc. odpor hmoty M.

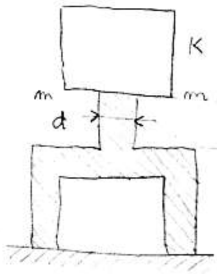
uzol kmitov:

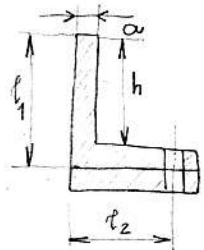
$$l_1 = l - \frac{v}{\omega}\,\mathrm{arctg}\,\frac{w_0}{\omega M}$$

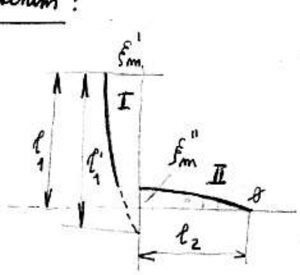
$$\Delta \le 0{,}05\, l$$

Štvrťvlnové opory: – aktívne.

– s nulovým uchytením:







– zaťaž. na I ~ pozdĺžne kmity
II ~ ohybové km.

Chýbajúca časť doplňuje súčiastku I na $\frac{\lambda_1}{4}$. Reaktívne zaťaženie tejto časti s dĺžkou Δl:

$$X_{vst} = X_{z1} = -iw_{01}\,\mathrm{ctg}\frac{2\pi}{\lambda_1}\Delta l_1 \qquad / \; w_{01} \text{ – vln. odpor časti I.} = w_1 S$$

„$\cos\frac{\pi}{\frac{\lambda}{2}}\cdot l = \xi$"

$$X_{vst.2} = (\text{má byť}) = X_{z1}$$

Pre $\frac{\xi''_m}{\xi'_m} = k < 1$ (0,1÷0,2) $\Rightarrow$ $k = \cos\frac{2\pi}{\lambda_1} l_1 = \cos\frac{\omega}{v_1} l_1$ / v – rýchl. ší. vĺn v I

„1 ="

$\lambda = \frac{c2\pi}{\omega}$

$$l_1 = \frac{v_1}{\omega}\arccos k \qquad / \; l_1' = \frac{v}{4f} = \frac{\lambda}{4}$$

$$\Delta l = l_1' - l_1 = \frac{v}{4f}\left[1 - \frac{2\arccos k}{\pi}\right] \checkmark$$

$$\Delta l \approx (0{,}065 \div 0{,}175)\lambda_1 \rightarrow$$

$$X_{z1} = -i(4{,}06 \div 9{,}78)w_0$$

Vst. odpor horizont. opory: $X_{vst.2} \approx -i\frac{3EJ}{\omega l_2^3} =$] $\rightarrow$ $l_2 = \sqrt[3]{\frac{3EJ}{(4{,}06 \div 9{,}78)\omega w_0}}$

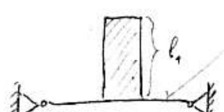
[kg/s]

tuhosť pružiny

– s pružným uchytením

Ako v predch. prípade, vyhľadáme $X_{vst.2}$ ..... $= \frac{D}{i\omega}$ .... $D = -w_{01}\omega\,\mathrm{ctg}\frac{\omega}{v}\Delta l$

↑ sústredená „pružnosť"
(pre malé prekr. – aj pružina)



$$\left[X = -i\frac{„k"}{\omega} \quad / \; „k" \sim \text{tuhosť} = \frac{\text{sila}}{\text{priehyb}}\right]$$

Uzlové opory – ich elementy majú „nekritické" rozmery. Sú viazané v uzloch.

Zmena tlaku na keramiku v dôsl. tep. rozťažnosti.

(podrobne na papieroch.)

$$\lambda_1 F_{prac} + \lambda_{1\Delta^1 t} - \lambda_1 F_{predp} \doteq \lambda_2 F_{predp} + \lambda_{2\Delta^2 t} - \lambda_2 F_{prac} \Rightarrow$$

$$F_{prac.} = F_{predp} + \frac{(\alpha_e l_e + \alpha_K 2 l_K)({}^2t_2 - {}^2t_1) - \alpha_1 (l_e + 2 l_K)({}^1t_2 - {}^1t_1)}{\frac{1}{E_1}\left(\frac{l_1}{S_1}\right)^* + \frac{1}{S_2}\left(\frac{l_e}{E_e} + \frac{2 l_K}{E_K}\right)}$$

Pevnosť lepeného spoja: keramika – elektróda.

Tra-bond 2211 : $\sigma_{Pt'} = 8000\ psi$

Al-Al 75°C ...... 25000 psi
125°C ..... 2000 psi
165°C ..... 1050 psi

$\Rightarrow$ zvol: 2000 psi $\triangleq$ 140 kp/cm²

$F = p.S = 140.(11,3\ cm^2) = 1580\ kp$ ~ ťahová sila

Obvodové skrutky: (keramika ⌀38.)

Predp: $\sigma_{Pt'\,dyn\,ker.} = 350\ kp/cm^2$, $S = 11,3\ cm^2$ $\Rightarrow$ $F_{k+} = 11,3 \cdot 350 = 3980\ kp$

$F_{predp} = 4030\ kp$ (opt.: 3915 kp)

$\Rightarrow$ max. A = 4030 + 3980 = 8010 kp (v ker.)

Na zákl. výpočt. a ex. zhodnôt zanedb. zmeny predpätia vypl. z ohr. meniča.

Z výpočtov matice: $\Delta l_{vnút} \doteq F.10^{-7}.20,68 = F.\ 0,2068.10^{-5}\ mm/kp$

(Cu elektr.)

Zvol. mat. skr. 14 260.7 ... $\sigma_K = 120\ kp/mm^2$

I. Zvol. $d_{skr} = 7\ mm$ / 3x ~ predp. účinná dĺžka skrutiek 25 mm

$$\Delta l_{3skr} \equiv \Delta l_{vonk.} = F \cdot \frac{25\ mm}{215.10^4 \frac{kp}{mm^2} \cdot 3 \cdot 3,5^2 \pi\ mm^2} = F \frac{25}{249 \cdot 10^4} \frac{mm}{kp}$$

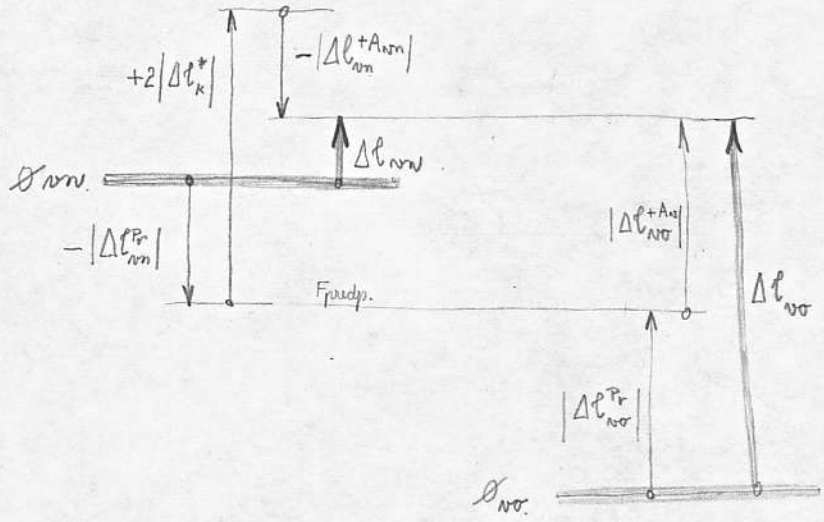
$$\Delta l_{vo} \doteq F.10^{-5}\ mm/kp$$

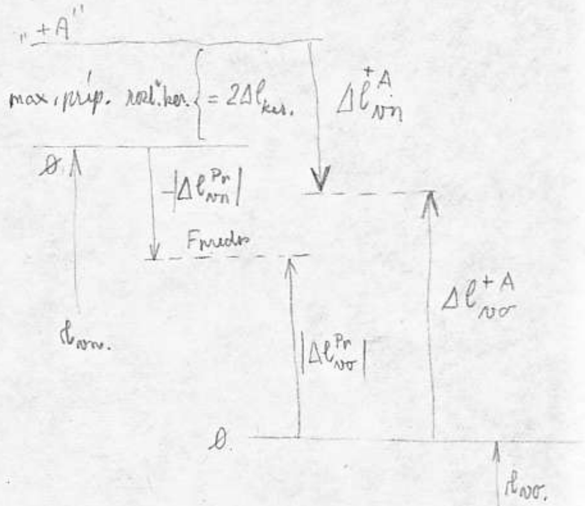
Zv. amplitúdu zaťaž. sily spoja: $A = F_{predp} + F_{lep.spoja} = 4030 + 1580 = 5610\ kp$

[$\Rightarrow$ ale konštr. - preveď, aby skr-y prebôli nam. na vzper!]

Určenie max. prípustných síl; pri $F_p = 4030\,kp$

1, Ker. „sa rozťahuje":





Poč. podľa 2. obr.:

Pri predp. $\sigma_{ker\,Pť\,dyn} = 350\,kp/cm^2$ ..... $_{max}F''_{ker} = 3980\,kp$

Tomu zodp. $2\Delta l_{ker} = 2\dfrac{_{max}F_{ker} \cdot l_K}{E_K\,S_K} = 2\,\dfrac{3980\,kp \cdot 6\,mm}{8.10^3\frac{kp}{mm^2} \cdot 1130\,mm^2} = 1{,}5 \cdot 3{,}52 \cdot 10^{-3}\,mm = 5{,}28\,\mu m$

$\Delta l_{vn}^{Pr} \doteq F_{pr} \cdot 0{,}2068 \cdot 10^{-5}\,mm = 4030 \cdot 0{,}2068 \cdot 10^{-5}\,mm \doteq 8{,}35\,\mu m$

$\Delta l_{vo}^{Pr} = F_{pr} \cdot 10^{-5}\,mm = 4030 \cdot 10^{-5}\,mm = 40{,}3\,\mu m$

$\Delta l_{vo}^{+A} + \Delta l_{vn}^{+A} = 2\Delta l_{ker\,príp.} + (\Delta l_{vn} + \Delta l_{vo})_{pr} = 5{,}28 + 8{,}35 + 40{,}3 = 53{,}93\,\mu m$

$F_{horné}^{+A}\left(0{,}2068 \cdot 10^{-5} + 10^{-5}\right) = 53{,}93 \cdot 10^{-3}\,mm \implies F_{horné}^{+A} = \dfrac{53{,}93 \cdot 10^{-3}}{1{,}2068 \cdot 10^{-5}} = \mathbf{4470\,kp}$ (len pre lieto! rozmery skrutiek)

2, Ker „sa sťahuje": [u skr. neprip. namáh. na vzper ($\Rightarrow$ lep. spoje nie sú nam. na ťah) $\sigma_{dyn\,d\,ker} > \sigma_{dyn\,ť\,ker}$]

Pri súmern. el. sign., predpokl. $\Delta l_{ker\,sť} \approx \Delta l_{ker\,rozť} \Rightarrow$

$\Delta l_{vo}^{-A} + \Delta l_{vn}^{-A} = (\underset{40{,}3}{\Delta l_{vo}} + \underset{8{,}35}{\Delta l_{vn}})_{pr} - \underbrace{2\Delta l_{ker}}_{5{,}28\,\mu m} = 48{,}65 - 5{,}28 = 43{,}37\,\mu m$

$F_{„h"}^{-A} = \dfrac{43{,}37 \cdot 10^{-3}\,mm}{1{,}2068 \cdot 10^{-5}\,mm/kp} = 3590\,kp$

Sily v 1 skr d_{skr} 7; $l_{úč.} = 25$: $F_h = \dfrac{4470}{3} = 1490\,kp$, $F_m = \dfrac{3590}{3} = 1190\,kp$ $\Big\}$ $F_m = \dfrac{1490 + 1190}{2} = \dfrac{2680}{2} = 1340\,kp$, $F_a = \dfrac{1490 - 1190}{2} = \dfrac{300}{2} = 150\,kp$

$\left(F_p = \dfrac{4030}{3} = 1343\,kp\right)$

$S_{skr} = 3{,}5^2\pi = 38{,}5\,mm^2$

→

$$\sigma_m = \frac{1340}{38{,}5} = 34{,}8\ kp/mm^2$$

$$\sigma_a = \frac{150}{38{,}5} = 3{,}9\ kp/mm^2$$

σ_{Pt} 14260.7 = min 145 kp/mm²

Z Heywoodových vz.:

predp. do živ.: 10^{11} cykl. $= N$

$$m = \log N = 11 \qquad A_0 = \frac{1+0{,}0038 \cdot 11^4}{1+0{,}008 \cdot 11^4} = \frac{1+0{,}0038 \cdot 1{,}46 \cdot 10^4}{1+0{,}008 \cdot 1{,}46 \cdot 10^4} = \frac{1+55{,}5}{1+117} = 0{,}479$$

$$\gamma = \left(2+\frac{34{,}8}{145}\right)\frac{34{,}8}{3 \cdot 145} = 2{,}24 \cdot 0{,}08 = 0{,}179$$

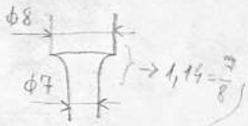
$$\sigma_{a\,vyp} = 145\left[1-\frac{34{,}8}{145}\right]\cdot\left[0{,}479+0{,}179(1-0{,}479)\right] =$$

$$= 145\left[1-0{,}23\right]\cdot\left[0{,}479+0{,}179 \cdot 0{,}521\right] = 145 \cdot 0{,}77(0{,}479+0{,}0931) = 112 \cdot 0{,}5721 = 64\ \frac{kp}{mm^2}$$

$\sigma_{a\,vyp} \gg \sigma_a \Rightarrow$ predimenz. skrutky, ale ešte nepoč. s vrubmi!

$$\left(\sigma_h = \frac{1490}{38{,}5} = 38{,}7\ kp/mm^2 \quad \ldots\ldots \sigma_k = 120\ kp/mm^2\right)$$

Iný výp.:



$\phi 8$

$\phi 7$

$\rightarrow 1{,}14 = \frac{7}{8}$

$\frac{r}{d} = \frac{0{,}5}{7} = 0{,}07$

$\alpha \approx 1{,}7$

$$\sigma_{ct} \approx 0{,}47\,\sigma_{Pt} = 0{,}47 \cdot 145 = 68{,}2\ kp/mm^2$$

$$\sigma_c^* = \frac{\sigma_{ct}\,\nu\,\eta}{\beta} = \frac{68{,}2 \cdot 1{,}7 \cdot 0{,}8}{1{,}51} \approx 61\ kp/mm^2$$

Černoch 476 (ohyb!) ? mam 1,7 .... 1,0 !

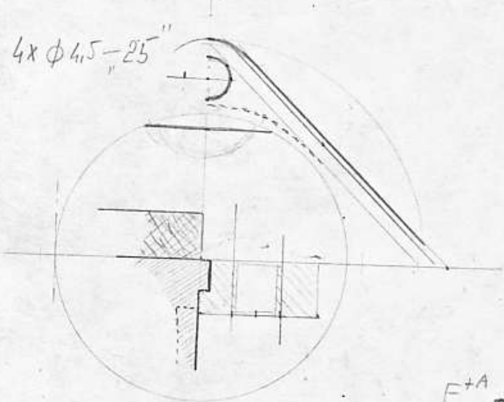
$k \approx \frac{28}{145} \doteq 0{,}2$

$\rho \approx 0{,}5$

$$\beta = \frac{1{,}7}{1+\frac{1{,}7-1}{1{,}7}\cdot\frac{0{,}2}{\sqrt{0{,}5}}} = \frac{1{,}7}{1+0{,}41 \cdot 0{,}3} = \frac{1{,}7}{1{,}123} = 1{,}51$$

predimenz. skr.

II. $4 \times \phi 4{,}5 - 25''$



$$\Delta l^{Tr}_{mul} \approx 8{,}35\ \mu m \qquad 2\Delta l_{ker} = 5{,}28\ \mu m$$

$$\Delta l^{Pr}_{Nr} = \frac{4030\,kp \cdot 25\,mm}{2{,}1 \cdot 10^4 \frac{kp}{mm^2} \cdot 4 \cdot \pi\, 2{,}25^2\, mm^2} = \frac{4030 \cdot 25}{10^4 \cdot 26{,}4 \cdot 707} = 1527{,}354 \cdot 10^{-4} =$$

$$= 540 \cdot 10^{-4}\,mm \doteq 54\ \mu m$$

$$\Delta l^{+A}_{Nr} + \Delta l^{+A}_{m} = 5{,}28 + 8{,}35 + 54 = 67{,}63\ \mu m$$

$$F^{+A} \cdot \left(0{,}2068 \cdot 10^{-5} + \frac{5400 \cdot 10^{-5}}{4030}\right) = F^{+A} \cdot 10^{-5}\left(0{,}2068 + 1{,}34\right)_{mm} \doteq F^{+A} \cdot 10^{-2} \cdot 1{,}5468\ \mu m$$

$$F^{+A} = \frac{67{,}63 \cdot 100}{1{,}5468} \doteq 4380\ kp$$

$$\Delta l^{Pr}_{Nr} + \Delta l^{Pr}_{m} - 2\Delta l_{ker} = 62{,}35 - 5{,}28 = 57{,}07\ \mu m$$

$$F^{-A} = \frac{57{,}07 \cdot 10^2}{1{,}5468} \doteq \underline{3700\ kp}$$

$$\sigma_h = \frac{4380:4\ kp}{2{,}25^2\,\pi\ mm^2} = \frac{1095}{15{,}9} = 68{,}8\ kp/mm^2$$

$$\sigma_n = \frac{3700:4}{2{,}25^2\,\pi} = \frac{925}{15{,}9} = 58{,}2\ kp/mm^2$$

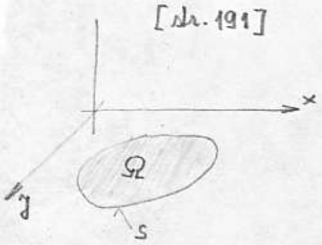
$$\sigma_m = \frac{58{,}2+68{,}8}{2} = \frac{127}{2} = 63{,}5\ kp/mm^2$$

$$\sigma_a = \frac{68{,}8-58{,}2}{2} = \frac{10{,}6}{2} = 5{,}3\ kp/mm^2$$

KMITY MEMBRÁN

Rovnica priečnych kmitov membrán (bez vonk. síl): $\Delta U = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = \frac{1}{a^2}\frac{\partial^2 U}{\partial t^2}$

[str. 191]



Vlastné kmity: $U(x,y,t) = u(x,y)\cos\omega t$, alebo $U(x,y,t) = u(x,y)\sin\omega t$

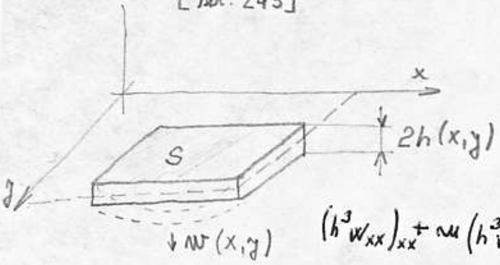
Votknutý okraj: $U\big|_s = 0$

$\rightarrow$ integrovanie rovnice $\Delta u + \lambda u = 0$ | $\lambda = \frac{\omega^2}{a^2}$

$u(x,y) \not\equiv 0 \rightarrow$ hľadanie vlastných čísiel, a vl. f-ií operátora Δ.

Vlastné frekvencie dosky s odrezným okrajom (hrúbka na okraji $\rightarrow 0$)

[str. 243]



(Pot. energ. def. dosky.
$W = \frac{E}{24(1-\mu)^2}\iint_S h^3\left[w_{xx}^2 + 2\mu\, w_{xx}w_{yy} + w_{yy}^2 + 2(1-\mu)w_{xy}^2\right]dx\,dy$)

mesúhl. s: Gonda: kmit pružn. telies! –

Rovnica kmitania dosky:

$$(h^3 w_{xx})_{xx} + \mu(h^3 w_{yy})_{xx} + \mu(h^3 w_{xx})_{yy} + (h^3 w_{yy})_{yy} + 2(1-\mu)(h^3 w_{xy})_{xy} = -\frac{12(1-\mu^2)\gamma}{E}h\frac{\partial^2 w}{\partial t^2}$$

γ – merná hm. mat-u dosky ; „index značí deriváciu".

$w = u(x,y)\,e^{i\omega t}$ $\quad\omega$ ~ fr. vl. kmitov.

Rovnica pre amplitúdu:

$$(h^3 u_{xx})_{xx} + \mu(h^3 u_{yy})_{xx} + \mu(h^3 u_{xx})_{yy} + (h^3 u_{yy})_{yy} + 2(1-\mu)(h^3 u_{xy})_{xy} = \lambda h u$$

$$\lambda = 12(1-\mu^2)\gamma\omega^2 E^{-1}$$

okraj. podm.: $u\big|_{L''} = \frac{\partial u}{\partial \nu}\Big|_{L''} = 0$

$u\big|_{L'} = 0 \quad 0 < \alpha < \frac{2}{3}$

$\frac{\partial u}{\partial \nu}\Big|_{L'} = 0 \quad 0 < \alpha < \frac{1}{3}$

$\Big\}\ \mathfrak{I}(u) - \lambda h u = 0$

Vlastné kmity pružných telies – všeobecná mat. teória.

[str. 247]

Vlastné čísla pre štvorcovú dosku, strana π, votknuté okraje. 13,28 ; 55,24 ; 120,00 ; 177,67 ; 178,3 ; (Weinstein)

[str. 336]

Vlastné kmity prútov premenného prierezu ; Radiálne vl. km-y pruž. valca [str. 385]

Kmity pružnej pravouhlej dosky v jej rovine. [str. 386]

↑ S. G. Michlin: Variačné metódy v matematickej fyzike, Alfa (Bratisl.) 1975.

Rovn. prieh. membrány: $-\Delta w = \frac{q}{T}$ $\quad u$ ~ prieh. $\quad T$ ~ napnutie $\quad \Delta$ – operátor $\quad q$ – priečn. zaťaž.

ohyb dosky pod nespoj. mem. sa priečn. zať.:

$$\frac{q(x,y)}{D} = \Delta^2 w = \frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}$$

Základné frekvencie membrány

$$n = \frac{\mathcal{L}}{2\pi}\sqrt{gS/Fq}$$

$\mathcal{L}$ – kruh	4,261	rovnostr. Δ	4,774
štvorec	4,443	polkruh	4,803
kruh. kvadrant	4,551	obdĺ. 2×1	4,967
60° kruh. výseč	4,616	obdĺ. 3×1	5,736
obdĺžnik 3×2 :	4,624		

S ~ rovnomerné napätie na jednotku dĺžky okraja
q ~ váha plošnej jednotky membrány
F ~ povrch membrány
g ~ gravit. zrýchl.

frekvencie vyšších rádov :

obdĺžnik a×b $\qquad n_{mi} = \frac{1}{2}\sqrt{\frac{gS}{q}\left(\frac{m^2}{a^2}+\frac{i^2}{b^2}\right)}$ $\qquad m = 1,2,3\ldots \quad i = 1,2,3\ldots$ ✓

kruh $\qquad n_{m,i} = \left(\mathcal{L}_{m,i}/2\pi r\right)\sqrt{gS/q}$

m ~ počet uzlových priemerov
i ~ počet uzl. kruhov, aj s okrajom dosky

$\mathcal{L}_{m,i}$:

i \ m	J_0	J_1	J_2	J_3	4	5
1	2,405	3,832	5,135	6,38 (79)	7,59	8,78
2	5,520	7,02 (16)	8,42 (17)	9,760	11,06	12,3
3	8,65 (564)	10,173	11,620	13,02 (17)	14,4	15,7
4	11,8 (792)	13,3	14,8	16,2	17,6	19,0
5	14,9	16,5	18,0	19,4	20,8	22,2
6	18,1	19,6	21,1	22,6	24,0	25,4
7	21,2	22,8	24,3	25,7	27,2	28,6
8	24,4	25,9	27,4	28,9	30,4	31,8

Kmitanie dosky

$(h \ll a, b)$, Tuhosť v ohybu: $D = Eh^3/12(1-\mu^2)$

μ ~ Poiss. konšt. $\quad \gamma$ ~ merná váha $\quad g$ ~ gravit. zrýchl. $\quad h$ – hrúbka dosky

a×b ..... oprelé na okrajoch : $\quad n_{m,i} = \frac{\pi}{2}\sqrt{\frac{gD}{\gamma h}}\left(\frac{m^2}{a^2}+\frac{i^2}{b^2}\right)$ $\quad m; i = 1,2,3\ldots$ ~ norm. frekv. (nie kruhová)

a×a .... zákl : $n = \frac{\pi}{a^2}\sqrt{gD/\gamma h}$

vyššie (s voľným okrajom) $\quad f_i = \left(\alpha_i/2\pi a^2\right)\sqrt{gD/\gamma h}$ / $\alpha_1 = 14{,}10 \quad \alpha_2 = 20{,}56 \quad \alpha_3 = 23{,}91$

Kruh ..... $n_i = \left(\alpha_i/2\pi r^2\right)\sqrt{gD/\gamma h}$

upnutý okraj

i	$m=0$	1	2
0	10,21	21,22	34,84
1	39,78	–	–
2	88,90	–	–

voľný okraj

	$m=0$	1	2	3
	–	–	5,251	12,23
$i=1$	9,076	20,52	5,24	52,91
$i=2$	38,52	59,86	–	–

oprelé uprostred : $\alpha_{i=0} = 3{,}75 \quad \alpha_2 = 20{,}91 \quad \alpha_3 = 119{,}7$

Dobrovoľný ... strojn. príručka.

Braddick : Vibrations, waves and diffraction : Stojaté vlnenie v 3 rozm. priest.

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} \quad \ldots \phi = \exp(\pm 2\pi i\kappa x)\exp(\pm 2\pi i\lambda y)\exp(\pm 2\pi i\mu z)\exp(\pm 2\pi i\nu t) \quad / \ \nu^2 = c^2(\kappa^2+\lambda^2+\mu^2)$$

pri $x = y = z = 0$ ... 0 ampl. $\rightarrow \phi = \sin 2\pi\kappa x \sin 2\pi\lambda y \sin 2\pi\mu z \exp \pm 2\pi i\nu t$

$x = a \quad y = b \quad z = c$ ... 0 ampl. $\rightarrow 2\pi\kappa a = k\pi \quad \rightarrow \kappa = \frac{k}{2a}$ ....

$$\nu_{k\ell m} = \frac{c}{2}\sqrt{\frac{k^2}{a^2}+\frac{\ell^2}{b^2}+\frac{m^2}{c^2}}$$

MENIČ „$\phi 50$"

$$c_1 = \lambda_{11} f_{11} = \lambda_{12} f_{12} \Rightarrow \lambda_{12} = \lambda_{11} \frac{f_{11}}{f_{12}} \Rightarrow l_{žiar.} = 65 \cdot \frac{20\,000}{18\,000} = 72{,}2 \text{ mm}$$

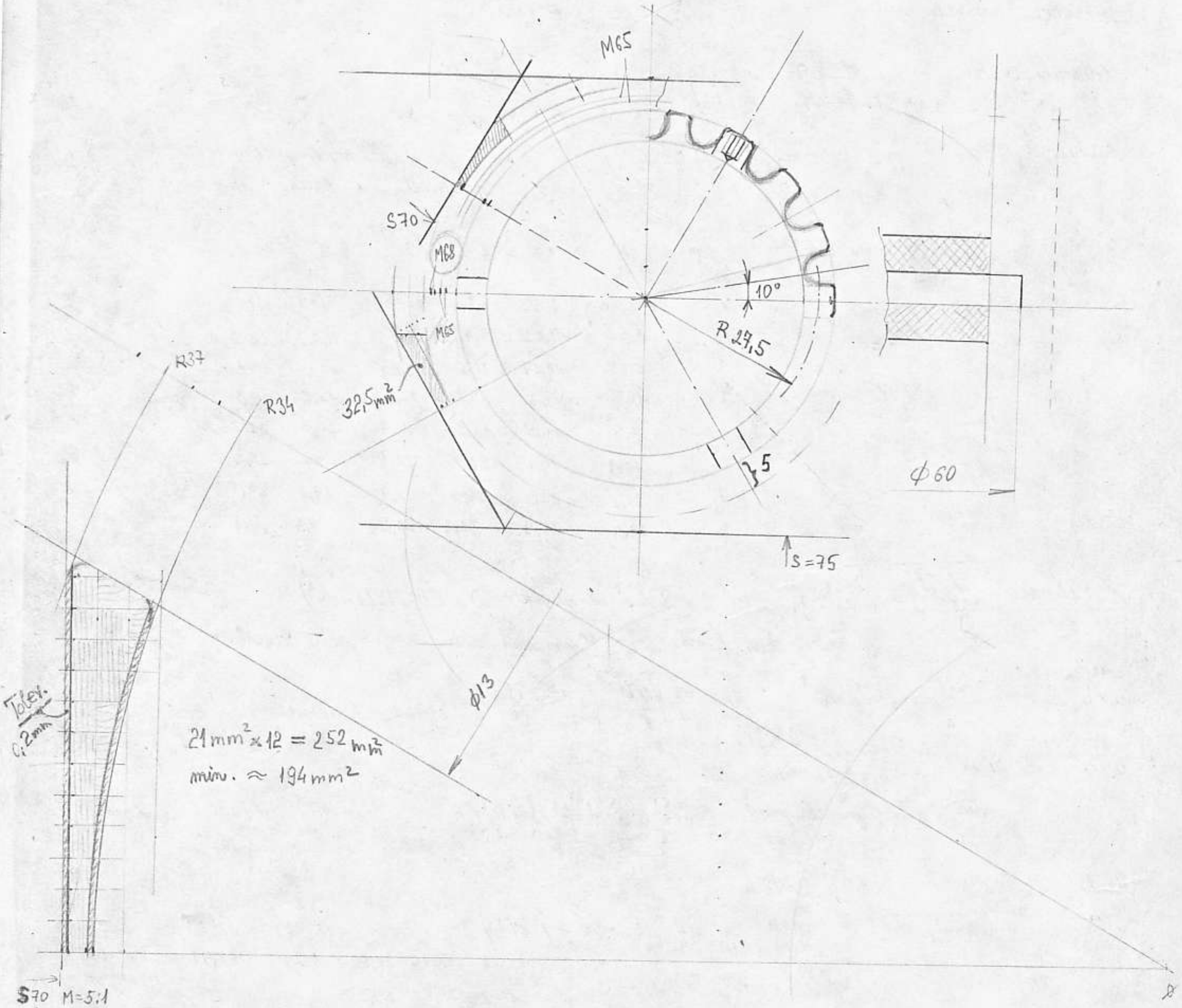
$$l_{odr.} = (117-65-17)\,\frac{20\,000}{18\,000} = 38{,}9 \text{ mm}$$

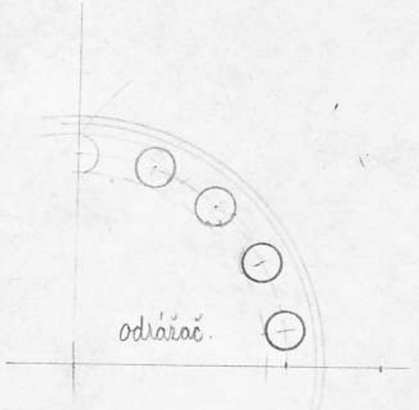
$$\Sigma l = 128{,}1 \text{ mm}$$

Predpätie keramiky ($\phi 50$):

$$S = r_k^2 \pi = 2{,}5^2 \pi = 19{,}62 \text{ cm}^2 \qquad p_{opt.} \approx 350 \text{ kp/cm}^2$$

$$F_{predp.} = S \cdot p_{opt.} = 19{,}62 \cdot 350 = 6880 \text{ kp}$$





oceľ 14 331 $\sim \sigma_{kt} \sim 85\,kp/mm^2$ $\quad \sigma_{oh\,10^7} \approx 48 \quad \tau_{\check{s}m\,10^7} \approx 28$ } z dil (vtah)

$$\Delta l_{Cu} = \frac{F.\,5\,mm}{12{,}5.10^3 \frac{kp}{mm^2} \cdot 1962\,mm^2} = F.10^{-6}\frac{5}{24{,}54} = F.2{,}04.10^{-7}\,mm\ [kp]$$

$$\Delta l_{2k} = \frac{F.\,12}{8.10^3.\,1962} = F.10^{-6}\frac{12}{15{,}7} = F.7{,}65.10^{-7}\,mm\ [kp]$$

$$\Delta l_{od.} \overset{predp.}{\approx} \frac{F.5}{2{,}1.10^4.\,1962} = F.10^{-7}\frac{5}{4{,}12} = F.1{,}21.10^{-7}\,mm\ [kp]$$

$$\Delta l_{\check{s}i} \approx \frac{F.\ 5}{7{,}22.10^3.\,3310} = F.10^{-6}\frac{5}{23{,}9} = F.2{,}09.10^{-7}\,mm\ [kp]$$

$$\Delta l_{vn} = F.(2{,}04 + 7{,}65 + 1{,}21 + 2{,}09).10^{-4}\,\mu m = F.12{,}99.10^{-4}\,\mu m\ [kp] = F.1{,}3.10^{-3}\,\mu m\ [kp]$$

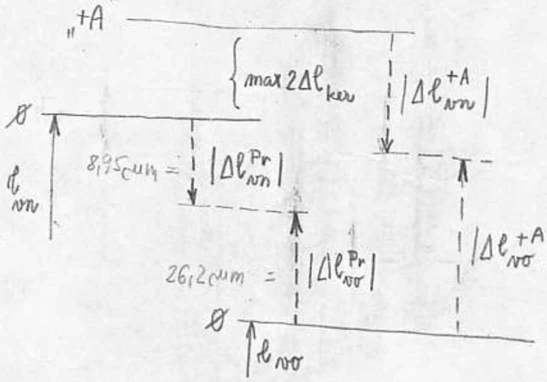
$$\Delta l_{vo} = \frac{F.[kp]}{2{,}1.10^4\left[\frac{kp}{mm^2}\right]} \text{"}\left(\frac{l}{S}\right)\text{"} = \frac{F}{2{,}1.10^4}\cdot\frac{8}{10^2} = 3{,}81\,F.10^{-6}\,mm\ [kp] \triangleq F.\,3{,}81.10^{-3}\,\mu m\ [kp]$$

odhad
$$\left\{\begin{array}{l} \text{"}S_{min}\text{"} = 252\,cm^2 \\ \text{"}l_{max}\text{"} = 5+6+5+6+4 = 26\,mm \\ \text{"}S_{max}\text{"} = 252+234 = 486\,mm^2 \\ \text{"}l_{min}\text{"} \doteq 2+6+5+6+3 = 22\,mm \end{array}\right.$$

$\frac{l}{S} = \frac{26\,mm}{252\,mm^2} = \underline{0{,}103\,\frac{1}{mm}}$ ... max. "$\left(\frac{l}{S}\right)$"

min "$\left(\frac{l}{S}\right)$" $= \frac{22}{486} = \underline{0{,}0452\,\frac{1}{mm}}$

uvaž. "$\left(\frac{l}{S}\right)$" $= \underline{0{,}08\,\frac{1}{mm}}$



Uvaž. σ_{Pt} dyn (dovol.) $= 350\,kp/cm^2$; $S_{ker} = 19{,}62\,cm^2$

$$\text{"}+A\text{"} = 350.\,19{,}62 = 6880\,kp \Rightarrow$$

$$2\Delta l_{ker} = 6880.\,7{,}65.10^{-4}\,\mu m = 5{,}26\,\mu m$$

$$\Delta l_{vn}^{Pr} = 6880.\,1{,}3.10^{-3}\,\mu m = 8{,}95\,\mu m$$

$$\Delta l_{vo}^{Pr} = 6880.\,3{,}81.10^{-3}\,\mu m = 26{,}2\,\mu m$$

$$\Delta l_{vn}^{+A} + \Delta l_{vo}^{+A} = 5{,}26 + 8{,}95 + 26{,}2 = \underline{40{,}41\,\mu m} = F^{+A}(1{,}3.10^{-3} + 3{,}81.10^{-3}) \Rightarrow$$

$$F^{+A} = \frac{40{,}41}{5{,}11}.10^3 = 7900\,kp$$

$$\Delta l_{vn}^{-A} + \Delta l_{vo}^{-A} = 26{,}2 + 8{,}95 - 5{,}26 = \underline{29{,}89\,\mu m} = F^{-A}(1{,}3.10^{-3} + 3{,}81.10^{-3})$$

$$F^{-A} = \frac{29{,}89\,\mu m}{5{,}11}.10^3 = 5850\,kp$$

Keď neuvaž. tep. zmeny; zaťaž. matice:

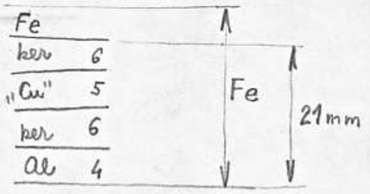
$F_{max} \approx 7900\,kp$
$F_{min} \approx 5850\,kp$
$F_{predp} \approx 6880\,kp$

$$F_{str} = \frac{7900+5850}{2} = \frac{13750}{2} = 6875\,kp$$

$$F_{ampl} = \frac{7900-5850}{2} = \frac{2050}{2} = 1025\,kp$$

Zmena zaťaž. matice (keram. ϕ50):

Predp.:



Fe: $\alpha = \frac{12.10^{-6}}{deg}$

bronz: $17{,}5.10^{-6}$

al: $23{,}8.10^{-6}$

ker: $\sim 2{,}5.10^{-6}$

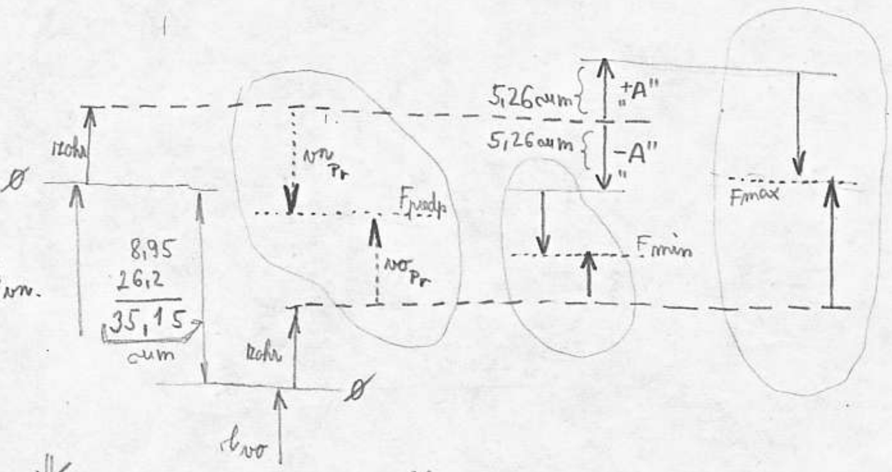
predĺženie: $\lambda_{2ker} = 2 \cdot l_K \cdot \alpha_K \cdot \Delta t_K = 2.6.10^3 \cdot 2{,}5.10^{-6} \cdot \Delta t_{ker}$ [μm]

$\lambda_{elektr} = 5.10^3 \cdot 17{,}5.10^{-6} \cdot \Delta t_e$ [μm]

$\lambda_{al\ žiar} \doteq 4.10^3 \cdot 23{,}8.10^{-6}\, \Delta t_{al}$ [μm]

$\lambda_{Fe} = 21.10^3 \cdot 12.10^{-6}\, \Delta t_{matic}$ [μm]

Δt_i ~ zohriatie jednotl. súč. pri práci (z mont. teploty, ≈ 20°C)



3 zákl. situácie pri práci meniča, s uvaž. aj teplotných zmien.

⇓

$\Delta l_{Eker} = \angle^{\pm 5{,}26}_{\emptyset}$

$$35{,}15 + \overbrace{\left(30.10^{-3}\Delta t_{ker} + 87{,}5.10^{-3}\Delta t_e + 95{,}2.10^{-3}\Delta t_{al}\right)}^{\Delta l_{t\ nn}} - \overbrace{\left(252.10^{-3} \cdot \Delta t_{ma}\right)}^{\Delta l_{t\ no}} + A =$$

$$= F_{[kp]}\left(\underbrace{1{,}3.10^{-3}}_{nn} + \underbrace{3{,}81.10^{-3}}_{no}\right) = F.5{,}11.10^{-3}$$

$${}^{t}F_{predp} = \frac{35{,}15 + 10^{-3}\left[30\Delta t_k + 87{,}5\Delta t_e + 95{,}2.\Delta t_{al} - 252\Delta t_{mal}\right]}{5{,}11 \cdot 10^{-3}}$$

$${}^{t}F_{max} = \frac{40{,}41 + 10^{-3}\left[30\Delta t_K + 87{,}5\Delta t_e + 95{,}2\Delta t_{al} - 252\,\Delta t_{ma}\right]}{5{,}11 \cdot 10^{-3}}$$

$${}^{t}F_{min} = \frac{29{,}89 + 10^{-3}\left[30\Delta t_K + 87{,}5\Delta t_e + 95{,}2\Delta t_{al} - 252\Delta t_{ma}\right]}{5{,}11 \cdot 10^{-3}}$$

Predp. krajné hodnoty:

$\Delta t_{ker} = 30°C\,(20°C)$ $\Delta t_{al\,(ži)} = 20°C\,(15°C)$ $\Delta t_{mal.} = 20°C\,(5°C)$

$\Delta t_{elektr.} = 40°C\,(20°C)$

$$\underbrace{30.20 + 87{,}5.20 + 95{,}2.15}_{} = \underbrace{3780 \qquad 3780 : 252 = 15}_{\Downarrow}$$

1.) Pri „min." hodnotách ker. 40°C, elektr. 40°C, žiar. 35°C } by vzn. vzper v malici pri jej teplote 35°C (a vyššie.)

2.) „Max". sily:

$$30.30 + 87{,}5.40 + 95{,}2.20 - 252.5 = 5044$$

$${}^tF_{predp.} = 6880 + \frac{5044}{5{,}11} = 6880 + 985 = 7865\ kp$$

$${}^tF_{max} = 7895 + 985 = 8880\ kp$$

$${}^tF_{min} = 5850 + 985 = 6835\ kp$$

$${}^tF_{str} = \frac{8880 + 6835}{2} = \frac{15715}{2} = 7857{,}5\ kp \approx {}^tF_{predp}$$

$${}^tF_{ampl} = \frac{8880 - 6835}{2} = \frac{2045}{2} = 1022{,}5\ kp$$

$$\left[\begin{array}{l} F_{str} = 6875\ kp \\ F_{amp} = 1025\ kp \end{array}\right\} \text{bez „}t\text{"} \Big]$$

Z Heywood-ových vzť. pre predp. živote.: 10^{11} cykl. $= N$ $\quad m = 11\ (= \log N)$

$$A_0 = \dots = 0{,}479$$

Z min. plochy (po toler.), $S = 194\ mm^2$:

$$\sigma_{m_{vyp}} = \frac{7857{,}5\ kp}{194\ mm^2} = 40{,}5 \qquad \sigma_a = \frac{1022{,}5\ kp}{194\ mm^2} = 5{,}27 \qquad \Big/\ \sigma_{Pt.14260.7} = 145\ \frac{kp}{mm^2}$$

$$\vartheta = \left(2 + \frac{\sigma_m}{\sigma_{Pt}}\right)\frac{\sigma_m}{3\sigma_{Pt}} = \left(2 + \frac{40{,}5}{145}\right)\frac{40{,}5}{3.145} = (2 + 0{,}279)\,0{,}093 \doteq 0{,}212$$

$$1 - \frac{\sigma_m}{\sigma_{Pt}} = \frac{\sigma_a / \sigma_{Pt}}{A_0 + \vartheta(1 - A_0)} - 1 \;\Rightarrow\; \sigma_m = \left(1 - \frac{\sigma_a/\sigma_{Pt}}{A_0 + \vartheta(1 - A_0)}\right)\sigma_{Pt} = \left(1 - \frac{5{,}27/145}{0{,}479 + 0{,}2(1 - 0{,}479)}\right)145$$

$$\sigma_m = \left(1 - \frac{0{,}0364}{0{,}479 + 0{,}104}\right).145 = (1 - 0{,}0625).145 = 136\ \frac{kp}{mm^2}$$

$$\sigma_{m_{vyp}} . \beta . k = \sigma_m \;\longrightarrow\; \text{„bezpečn." } k = \frac{136}{40{,}5.\beta} \quad \Big|\quad \beta_{k=1} = 3{,}36 \sim \text{max. prípustný vrub. súč. otvoru.}$$

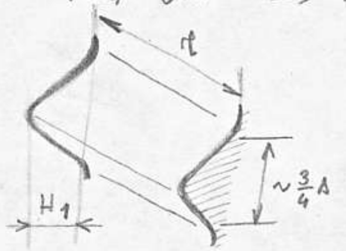
Vzhľ. na posiln. okraje ≈ reálne!

Závit : matica - odrážač (M65×1,5) $d_2 = 64{,}026$ nosná hl. $H_1 = 0{,}812$

$D_1 = 63{,}376$

$$P_{max} = 8880\ kp$$

Predp. 6 mm ⟹ 4 závity - zrazenie ... poč. 3 závity.



$$t \approx n . d_2 \pi = 3 . 64 . \pi \doteq 604\ mm$$

$$\frac{3}{4} s \approx 1{,}13$$

$$\text{strih} \approx \frac{8880\ kp}{604 . 1{,}13\ mm^2} \doteq \frac{8880\ kp}{6{,}83\ cm^2} = 1300\ kp/cm^2$$

(974 ~ pre 4 záv.)

$$\text{otlačenie} \approx \frac{8880\ kp}{604 . 0{,}812\ mm^2} = \frac{8880}{4{,}90} \doteq 1810\ kp/cm^2$$

(1360 ~ pre 4 záv.)

---

nepoužiť !

(M65×2)

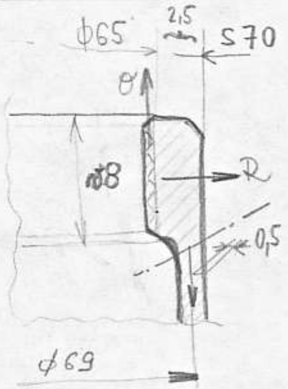
$d_2 = 63{,}701$
$d_3 = 62{,}546$
$D_1 = 62{,}835$
$H_1 = 1{,}083$
$\frac{3}{4} s \approx 1{,}5$

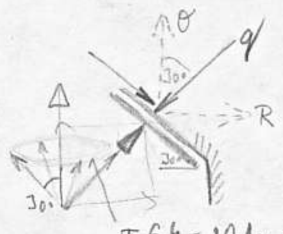
Predp. 6 mm ; 3 záv.

$$t \approx 3 . 63{,}7\pi \approx 600$$

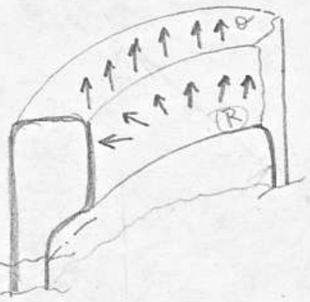
$$\tau \approx \frac{8880}{6 . 1{,}5} = 987\ kp/cm^2$$

$$p_{ot} \approx \frac{8880}{6 . 1{,}08} = 1360\ kp/cm^2$$





$\pi . 6{,}4 = 20{,}1\ cm$

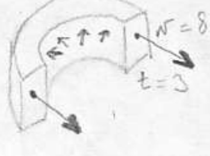


$$q . 20{,}1\ cm . \cos 30° = P_{max} \rightarrow q = \frac{8880\ kp}{0{,}866 . 20{,}1\ cm} = \frac{8880}{17{,}4} = 510\ kp/cm$$

$$\sigma = q . \cos 30° = 510 . 0{,}866 = 442\ kp/cm$$

$$r = q . \sin 30° = 510 . 0{,}5 = 255\ kp/cm$$

Prstenec ~ tlak z vnútra :

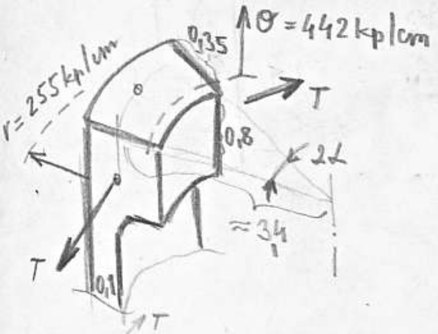


$$p = \frac{q . r\pi}{r\pi s} = \frac{q}{s} = \frac{510}{0{,}8} = 638\ kp/cm^2$$

$$2P = p . r\pi s \rightarrow P = \frac{p r \pi s}{2} \rightarrow \sigma_t = \frac{P}{s t} = \frac{p r \pi s}{2 s t} = \frac{638 . 3{,}3\pi}{2 . 0{,}3}$$

$$\sigma_t = 11000\ kp/cm^2$$

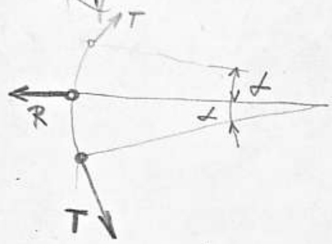
⟹ vyšší veniec je potr. ! (pre celé vyťaž. výkonu z kor.)



$i = r\,2\alpha = 3{,}4\,cm \cdot 2\alpha$

$R = r \cdot i = 255\,\frac{kp}{cm} \cdot 3{,}4\,cm \cdot 2\alpha = 1730 \cdot \alpha\,\frac{kp}{cm}$ | α, v rad.

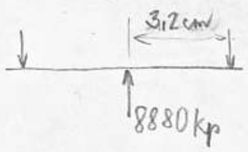
$\sigma = \sigma \cdot i = 442\,\frac{kp}{cm} \cdot 3{,}3\,cm \cdot 2\alpha = 2920\,\alpha\,\frac{kp}{cm}$



$R_{skut} = R - 2 \cdot T \sin\alpha$

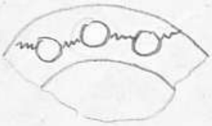
keď sa rad. prenesie na T (veniec s tlakom) a „upnutie nie je tuhé – ohyb od R nebude".

Reflektor, hrúbka priruby ≈ 7,



Konk. preruš. od posúv. sily – pri predp. nosníkového zaťaženia (medzi malými dierami)

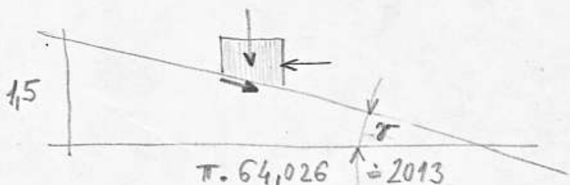
Predp.



$\Rightarrow \tau = \frac{4440\,kp}{1{,}5\,cm \cdot 0{,}7\,cm} = \frac{4440}{105} = 4230\,kp/cm^2$

$14331 \sim \sigma_{kt} = 8500\,kp/cm^2$ — hraničné!

Krútiaci moment k predpätiu.



$tg\,\gamma = \frac{1{,}5}{201{,}3} = 0{,}00746$ | $0{,}00314 \approx 0{,}18°$

$f' = \frac{f}{\cos\frac{\alpha}{2}} = \frac{f}{\cos 30°} = \frac{0{,}1 \div 0{,}15}{0{,}866} = \begin{cases} 0{,}1154 \\ 0{,}173 \end{cases}$

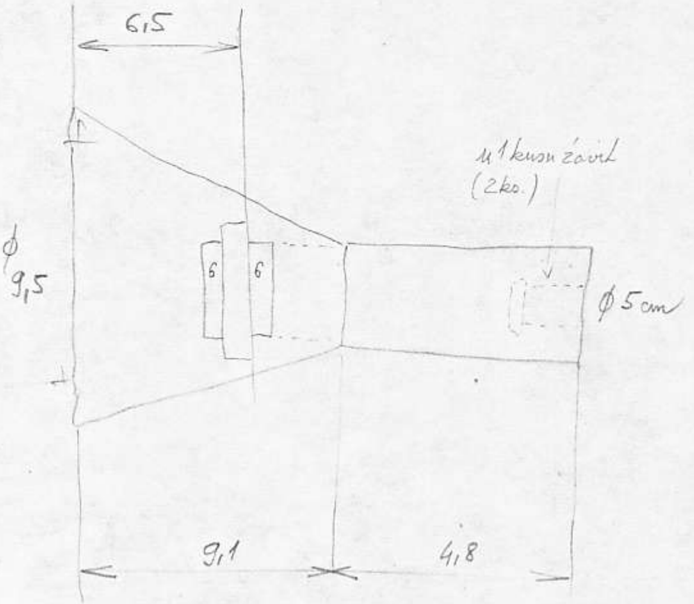
$M \doteq F_{\infty} \cdot \frac{d_2}{2} \cdot (tg\,\gamma + f') = 6880 \cdot \frac{6{,}4026}{2} \cdot \left(0{,}00746 + \begin{matrix} 0{,}1154 \\ 0{,}173 \end{matrix}\right) = 22020 \cdot \begin{pmatrix} 0{,}12286 \\ 0{,}18046 \end{pmatrix} = \begin{cases} 2704\,kpm \\ 398\,kpm \end{cases}$

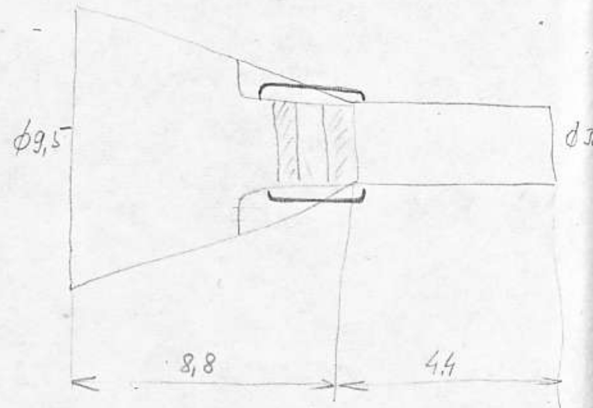
$M \approx 36\,kpm$! (keď uvaž. ↑ síl s otepl. menica : $M \approx 30\,kpm$)

$\frac{6880}{2{,}52} = 2730\,kp/cm^2$ $\left(max - s\ toler. - \frac{6880}{194} = 3546\,kp/cm^2\right)$

„stredné" napätie čisto od predpätia.

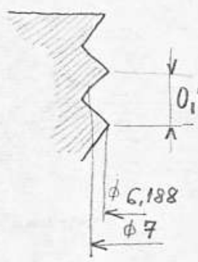
MENIČE NA ČISTENIE





(fól. al./mos. 2 mm elektr. — ~ 200W
po zvár. ——— ~ 1200W

Matice [pri spoj s 3 skrutkami M7 (⌀5)]



norm. hĺbka: $H_1 = 0,406$

stred. hr. rozb. závitu $\approx 0,595$

Výška norm. matice: 5

Napätia pri 1 závite (od predpätia ≈ 4000 kp)

Otlač.: $p \doteq \frac{1333\,kp}{6,513\pi \cdot 0,406\,mm^2} = \frac{1333}{8,32} = 161\,kp/mm^2$

Strih.: $\tau \doteq \frac{1333}{7 \cdot \pi \cdot 0,595} = \frac{1333}{13,1} = 102\,kp/mm^2$

$P_{pr} = 4000\,kp$; $h_{ma} = 5\,mm \Rightarrow p_{ot} \approx \frac{0,75}{5} \cdot 161 = 24,1\,kp/mm^2 \mathrel{\hat{=}} 2410\,\frac{kp}{cm^2}$

$\tau_{str} \approx \frac{0,75}{5} \cdot 102 = 15,3\,kp/mm^2 \mathrel{\hat{=}} 1530\,\frac{kp}{cm^2}$

M7×0,75

M7×1 ~ základný závit $d_2 = 6,35$ $H_1 = 0,541$ $\rightarrow t_j \approx 0,795$

$P_{pr} \approx 4000\,kp$; $p_{ot1} = \frac{1333}{6,35 \cdot \pi \cdot 0,541} = \frac{1333}{10,8} = 123\,kp/mm^2$

$\tau_{str1} = \frac{1333}{7 \cdot \pi \cdot 0,795} = \frac{1333}{17,5} = 76,2\,kp/mm^2$

$h_{ma} = 5 \Rightarrow p_{ot} \doteq \frac{123}{5} \mathrel{\hat{=}} 2400\,kp/cm^2$

$\tau_{str} = \frac{76,2}{5} \mathrel{\hat{=}} 1520\,kp/cm^2$

M7 (×1) – zákl.

Krút. moment pri uťah. spojovacích skrutiek (meničov)

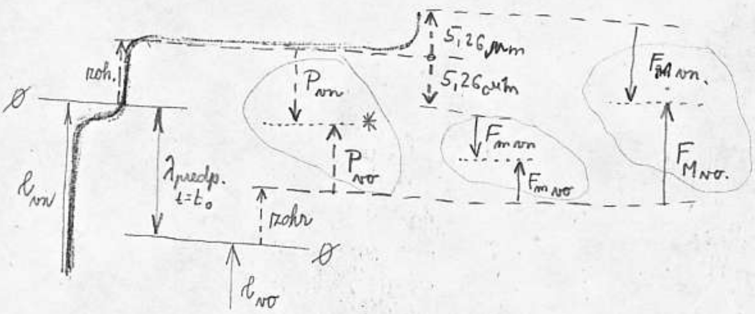
M7 x 1 ; $P_{pr.1} \doteq 1000\,kp$

$$M \approx F_{os} \frac{d_2}{2} \left(tg\gamma + f' \right) \qquad f' = \frac{f}{\cos \frac{\varepsilon}{2}} = \frac{0{,}1 \div 0{,}15}{0{,}866} = \left\langle \begin{matrix} 0{,}1154 \\ 0{,}173 \end{matrix} \right.$$

$$tg\gamma = \frac{1}{\pi \cdot 6{,}35} = \frac{1}{19{,}93} = 0{,}05$$

$$M \approx 1000\,kp \cdot \frac{0{,}635\,cm}{2} \left(0{,}05 + \begin{matrix} 0{,}1154 \\ (0{,}173) \end{matrix} \right) = 317 \cdot \begin{matrix} 0{,}165 \\ (0{,}22) \end{matrix} = \left\langle \begin{matrix} 52{,}4\,kpcm \triangleq 0{,}524\,kpm \\ 69{,}8\,kpcm \triangleq 0{,}698\,kpm \end{matrix} \right\} \phi 38''$$

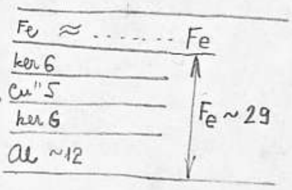
Zohľadňovať aj predp. od prac. teploty! –



Max def. ker-y, pri $\sigma_{dyn\,t'} = 350\,kp/cm^2$:

$$\Delta l_{2k} = \frac{F\, l_{2k}}{ES} = \sigma_{dyn\,t'} \cdot \frac{l_{2k}}{E} = 350 \frac{kp}{cm^2} \cdot \frac{1{,}2\,cm}{8 \cdot 10^5 \frac{kp}{cm^2}} = 52{,}5 \cdot 10^{-5}\,cm \triangleq 5{,}26\,\mu m$$

Zmena síl pri pracovnej teplote:



$$\lambda_{2ker} = 2 l_k \alpha_k \Delta t_k = 2 \cdot 6 \cdot 10^3 \cdot 2{,}5 \cdot 10^{-6} \Delta t_k \quad [\mu m]$$

$$\lambda_{„Cu"} = 5 \cdot 10^3 \cdot 17{,}5 \cdot 10^{-6} \Delta t_{elektr.} \quad [\mu m]$$

$$\lambda_{al} = 12 \cdot 10^3 \cdot 23{,}8 \cdot 10^{-6} \Delta t_{al} \quad [\mu m]$$

$$\lambda_{Fe} = 29 \cdot 10^3 \cdot 12 \cdot 10^{-6} \Delta t_{Fe} \quad [\mu m]$$

$\lambda_{predp.}$ $t = t_0$ „$\phi 38$"

$$= \Delta l_{sp.\,skr.} + \Delta l_{vn} \doteq 3870\,kp \left(\overbrace{\frac{4\,cm}{21 \cdot 10^5 \frac{kp}{cm^2} \cdot (4 \cdot 0{,}196)\,cm^2}}^{skr.} + \overbrace{\frac{1{,}2}{8 \cdot 10^5 \cdot 11{,}35}}^{ker.} + \overbrace{\frac{0{,}5}{12{,}5 \cdot 10^5 \cdot 11{,}35}}^{elektr.} + \overbrace{\frac{1{,}2}{7{,}22 \cdot 10^5 \cdot 11{,}35}}^{al} + \overbrace{\frac{0{,}9}{21 \cdot 10^5 \cdot 11{,}35}}^{Fe} \right)$$

$$\overset{[\mu m]}{\triangleq} 387 \left[\frac{4}{21 \cdot 4 \cdot 0{,}196} + \frac{1}{11{,}35} \left(\frac{1{,}2}{8} + \frac{0{,}5}{12{,}5} + \frac{1{,}2}{7{,}22} + \frac{0{,}9}{21} \right) \right] = 387 \left[0{,}25 + \frac{0{,}15 + 0{,}04 + 0{,}166 + 0{,}0428}{11{,}35} \right]$$

$$= 387 \left[0{,}25 + \frac{0{,}3988}{11{,}35} \right] = 387 \left[0{,}25 + 0{,}0351 \right] = 387 \cdot 0{,}2851 \doteq 110\,\mu m$$

λ_{predp} $t = t_0$ „$\phi 50$"

$$= \Delta l_{vo} + \Delta l_{vn} = 688 \left[0{,}167 + \frac{0{,}3988}{19{,}6} \right] = 688 \left[0{,}167 + 0{,}0204 \right] = 688 \cdot 0{,}1874 = 129\,\mu m$$

$\lambda_{predp.} + rozt._{nn} - rozt._{no} = \Delta l_{nn} + \Delta l_{no}$

$\lambda_{predp} / F_{pr.}$ ($t = t_0$)

$\phi 38$: $110 + (12 \cdot 2{,}5 \cdot 10^{-3} \Delta t_k + 5 \cdot 17{,}5 \cdot 10^{-3} \Delta t_{elektr} + 12 \cdot 23{,}8 \cdot 10^{-3} \Delta t_{al}) - 29 \cdot 12 \cdot 10^{-3} \Delta t_{Fe} = F \cdot 0{,}02851$

$110 + 10^{-3}(30 \Delta t_k + 87{,}5 \Delta t_{"Cu"} + 286 \Delta t_{al} - 348 \Delta t_{Fe}) = 0{,}02851 \cdot F$

odh.: $\Delta t_k = 50 - 20 = 30°C$

$\Delta t_{"Cu"} = 60 - 20 = 40°C$

$\Delta t_{al} \doteq \Delta t_{Fe} = 40 - 20 = 20°C$

$110 + 10^{-3}(30 \cdot 30 + 87{,}5 \cdot 40 + \overbrace{286 \cdot 20}^{al} - \overbrace{348 \cdot 20}^{Fe}) = 0{,}02851 F$

$110 + 10^{-3}(900 + 3500 + 5720 - 6960) = 0{,}02851 \cdot F$

$110 + 10^{-3} \cdot (10120 - 6960) = 0{,}02851 F$

$110 + 3{,}160 = 0{,}2851 F \rightarrow F \doteq 3{,}97\ Mp$

Pri uťah.: $\underbrace{0{,}524\ (\rightarrow 0{,}698)\ kpm}$ ..... F_{predp}: $968\ kp \xrightarrow{t°C} 994\ kp$

$M_k \approx (0{,}41 \div 0{,}545)\ kpm$

$\left.\begin{array}{l} \sum \frac{\Delta l_{al}}{\Delta t_{al}} \approx \sum \frac{\Delta l_{Fe}}{\Delta t_{Fe}} \\ \Delta t_{al} \approx \Delta t_{Fe} \end{array}\right\}$ ⇒ predp. v skut. lepšie pomery než

$\phi 50$: $129 + \overbrace{3{,}160}^{k, Cu, al} = 0{,}1874 F \rightarrow F_{1skr} = \frac{7060}{6} = 1178\ kp$

Pri uťah.: $0{,}599\ (\rightarrow 0{,}797)\ kpm$ ..... F_{predp}: $1140\ kp \xrightarrow{t°C} 1178\ kp$ .... zanedb. ramena.

! $+\Delta M_k \doteq$ (matica, príruba) $\overbrace{\begin{pmatrix} 968 \\ 1140 \end{pmatrix}}^{kp} \cdot \begin{pmatrix} 0{,}1 \\ 0{,}15 \end{pmatrix} \cdot \overbrace{0{,}9}^{cm} =$ 87,2 kpcm ≐ 0,87 kpm; 131 kpcm ≐ 1,3 kpm; 1,03 kpm; 1,55 kpm } pripočítať ku M_k 0,41 ÷ 0,545 resp. M_k 0,599 ÷ 0,797

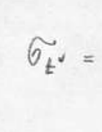
Kombinované namáhanie drieku skrutky pri uťah. (ťah + krut.)

$F = 1178\ kp$; $M_k = M_{k\,max,max} = 0{,}797\ kpm$ (zavit)

$\sigma_t = \frac{F}{S} = \frac{1178\ kp}{19{,}6\ mm^2} \doteq 60\ kp/mm^2$

$W_k \approx \frac{\pi d^3}{16} = \frac{\pi \cdot 5^3}{16} = \frac{\pi \cdot 125}{16} = 24{,}55\ mm^3$

$\tau_k = \frac{M_k}{W_k} = \frac{0{,}797 \cdot 10^3\ kpmm}{24{,}55\ mm^3} = \frac{797}{24{,}55} = 32{,}4\ kp/mm^2$

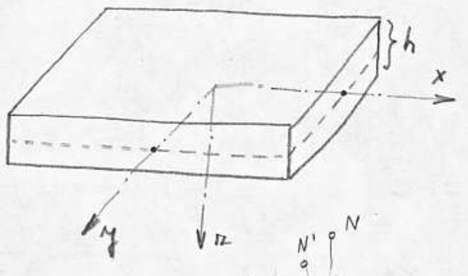


$\sigma_N = \sqrt{\sigma_t^2 + \tau_k^2} = \sqrt{60^2 + 32{,}4^2} \doteq \sqrt{3600 + 1050} = \sqrt{4650} \approx 68{,}0\ kp/cm^2$

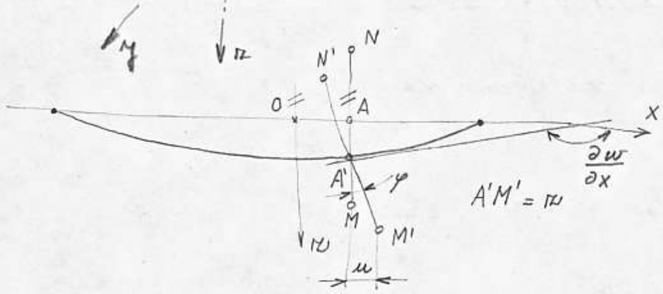
$\sigma_{kt} (14260.7) = 120\ kp/cm^2$

Priečne kmitanie dosák
I. M. Babakov: Teorija kolebanij, Nauka 1965.

Predpoklady: 1, ohybové deformácie pri kmitoch – malé, pružné deformácie; podliehajú Hukovmu zákonu.
2, exist. neutr. vrstva – ktorá pri malých def. sa nemení. (stredná výška v rovnorodej doske)



3,



– predp. o priebehu def.-ií.

4, Posunutia bodov na normálach neutr. vrstvy sú rovnaké (na tej istej normále!)

$u_0 = v_0 = 0$; $w = w_0 = f(x,y,t)$ ~ nezáv. od z. (9.1)

$$u = z\sin\varphi = z\varphi = -z\frac{\partial w_0}{\partial x}$$
$$v = z\sin\psi = z\psi = -z\frac{\partial w_0}{\partial y}$$

| φ, ψ – ∡ dotyčnice v bode A' s koord. osami.

Deformácie: (9.2)
$$e_{xx} = \frac{\partial u}{\partial x} = -z\frac{\partial^2 w_0}{\partial x^2} \qquad e_{yy} = \frac{\partial v}{\partial y} = -z\frac{\partial^2 w_0}{\partial y^2} \qquad e_{zz} = 0$$

skos → $$e_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = -2z\frac{\partial^2 w_0}{\partial x \partial y} \qquad e_{xz} = e_{yz} = 0$$ } (9.3)

Zložky napätia

$$\left.\begin{aligned} X_x &= \frac{E}{1-\sigma^2}(e_{xx} + \sigma e_{yy}) = -\frac{Ez}{1-\sigma^2}\left(\frac{\partial^2 w_0}{\partial x^2} + \sigma\frac{\partial^2 w_0}{\partial y^2}\right) \\ Y_y &= \frac{E}{1-\sigma^2}(e_{yy} + \sigma e_{xx}) = -\frac{Ez}{1-\sigma^2}\left(\frac{\partial^2 w_0}{\partial y^2} + \sigma\frac{\partial^2 w_0}{\partial x^2}\right) \\ X_y &= Y_x = -\frac{Ez}{1+\sigma}\frac{\partial^2 w_0}{\partial x \partial y} = -G e_{xy} \end{aligned}\right\} \quad (9.4)$$

šmyk. nap.

E – mod. pruž. G – mod. pr. v šmyku σ – Poissonovo číslo

Potenciálna a kinetická energia dosky

$$d\Pi = \frac{1}{2}\left(X_x e_{xx} + Y_y e_{yy} + X_y e_{xy}\right) dx\, dy\, dz$$ ~ pot. energ.

po dosad. z (9.3), (9.4):

$$d\Pi = \frac{1}{2}\Big[\frac{Ez^2}{1-\sigma^2}\frac{\partial^2 w_0}{\partial x^2}\left(\frac{\partial^2 w_0}{\partial x^2} + \sigma\frac{\partial^2 w_0}{\partial y^2}\right) + \frac{Ez^2}{\partial y^2}\frac{\partial^2 w_0}{1-\sigma^2}\left(\frac{\partial^2 w_0}{\partial y^2} + \sigma\frac{\partial^2 w_0}{\partial x^2}\right) + $$
$$+ \frac{2Ez^2}{1+\sigma}\left(\frac{\partial^2 w_0}{\partial x \partial y}\right)^2\Big] dx\, dy\, dz$$

$$2\Pi = \iiint \frac{Ez^2}{1-\sigma^2}\left[\left(\frac{\partial^2 w_0}{\partial x^2}\right)^2 + \left(\frac{\partial^2 w_0}{\partial y^2}\right)^2 + 2\sigma\frac{\partial^2 w_0}{\partial x^2}\frac{\partial^2 w_0}{\partial y^2} + 2(1-\sigma)\left(\frac{\partial^2 w_0}{\partial x \partial y}\right)^2\right] dx\, dy\, dz$$

→

Pot. energ. dosky s konšt. hrúbkou h : integr. $\kappa: 0 \to \frac{h}{2}$ / $2x$

$$2\Pi = \underbrace{\frac{Eh^3}{12(1-\sigma^2)}}_{D} \iint \left[\left(\frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 w_0}{\partial y^2} \right)^2 - 2(1-\sigma) \left\{ \frac{\partial^2 w_0}{\partial x^2} \frac{\partial^2 w_0}{\partial y^2} - \left(\frac{\partial^2 w_0}{\partial x \partial y} \right)^2 \right\} \right] dx\,dy$$

kontrolované !!!

Vonkajšie zaťaženie dosky, intenzity $q(x,y,t)$ (na plochu) : $\Rightarrow$

$$2\Pi = D \iint \left[\left(\frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 (w_0)}{\partial y^2} \right)^2 - 2(1-\sigma) \left\{ \frac{\partial^2 w_0}{\partial x^2} \frac{\partial^2 w_0}{\partial y^2} - \left(\frac{\partial^2 w_0}{\partial x \partial y} \right)^2 \right\} \right] dx\,dy - 2 \iint q(x,y,t)\, w_0\, dx\,dy. \quad (9.7)$$

Integr. sa po povrchu neutr. vrstvy.

Kinet. energia elementu $h \cdot dx \cdot dy$; hustoty $\frac{\gamma}{g}$ je:

$$\frac{\gamma h}{2g} \left(\frac{\partial w_0}{\partial t} \right)^2 dx\,dy. \qquad \left[\cdots \text{"} \frac{1}{2} \rho (h\,dx\,dy)\, v^2 \text{"} \right]$$

Kinet. energ. celej dosky :

$$T = \frac{\gamma h}{2g} \iint \left(\frac{\partial w_0}{dt} \right)^2 dx\,dy$$

– keď doska vykonáva pohyb $w_0(x,y,t) = w(x,y) \sin(pt + \alpha)$

Pre také kmity :

$$\Pi_{max} = \frac{D}{2} \iint \left[\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 - 2(1-\sigma) \left\{ \frac{\partial^2 w}{\partial x^2} \cdot \frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right\} \right] dx\,dy.$$

$$T_{max} = \frac{\gamma h p^2}{2g} \iint w^2\, dx\,dy \qquad [p \text{ — kruhová frekv.}]$$

už sa vyprchalo vonk. zaťaž. (netreba "minimalizovať") "(9.9.)"

Variačné rovnice priečnych kmitov doštičiek

$$S = \int_{t_A}^{t_B} (T - \Pi)\, dt \qquad (9.10)$$

– po množine hlavných kmitov, periódy $2\pi/p$; integrov. na úseku $t_B - t_A = 2\pi/p$ $\Rightarrow$

$$\delta(T_{max} - \Pi_{max}) = 0 \qquad (9.11)$$

... tieto rovn. musia vyhovieť hlavné formy kmitov doštičky.

[rytm. premena pot. na kin. energiu]

Variácia pot. energie ($\neq$ maxima) :

$$\delta \Pi_{max} = D \iint \left\{ \Delta w \left[\delta \left(\frac{\partial^2 w}{\partial x^2} \right) + \delta \left(\frac{\partial^2 w}{\partial y^2} \right) \right] - (1-\sigma) \left[\underbrace{\frac{\partial^2 w}{\partial x^2} \delta \left(\frac{\partial^2 w}{\partial y^2} \right)} + \right. \right.$$

$$\left. \left. + \underbrace{\frac{\partial^2 w}{\partial y^2} \delta \left(\frac{\partial^2 w}{\partial x^2} \right)} - 2 \frac{\partial^2 w}{\partial x \partial y} \delta \left(\frac{\partial^2 w}{\partial x \partial y} \right) \right] \right\} dx\,dy.$$

Δ ~ Laplaceov operátor.

Int. po častiach :
$$\iint \frac{\partial^2 w}{\partial x^2} \delta \left(\frac{\partial^2 w}{\partial y^2} \right) dx\,dy = \oint \frac{\partial^2 w}{\partial x^2} \delta \left(\frac{\partial w}{\partial y} \right) dx - \oint \frac{\partial^3 w}{\partial x^2 \partial y} \delta w\, dx + \iint \frac{\partial^4 w}{\partial x^2 \partial y^2} \delta w\, dx\,dy \quad (9.12)$$

analogicky :
$$\iint \frac{\partial^2 w}{\partial y^2} \delta \left(\frac{\partial^2 w}{\partial x^2} \right) dx\,dy = \oint \frac{\partial^2 w}{\partial y^2} \delta \left(\frac{\partial w}{\partial x} \right) dy - \oint \frac{\partial^3 w}{\partial x \partial y^2} \delta w\, dy + \iint \frac{\partial^4 w}{\partial x^2 \partial y^2} \delta w\, dx\,dy \quad (9.13)$$

$$\iint_\Omega u \left(\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} \right) d\Omega = \int_S u \cdot v\, dx + \int_S u\, v\, dy - \iint_\Omega v \left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \right) d\Omega$$

z (9.12), (9.13) – výmenou indexov:

$$\iint \frac{\partial^2 w}{\partial x^2}\delta\left(\frac{\partial^2 w}{\partial x^2}\right)dx\,dy = \oint \frac{\partial^2 w}{\partial x^2}\delta\left(\frac{\partial w}{\partial x}\right)\underset{dy}{\cancel{dx}} - \oint \frac{\partial^3 w}{\partial x^3}\delta w\, dy + \iint \frac{\partial^4 w}{\partial x^4}\delta w\, dx\,dy \quad ✓ \qquad (9.14)$$

$$\iint \frac{\partial^2 w}{\partial y^2}\delta\left(\frac{\partial^2 w}{\partial y^2}\right)dx\,dy = \oint \frac{\partial^2 w}{\partial y^2}\delta\left(\frac{\partial w}{\partial y}\right)\underset{dx}{\cancel{dy}} - \oint \frac{\partial^3 w}{\partial y^3}\delta w\, \underset{dx}{\cancel{dy}} + \iint \frac{\partial^4 w}{\partial y^4}\delta w\, dx\,dy \quad ✓ \qquad (9.15)$$

Podobne:

$$\iint \frac{\partial^2 w}{\partial x\partial y}\delta\left(\frac{\partial^2 w}{\partial x\partial y}\right)dx\,dy = \iint \frac{\partial^2 w}{\partial x\partial y}\cdot\frac{\partial}{\partial x}\left(\delta\frac{\partial w}{\partial y}\right)dx\,dy =$$

$$= \oint \frac{\partial^2 w}{\partial x\partial y}\delta\left(\frac{\partial w}{\partial y}\right)dy - \oint \frac{\partial^3 w}{\partial x^2\partial y}\delta w\, dx + \iint \frac{\partial^4 w}{\partial x^2\partial y^2}\delta w\, dx\,dy \qquad (9.16) \quad ✓$$

Pomoc. horeuv. formúl.:

$$\iint \underbrace{\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)}_{\Delta w}\left[\delta\left(\frac{\partial^2 w}{\partial x^2}\right)+\delta\left(\frac{\partial^2 w}{\partial y^2}\right)\right]dx\,dy = \qquad (9.17)$$

$$= \oint \Delta w\,\delta\left(\frac{\partial w}{\partial x}\right)dy \;\overset{+}{\cancel{-}}\; \oint \Delta w\,\delta\left(\frac{\partial w}{\partial y}\right)dx - \oint\left(\frac{\partial \Delta w}{\partial x}dy \;\overset{+}{\cancel{-}}\; \frac{\partial \Delta w}{\partial y}dx\right)\delta w + \iint \Delta^2 w\,\delta w\,dx\,dy$$

$$/\ \Delta^2 w = \frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2\partial y^2} + \frac{\partial^4 w}{\partial y^4} \qquad (9.18)$$

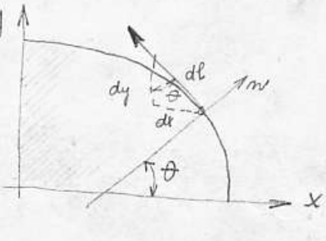
Po dos. (9.17) (9.12) (9.13) (9.16) do $\delta\Pi_{max}$ – a preskupení:

$$\frac{1}{D}\delta\Pi_{max} = \oint\left[\left(\frac{\partial^2 w}{\partial x^2}+\sigma\frac{\partial^2 w}{\partial y^2}\right)dy - (1-\sigma)\frac{\partial^2 w}{\partial x\partial y}dx\right]\delta\left(\frac{\partial w}{\partial x}\right) +$$

spolu „$\oint M_L$" a $\oint$ „H_L"

$$+\oint\left[\left(\frac{\partial^2 w}{\partial y^2}+\sigma\frac{\partial^2 w}{\partial x^2}\right)dx + (1-\sigma)\frac{\partial^2 w}{\partial x\partial y}dy\right]\delta\left(\frac{\partial w}{\partial y}\right) -$$

$$+\oint 2\left(\frac{\partial^2 w}{\partial x\partial y}\delta\left(\frac{\partial w}{\partial y}\right)(1-\sigma)\right)dy\ ? \quad - \oint\left(\frac{\partial \Delta w}{\partial x}dy \;\overset{+}{\cancel{-}}\; \frac{\partial \Delta w}{\partial y}dx\right)\delta w + \boxed{\iint \Delta^2 w\,\delta w\,dx\,dy.} \qquad (9.19)$$



$$\left.\begin{aligned} dx &= -dl\sin\theta \qquad\qquad dy = dl\cos\theta \\ \frac{\partial}{\partial x} &= \cos\theta\frac{\partial}{\partial n} - \sin\theta\frac{\partial}{\partial l} \\ \frac{\partial}{\partial y} &= \sin\theta\frac{\partial}{\partial n} + \cos\theta\frac{\partial}{\partial l}\end{aligned}\right\} \qquad (9.20)$$

(9.19) – s (9.20) →

„plus" keď $dx = dl\sin\theta$

$$\frac{1}{D}\delta\Pi_{max} = \iint_{(S)}\Delta^2 w\,\delta w\,ds - \oint\left[\frac{\partial\Delta w}{\partial n}\right]\delta w\,dl + \oint\left[\left(\frac{\partial^2 w}{\partial x^2}+\sigma\frac{\partial^2 w}{\partial y^2}\right)\cos^2\theta + \left(\frac{\partial^2 w}{\partial y^2}+\sigma\frac{\partial^2 w}{\partial x^2}\right)\sin^2\theta +\right.$$

$$\left.+(1-\sigma)\frac{\partial^2 w}{\partial x\partial y}\sin 2\theta\right]\delta\left(\frac{\partial w}{\partial n}\right)dl + \oint\left[(1-\sigma)\left(\frac{\partial^2 w}{\partial y^2}-\frac{\partial^2 w}{\partial x^2}\right)\frac{\sin 2\theta}{2} + \frac{\partial^2 w}{\partial x\partial y}\cos 2\theta\right]\delta\left(\frac{\partial w}{\partial l}\right)dl$$

$\cdot(1-\sigma)$

„mínus" keď $dx = dl\sin\theta$ [mala by ⊖ pred 3 člen. súčin]

Ohybový moment na okraji dosky:

$$M_L = -D\left[\left(\frac{\partial^2 w}{\partial x^2}+\sigma\frac{\partial^2 w}{\partial y^2}\right)\cos^2\theta + \left(\frac{\partial^2 w}{\partial y^2}+\sigma\frac{\partial^2 w}{\partial x^2}\right)\sin^2\theta + (1-\sigma)\frac{\partial^2 w}{\partial x\partial y}\sin 2\theta\right] \qquad (9.21)$$

Krútiaci moment na okraji dosky:

$$H_L = D(1-\sigma)\left[\left(\frac{\partial^2 w}{\partial y^2}-\frac{\partial^2 w}{\partial x^2}\right)\sin\theta\cos\theta + \frac{\partial^2 w}{\partial x\partial y}\cos 2\theta\right] \qquad (9.22)$$

Priečna sila na obvode dosky:

$$-D\frac{\partial \Delta w}{\partial n} = N_L \qquad (9.23)$$

Integrovaním per partes 3. okrajového integrálu .... prejde na tvar:

$$\oint H_L \frac{\partial}{\partial l}(\delta w)\, dl = -\oint \frac{\partial H_L}{\partial l}\delta w\, dl \qquad (9.24)$$

Variácia potenciálnej energie:

$$\delta \Pi_{max} = D\iint_{(s)} \Delta^2 w\, \delta w\, ds - \oint M_L \delta\left(\frac{\partial w}{\partial n}\right) dl + \oint\left(N_L - \frac{\partial H_L}{\partial l}\right)\delta w\, dl. \qquad (9.25)$$

Variácia kinetickej energie:

$$\delta T_{max} = \frac{\gamma h p^2}{g}\iint_{(s)} w.\delta w.ds$$

Var. rovn. priečn. kmitov dosky (z 9.25), (9.26):

$$\iint_{(s)}\left(D\Delta^2 w - \frac{\gamma h}{g}p^2 w\right)\delta w.\, ds - \oint M_L\frac{\partial}{\partial n}(\delta w)\, dl + \oint\left(N_L - \frac{\partial H_L}{\partial l}\right)\delta w\, dl = 0 \qquad (9.27)$$

/ ds ~ element povrchu neutr. vrstvy.

Dif. rovnica foriem priečnych kmitov dosky a okrajové podmienky:

(9.27) – prejde v 0 (1. variácia), bude splnené s $w(x,y)$ – kt. vyhovie rovnici:

$$D\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) - \frac{\gamma h}{g}p^2 w = 0 \qquad (9.28)$$

a prevedie v 0 „okrajové" integrály (z 9.27); čiže vyhovie podmienke:

$$\oint M_L \frac{\partial}{\partial n}(\delta w)\, dl - \oint\left(N_L - \frac{\partial H_L}{\partial l}\right)\delta w\, dl = 0 \qquad (9.29)$$

(9.28) je Eulerova rovnica, pre funkcionál:

$$\iint_{(s)}\left[\frac{\gamma h p^2}{g}w^2 - D\left\{\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)^2 - 2(1-\sigma)\left(\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right)\right\}\right] ds$$

(9.29) vyjadr. okrajové podmienky úlohy. Bude splnené:

1, Pri (pružne / pevne) uchyt. okraji $\qquad (\delta w)_{(L)} = \delta\left(\frac{\partial w}{\partial n}\right)_{(L)} = 0 \qquad (9.30)$

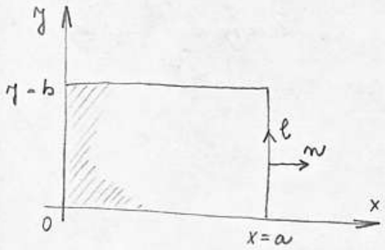
3, Voľné ~~oprete~~ okraje : $\qquad M_L = 0 \quad N_L = \frac{\partial H_L}{\partial l}$

2, Voľne oprete okraje : $\qquad (\delta w)_{(L)} = 0 \quad M_L = 0.$

Podmienky na M_L, $N_L - \frac{\partial H_L}{\partial l}$ ~ dynamické

δw, $\delta\left(\frac{\partial w}{\partial n}\right)$ ~ geometrické

Pravouhlá doska:



(9.20) ÷ (9.23)

$$\theta = 0, \quad \frac{\partial}{\partial x} = \frac{\partial}{\partial n}, \quad \frac{\partial}{\partial y} = \frac{\partial}{\partial l}$$

$$M_L = -D\left(\frac{\partial^2 w}{\partial x^2} + \sigma \frac{\partial^2 w}{\partial y^2}\right)$$

$$H_L = D(1-\sigma)\frac{\partial^2 w}{\partial x \partial y}$$

$$N_L = -D\left(\frac{\partial^3 w}{\partial x^3} + \frac{\partial^3 w}{\partial x \partial y^2}\right)$$

Okraje, rovnobežné s Oy – voľne opreté : $w = 0 \qquad \frac{\partial^2 w}{\partial x^2} + \sigma \frac{\partial^2 w}{\partial y^2} = 0$ (9.31)

Pevné (pružné) uchyt. : $w = \frac{\partial w}{\partial x} = 0$ (9.32)

Voľné okraje (∥ s Oy) : $\frac{\partial^2 w}{\partial x^2} + \sigma \frac{\partial^2 w}{\partial y^2} = 0 \qquad \frac{\partial^3 w}{\partial x^3} + (2-\sigma)\frac{\partial^3 w}{\partial x \partial y^2} = 0$ (9.33)

Podmienky na okraje, rovnobežné s Ox dostaneme zámenou $x \div y$.

Intenzita vonk. zaťaženia dosky: $F(x,y,t) = f(x,y) \sin pt \longrightarrow$

forma vynútených priečnych kmitov dosky, daná rovnicou:

$$D\Delta^2 w - \frac{\gamma h p^2}{g} w - f(x,y) = 0 \qquad (9.34)$$

w orientované „dole"

Niektoré vlastnosti foriem kmitania dosky

Rovnicu (9.28) možno chápať ako rovn. stat. priehybu pod zaťažením:

$$\mu(x,y)\, p^2 w(x,y) \qquad (9.35)$$

a) Ortogonálnosť foriem plynie z princípu vzájomnosti pohybov.

zaťaž: $\mu p_i^2 w_i \longrightarrow$ priehyb w_i

$\mu p_k^2 w_k \longrightarrow w_k$

Práce sú rovnaké $\rightarrow \iint_{(S)} \mu p_i^2 w_i w_k \, ds = \iint_{(S)} \mu p_k^2 w_i w_k \, ds \Rightarrow (p_i^2 - p_k^2)\iint_{(S)} \mu w_i w_k \, ds = 0$

keď $p_i^2 \neq p_k^2$ .... $\iint_{(S)} \mu w_i w_k ds = 0$ (9.36)

b) Rozložiteľnosť do radov : $D\Delta^2 w_i - p_i^2 \mu w_i = 0 \qquad (i = 1, 2, 3, \dots)$

$$w(x,y) = \sum_{i=1}^{\infty} a_i w_i(x,y) \qquad (9.37)$$

$$a_i = \frac{\iint_{(S)} \mu w(x,y)\, w_i(x,y) ds}{\iint_{(S)} \mu w_i^2(x,y)\, ds} \qquad (9.38)$$

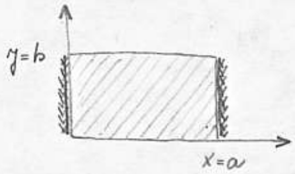
Pre normované formy - $\iint_{(S)} \mu w_i^2(x,y) ds = 1$ .... $a_i = \iint_{(S)} \mu w_i(x,y)\, w(x,y)\, ds$ (9.39)

Pre pravouhlé dosky obyčajne $w_{ik}(x,y) = X_i(x) \cdot Y_k(y)$, potom

rad (9.37) : $w(x,y) = \sum_{i,k=1}^{\infty} a_{ik} X_i(x) Y_k(y)$ (9.40)

Pri hlavných kmitoch povrch dosky rozdelí sa čiarami $w_i(x,y)=0$ na časti, kmitajúcich v jednej alebo opačnej fáze. — Uzlové čiary. (Chladny-ove figury)

Pr. Priečne kmity pravouhlej dosky – voľne opretej na dvoch rovnobežných okrajoch



$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} - k^4 w = 0 \qquad (9.41)$$

$$\Big/ \ k^4 = \frac{\gamma h p^2}{D g}$$

Riešenie hľadáme vo forme rada: $w(x,y) = \sum_{i=1}^{\infty} F_i(y) \sin\frac{i\pi x}{a}$ (9.42)

$\Big/ \ F_i(y)$ – neznáma funkcia

Zač. podm.: $x=0,\ x=a$ .... $w(x,y)=0$, $\frac{\partial^2 w}{\partial x^2}=0$ ..... nie je sila na väzbách

$(9.42) \rightarrow (9.41) \quad \Rightarrow \quad F_i^{(IV)} - \frac{2i^2\pi^2}{a^2}F_i'' + \left(\frac{i^4\pi^4}{a^4} - k^4\right)F_i = 0$ (9.43)

Charakterist. rovn.: $r^4 - \frac{2i^2\pi^2}{a^2}r^2 + \left(\frac{i^4\pi^4}{a^4} - k^4\right) = 0$ (9.44)

$r_1 = -r_3 = \sqrt{\frac{i^2\pi^2}{a^2} - k^2} \qquad r_2 = -r_4 = \sqrt{\frac{i^2\pi^2}{a^2} + k^2}$

Obecný integrál: $F_i(y) = A_i \,\mathrm{sh}\, r_1 y + B_i \,\mathrm{ch}\, r_1 y + C_i \,\mathrm{sh}\, r_2 y + D_i \,\mathrm{ch}\, r_2 y$

1.) $y=0$ voľne opreté, $y=b$ pevne uchyt.

$y=0$: $w(x,0)=0$; $\frac{\partial^2 w}{\partial y^2}=0 \rightarrow B_i = D_i = 0$

$y=b$: $w(x,b)=0$; $\frac{\partial w}{\partial y}=0 \rightarrow$

$$A_i \,\mathrm{sh}\, r_1 b + C_i \,\mathrm{sh}\, r_2 b = 0$$
$$A_i r_1 \,\mathrm{ch}\, r_1 b + C_i r_2 \,\mathrm{ch}\, r_2 b = 0$$

$$\begin{vmatrix} \mathrm{sh}\, r_1 b & \mathrm{sh}\, r_2 b \\ r_1 \,\mathrm{ch}\, r_1 b & r_2 \,\mathrm{ch}\, r_2 b \end{vmatrix} = 0$$

$$\frac{1}{r_1}\,\mathrm{th}\, r_1 b = \frac{1}{r_2}\,\mathrm{th}\, r_2 b$$

2.) $y=0$, $y=b$ } voľne opreté

$$F_i(y) = A_i \sin k_1 y + B_i \cos k_1 y + C_i \,\mathrm{sh}\, r_2 y + D_i \,\mathrm{ch}\, r_2 y$$

$$\Big/ \ k_1 = \sqrt{k^2 - \frac{i^2\pi^2}{a^2}}$$

Z

$w(x,0)=0 \quad \frac{\partial^2 w(x,0)}{\partial y^2}=0$

$w(x,b)=0 \quad \frac{\partial^2 w(x,b)}{\partial y^2}=0$

$\rightarrow B_i = C_i = D_i = 0$

$\sin k_1 b = 0$ ~ rovnica frekv.

$k_1 b = j\pi \quad (j=1,2,3\ldots)$

$k^2 = \left(\frac{i^2}{a^2} + \frac{j^2}{b^2}\right)\pi^2 \qquad p_{ij} = \pi^2\left(\frac{i^2}{a^2} + \frac{j^2}{b^2}\right)\sqrt{\frac{Dg}{\gamma h}} \qquad (i,j = 1,2,3\ldots)$

$F_j(y) = A_j \sin\frac{j\pi y}{b}$ .... $w(x,y) = \sum_{i,j=1}^{\infty} a_{ij} \sin\frac{i\pi x}{a}\sin\frac{j\pi y}{b}$ (9.45)

Prvý člen radu (9.45) : $w_{11}(x,y) = a_{11} \sin \frac{\pi x}{a} \sin \frac{\pi y}{b}$

$w(x,y;t) = w_{11}(x,y) \sin(pt+\alpha)$ ~ prvý hlavný kmit.

$p_{11} = \frac{\pi^2(a^2+b^2)}{a^2b^2} \sqrt{\frac{Dg}{\gamma h}}$ ~ frekvencia

Približné metódy výpočtu foriem a frekvencií priečnych kmitov dosky (Ritz - Galerkin)

Reley, Galerkin, Lejbenzon, Ritz, Michlin, Sorokin.

Pre riešenie variačnej rovnice $\delta(T_{max} - \Pi_{max}) = 0$ (9.46)

rozloží sa minimalizujúca formula vo forme rada (postupnosti)

$$w(x,y) = \sum_{i=1}^{n} a_i w_i(x,y) \qquad (9.47)$$

w_i - známe funkcie (z okraj. podmienok)

a_i - parametre z $\frac{\partial}{\partial a_i}(T_{max} - \Pi_{max}) = 0 \qquad (i=1,2,3 \dots) \quad (9.48)$

(9.47) → (9.46) ; z extrému ⇑

kde $T_{max} = \frac{\gamma p^2}{2g} \iint_{(S)} h\, w^2\, dx\, dy$

0 pre voľne opreté dosky.

$$\Pi_{max} = \frac{1}{2} \iint_{(S)} D \left\{ (\Delta w)^2 - 2(1-\sigma)\left[\frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] \right\} ds \qquad (9.49)$$

Pri výbere funkcií w_i možno vych. aj len zo geometrických podmienok. Vyhovujúce podmienke (9.46) - vyhovujú aj dynamickým podmienkam. (viď. 9.27).
Napr. pri voľnom okraji δw a $\delta\left(\frac{\partial w}{\partial n}\right)$ sa menia [?]. Preto keď $w_i(x,y)$ vyhov. (9.46), alebo (9.27), bude splnené aj $D\Delta^2 w - \mu p^2 w = 0$ a dyn. okr. podmienky: $M_L = 0 \quad N_L - \frac{\partial H_L}{\partial l} = 0$

Obyčajne $w_{ij}(x,y) = X_i(x) \cdot Y_j(y)$.

$$\left.\begin{aligned} X_i(x) &= \sin\frac{\lambda_i x}{a} + A_i \cos\frac{\lambda_i x}{a} + B_i \operatorname{sh}\frac{\lambda_i x}{a} + C_i \operatorname{ch}\frac{\lambda_i x}{a} \\ Y_j(y) &= \sin\frac{\lambda_j y}{b} + A_j \cos\frac{\lambda_j y}{b} + B_j \operatorname{sh}\frac{\lambda_j y}{b} + C_j \operatorname{ch}\frac{\lambda_j y}{b} \end{aligned}\right\} \qquad (9.52)$$

/ $\lambda_i = k_i a \qquad \lambda_j = k_j b \qquad k^4 = \frac{\mu p^2}{EJ}$

a, b - dĺžky okrajov

$A_i, B_i \dots A_j, B_j \dots \lambda_i, \lambda_j$ ~ z okr. podm. pre $X_i \quad Y_j$

Potom w dosad. do (9.49)

Vonkajšie typy integrálov: $\int_0^l X_i^2\, dx \quad \int_0^l X_i'^2\, dx \quad \int_0^l X_i''^2\, dx \quad \int_0^l X_i' X_j'\, dx = -\int_0^l X_i X_j''\, dx$

[hodnoty v tab. I, II]

Pr.: Zákl. fr. pravouhlej dosky, pevne uchyt. na okrajoch:

Okrajové podmienky: $w = 0 \quad \frac{\partial w}{\partial x} = 0$

$w = 0 \quad \frac{\partial w}{\partial y} = 0 \quad \ldots [\parallel s\ x]$

Pre 1. jednočlenné priblíženie v úlohe minimalizačnej funkcie vezmeme si

$w_1(x,y) = X_1(x)\,Y_1(y)$, vyhov. okraj. podmienkam.

$X_1(0) = X_1(a) = 0 \qquad X_1'(0) = X_1'(a) = 0$

$Y_1(0) = Y(b) = 0 \qquad Y_1'(0) = Y_1'(b) = 0$

Pre max. kin. energ.: $T_{max} = \frac{\vartheta h p^2}{2g} \iint w^2\,dx\,dy = \frac{\vartheta h p^2}{2g} \int_0^a X_1^2 dx \int_0^b Y_1^2 dy$

z tab II: $T_{max} = 1{,}0359^2 \frac{\vartheta h p^2 a b}{g}$

Pre potenc. energ.: $2\Pi_{max} = D \int_0^a\int_0^b \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 dx\,dy =$

$= D\int_0^a X_1''^2 dx \int_0^b Y_1^2 dy + 2D\int_0^a X_1 X_1'' dx \int_0^b Y_1 Y_1'' dy + D\int_0^a X_1^2 dx \int_0^b Y_1''^2 dy =$

$= D\left[\frac{518{,}52 \cdot 1{,}0359 b}{a^3} + 2\frac{12{,}775^2}{ab} + \frac{518{,}52 \cdot 1{,}0359\, a}{b^2} \right].$

Pre $a = b$: $2\Pi_{max} = \frac{1400{,}672 D}{a^2} \qquad 2T_{max} = 1{,}0731 \frac{\vartheta h p^2 a^2}{g}$

$$p_1 = \frac{36}{a^2}\sqrt{\frac{Dg}{h\vartheta}}$$

Pr.: Zákl. fr. pravouhlej dosky – 1 okraj pevne uchyt., ostatné voľné.

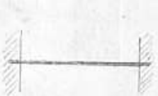
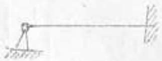
vetknutie: $x = 0$

Prvá forma kmitov: $Y_1(y) = $ konšt. (položíme $=1$), prvá frekvencia $= 0$.

$\frac{\partial w}{\partial y} = \frac{\partial^2 w}{\partial y^2} = \frac{\partial^2 w}{\partial x \partial y} = 0$

$$\left.\begin{aligned} \Pi_{max} &= \frac{D}{2}\int_0^a\int_0^b \left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx\,dy = \frac{D}{2}\int_0^b Y_1^2 dy \int_0^a X_1''^2 dx = 22{,}933\frac{Db}{a^3} \\ T_{max} &= \frac{\vartheta h p^2}{2g}\int_0^a\int_0^b X_1^2 Y_1^2\,dx\,dy = 1{,}8556\frac{\vartheta h p^2 a b}{2g} \end{aligned}\right\}$$

$$p_1 = \frac{3{,}52}{a^2}\sqrt{\frac{Dg}{\vartheta h}}$$

lab. I.	i	$\frac{1}{\ell}\int_0^\ell X_i^2\,dx$	$\ell\int_0^\ell X_i'^2\,dx$	$\ell^3\int_0^\ell X_i''^2\,dx$	$\frac{1}{\ell}\int_0^\ell X_i\,dx$
	1	0,5	4,9343	48,705	0,6366
	2	0,5	19,739	779,28	Ø
	3	0,5	44,413	3945,1	0,2122
	4	0,5	78,955	12 468	Ø
	5	0,5	123,37	30 440	0,1273
	1	1,0359	12,775	518,52	0,8445
	2	0,9984	45,977	3797,1	Ø
	3	1,000	98,920	14 619	0,3637
	4	1,000	171,58	39 940	Ø
	5	1,000	264,01	89 138	0,2314
	1	0,4996	5,5724	118,80	0,6147
	2	0,5010	21,451	1250,4	-0,0586
	3	0,5000	47,017	5433,0	0,2364
	4	0,5000	82,462	15 892	-0,0310
	5	0,5000	127,79	36998	0,1464
	1	1,8556	8,6299	22,933	1,0667
	2	0,9639	31,24	467,97	0,4252
	3	1,0014	77,763 152,83	3808,5	0,2549
	4	1,0000	↓	14 619	0,1819
	5	1,0000	205,521	39 940	0,1415

lab. II. $\ell\int_0^\ell X_i' X_j'\,dx = -\ell\int_0^\ell X_i X_j''\,dx$

	$i \backslash j$	1	2	3	4	5
	1	4,9343	Ø	Ø	Ø	Ø
	2	Ø	19,739	Ø	Ø	Ø
	3	Ø	Ø	44,413	Ø	Ø
	4	Ø	Ø	Ø	78,955	Ø
	5	Ø	Ø	Ø	Ø	123,37
	$i \backslash j$	1	2	3	4	5
	1	12,775	Ø	-9,9065	Ø	-7,7511
	2	Ø	45,977	Ø	-17,114	Ø
	3	-9,9065	Ø	98,920	Ø	-6,2833
	4	Ø	-17,114	Ø	171,58	Ø
	5	-7,7511	Ø	-6,2833	Ø	246,01
	$i \backslash j$	1	2	3	4	5
	1	5,5724	2,1424	-1,9001	1,6426	-1,4291
	2	2,1424	21,451	3,9008	-3,8226	3,5832
	3	-1,9001	3,9098	47,017	5,5836	-5,6440
	4	1,6426	-3,8226	5,5836	82,462	7,2171
	5	-1,4291	3,5832	-5,6440	7,2171	127,79

Pr.: 1. frekv. priečnych kmitov rovnomernej štvorcovej dosky, voľne opretej po okrajoch a zaťaž. aj 4 silami M v koord $\left(\frac{a}{4},\frac{a}{4}\right)$ $\left(\frac{a}{4},\frac{3a}{4}\right)$ $\left(\frac{3a}{4},\frac{a}{4}\right)$ $\cdot\left(\frac{3a}{4},\frac{3a}{4}\right)$.

Hmota dosky ~ $\mu = konšt.$

Sústredené hmoty ovplyvňujú člen na kinet. energiu.

1. priblíženie: $w = \sin\frac{\pi x}{a}\sin\frac{\pi y}{a}$

$$\Pi_{max} = \frac{D}{2}\int_0^a\int_0^b\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)^2 dx\,dy \qquad \Big/ \iint_{(S)}\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2\right]dx\,dy = 0.$$

$$\frac{\partial^2 w}{\partial x^2} = \frac{\partial^2 w}{\partial y^2} = -\frac{\pi^2}{a^2}\sin\frac{\pi x}{a}\sin\frac{\pi y}{a}$$

$$\Pi_{max} = \frac{D\pi^4}{2a^2}$$

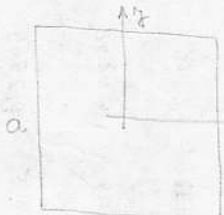
Kinet. energ. dosky: $\frac{\mu p^2 a^2}{8}$

sústredených hmôt: $\frac{Mp^2}{2}\sum_{i=1}^{4} w_i^2$ $\left[\; w_1 = \frac{1}{2} \;;\; atď. \right]$

$$2T_{max} = \frac{\mu p^2 a^2}{4} + Mp^2$$

$$p_1 = \frac{2\pi^2}{a^2}\sqrt{\frac{D}{\mu + \frac{4M}{a^2}}}$$

Pr.: 1. frekv. štvorcovej dosky s voľnými okrajmi.



predp.: $w = xy$, potom

$$\frac{\partial^2 w}{\partial x^2} = \frac{\partial^2 w}{\partial y^2} = 0 \qquad \frac{\partial^2 w}{\partial x\partial y} = 1$$

$$\Pi_{max} = D(1-\sigma)a^2 \quad \checkmark$$

$$T_{max} = \frac{\gamma h p^2}{2g}\int_{-a/2}^{a/2}\int_{-a/2}^{a/2} x^2 y^2\,dx\,dy = \frac{\gamma h p^2 a^6}{288 g} \quad \checkmark$$

pre $\sigma = 0{,}225$: $p_1 = \frac{14{,}9}{a^2}\sqrt{\frac{Dg}{\gamma h}}$

$$a_1 = \sqrt{\frac{h}{\pi f_1}\sqrt{\frac{6E}{\rho(1+\sigma)}}}$$

Pr.: 1. fr. štvorc. dosky (strana $2a$), upnutej po okraji.

P.W.: $w(x,y) = (x^2-a^2)^2(y^2-a^2)^2$, — vyhovuje okr. podmienkam

$w,\ \frac{\partial w}{\partial x},\ \frac{\partial w}{\partial y}$ pri $x = \pm a$ $y = \pm a$ .... $= 0$

$$A_{ik} = \iint_{(S)} D\Delta^2 w_k w_i\,ds \;\longrightarrow\; A_{11} = D\int_{-a}^{a}\int_{-a}^{a}\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)w\,dx\,dy = \frac{65536\,Da^{14}}{1225}$$

$$B_{11} = \frac{\gamma h}{g}\iint_{(S)} w_1\cdot w_1\,ds = \frac{\gamma h}{g}\int_{-a}^{a}\int_{-a}^{a}(x^2-a^2)^4(y^2-a^2)^4\,dx\,dy = \frac{65536\,a^{18}\gamma h}{99225\,g}$$

$A_{11} - p^2 B_{11} = 0$. $\rightarrow p_1 = \frac{9}{a^2}\sqrt{(Dg)/(\gamma h)}$

Metóda Galerkina:

$$w(x,y) = \sum_{i=1}^{m} a_i w_i(x,y) \qquad (9.53)$$, musí vyhovieť:

$$\iint_{(s)} \left(D\Delta^2 w - \mu p^2 w\right) \delta w \, ds = 0$$ / var. rovn. , a vyplniť poč. podmienky:

$$\oint M_L \,\delta\left(\frac{\partial w}{\partial n}\right) dl - \oint \left(N_L - \frac{\partial H_L}{\partial l}\right) \delta w \, dl = 0 \qquad (9.55)$$

v (9.54):

$$w(x,y) = \sum_{i=1}^{m} a_i w_i(x,y) \qquad \delta w(x,y) = \sum_{i=1}^{m} w_i(x,y)\,\delta a_i$$

prepíš. na rovn.:

$$\sum_{i=1}^{m} \left[\sum_{k=1}^{m} a_k \iint_{(s)} \left(D\Delta^2 w_k - \mu p^2 w_k\right) w_i \, ds \right] \delta a_i = 0$$, skade:

$$\sum_{k=1}^{m} a_k \iint_{(s)} \left(D\Delta^2 w_k - \mu p^2 w_k\right) w_i \, ds = 0 \qquad (i = 1, 2, \dots m) \qquad (9.56)$$

Zavedením ozn.:

$$\left.\begin{aligned} \iint_{(s)} D\Delta^2 w_k w_i \, ds &= A_{ik} \\ \iint_{(s)} \mu w_k w_i \, ds &= B_{ik} \end{aligned}\right\} \qquad (9.57)$$, potom

(9.56) možno písať:

$$(A_{i1} - p^2 B_{i1}) a_1 + \dots + (A_{im} - p^2 B_{im}) a_m = 0 \qquad / \; i = 1, 2 \dots m$$, z toho

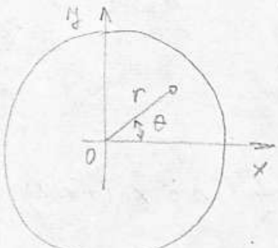
$$\begin{vmatrix} A_{11} - p^2 B_{11} & \dots & A_{1m} - p^2 B_{1m} \\ A_{21} - p^2 B_{21} & \dots & A_{2m} - p^2 B_{2m} \\ \dots & \dots & \dots \\ A_{m1} - p^2 B_{m1} & \dots & A_{mm} - p^2 B_{mm} \end{vmatrix} = 0$$

– z toho možno vypoč. m - hodnôt p^2, približne.

Najmenšie dá ≈ 1. frekvenciu.

→

Rovnice priečnych kmitov kruhovej dosky.



Tranš. operátory:

$$\left.\begin{aligned} \frac{\partial}{\partial x} &= \cos\theta\frac{\partial}{\partial r} - \frac{\sin\theta}{r}\frac{\partial}{\partial\theta} \\ \frac{\partial}{\partial y} &= \sin\theta\frac{\partial}{\partial r} + \frac{\cos\theta}{r}\frac{\partial}{\partial\theta} \end{aligned}\right\} \quad (9.59)$$

$$\frac{\partial^2 w}{\partial x^2} = \frac{\partial^2 w}{\partial r^2}\cos^2\theta - \frac{\sin 2\theta}{r}\frac{\partial^2 w}{\partial r\partial\theta} + \frac{\sin 2\theta}{r^2}\frac{\partial w}{\partial\theta} + \frac{\sin^2\theta}{r}\frac{\partial w}{\partial r} + \frac{\sin^2\theta}{r^2}\frac{\partial^2 w}{\partial\theta^2}$$

$$\frac{\partial^2 w}{\partial y^2} = \frac{\partial^2 w}{\partial r^2}\sin^2\theta + \frac{\sin 2\theta}{r}\frac{\partial^2 w}{\partial r\partial\theta} - \frac{\sin 2\theta}{r^2}\frac{\partial w}{\partial\theta} + \frac{\cos^2\theta}{r}\frac{\partial w}{\partial r} + \frac{\cos^2\theta}{r^2}\frac{\partial^2 w}{\partial\theta^2}$$

$$\frac{\partial^2 w}{\partial x\partial y} = \frac{\partial^2 w}{\partial r^2}\sin\theta\cos\theta + \frac{\cos 2\theta}{r}\frac{\partial^2 w}{\partial r\partial\theta} - \frac{\cos 2\theta}{r^2}\frac{\partial w}{\partial\theta} - \frac{\sin\theta\cos\theta}{r}\frac{\partial w}{\partial r} - \frac{\sin\theta\cos\theta}{r^2}\frac{\partial^2 w}{\partial\theta^2}.$$

[prechod na polárne koordin.]

$$\Pi_{max} = \frac{D}{2}\int_0^{2\pi}\int_0^{a}\Big[\underbrace{\Big(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2}\Big)^2}_{\text{overené } \checkmark} - 2(1-\sigma)\Big\{\frac{\partial^2 w}{\partial r^2}\Big(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2}\Big) - \Big[\frac{\partial}{\partial r}\Big(\frac{1}{r}\frac{\partial w}{\partial\theta}\Big)\Big]^2\Big\}\Big]\, r\,d\theta\,dr \quad (9.60)$$

$$\left.\begin{aligned} \Big(\frac{\partial^2 w}{\partial x^2}\Big)_{\theta=0} &= \frac{\partial^2 w}{\partial r^2} \\ \Big(\frac{\partial^2 w}{\partial y^2}\Big)_{\theta=0} &= \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2} \\ \Big(\frac{\partial^2 w}{\partial x\partial y}\Big)_{\theta=0} &= \frac{\partial}{\partial r}\Big(\frac{1}{r}\frac{\partial w}{\partial\theta}\Big) \end{aligned}\right\} \quad (9.61)$$

$$T_{max.} = \frac{\gamma h p^2}{2g}\int_0^{2\pi}\int_0^{a} w^2 r\,d\theta\,dr \quad (9.62)$$

Rovnica voľných priečnych kmitov rovnorodej kruhovej dosky:

$$\left.\begin{aligned} &\underbrace{\Big(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\Big)}_{\Delta}\Big(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2}\Big) - \frac{\gamma h p^2}{Dg}w = 0 \\ &\Delta(\Delta w) - \frac{\gamma h p^2}{Dg}w = 0 \end{aligned}\right\} \quad (9.63)$$

súhlasí s: (Matsuzawa Acustica 77.)

Na okraji $\frac{\partial}{\partial n} = \frac{\partial}{\partial r}$ $\quad \frac{\partial}{\partial l} = \frac{\partial}{r\partial\theta}$, potom var. rovn.:

$$\iint_{(s)}\Big(D\Delta^2 w - \frac{\gamma h p^2}{g}w\Big)\delta w\, r\,d\theta\,dr - \oint M_L\,\delta\Big(\frac{\partial w}{\partial r}\Big) r\,d\theta + \oint\Big(N_L - \frac{1}{r}\frac{\partial H_L}{\partial\theta}\Big)\delta w\, r\,d\theta = 0 \quad (9.66)$$

Na okraji:

$$\left.\begin{aligned} M_L &= -D\Big[\frac{\partial^2 w}{\partial r^2} + \sigma\Big(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2}\Big)\Big] && \sim \text{ohyb. mom.} \\ H_L &= D(1-\sigma)\frac{\partial}{\partial r}\Big(\frac{1}{r}\frac{\partial w}{\partial\theta}\Big) && \sim \text{krút. mom.} \\ N_L &= -D\frac{\partial}{\partial r}\Big(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2}\Big) && \sim \text{priečna sila} \end{aligned}\right\} \quad (9.67)$$

$$\Delta^2 w - k^4 w = 0$$ ~ bude splnené pre formy, kt. vyhov. rovniciam:

$$\Delta w + k^2 w = 0 \qquad \Delta w - k^2 w = 0, \quad \text{alebo}$$

$$\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} \pm k^2 w = 0 \qquad / \quad k^4 = \frac{\gamma h p^2}{D g} \qquad (9.68).$$

Funkciu, vyhov. (9.68) hľadáme vo tvare: $w(r,\theta) = R(r)\,\phi(\theta)$ ... po dosadení

$$\left(\frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr}\right)\phi + \frac{1}{r^2}R\frac{d^2\phi}{d\theta^2} \pm k^2 R\phi = 0 \rightarrow$$

$$\frac{\frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr} \pm k^2 R}{-\frac{R}{r^2}} = \frac{\frac{d^2\phi}{d\theta^2}}{\phi} = -m^2 \qquad (9.70)$$

Rozpísaním:

$$\left.\begin{aligned} &\frac{d^2\phi}{d\theta^2} + m^2\phi = 0 \quad \rightarrow \overset{(9.72)}{\phi(\theta) = \alpha \sin m\theta} \\ &\frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr} + \left(\pm k^2 - \frac{m^2}{r^2}\right)R = 0 \end{aligned}\right\} \qquad (9.71)$$

Besselova rovnica

Z Bes. rovn., pri $(+)k^2$, keď zam. $x = kr$:

$$\frac{d^2R}{dx^2} + \frac{1}{x}\frac{dR}{dx} + \left(1 - \frac{m^2}{x^2}\right)R = 0$$ ... Riešením je Besselova funkcia 1. druhu m. rádu. : $R_1(r) = J_m(kr)$.

[Riešením je aj Neumanova f. – ale pri $r = 0$: $\rightarrow \infty$]

Z Bes. rovn., pri $(-)k^2$, keď $y = ikr$ $/\ i = \sqrt{-1}$

$$\frac{d^2R}{dy^2} + \frac{1}{y}\frac{dR}{dy} + \left(1 - \frac{m^2}{y^2}\right)R = 0$$ ..... Riešenie: $R_2(r) = J_m(ikr)$

Všeobecný integrál: $R = A J_m(kr) + B J_m(ikr)$

Všeob. integrál rovnice (9.68) potom má tvar:

$$w(r,\theta) = \alpha \sin m\theta \left[J_m(kr) + \lambda J_m(ikr)\right] \qquad (9.74)$$

Do 2 okraj. podm. dosaď (9.74); α, λ – sa vyp. z okraj. podmienok (uchyt. okrajov). po vyhl. α, λ dost. transcendentnú rovnicu s $\underline{k}$. Jej korene nájdeme z tab. Bess. funkcií. – dávajú frekvencie kmitov.

Uzlové čiary na doske .... geom. miesta nulových hodnôt funkcie (9.74)

1) $\sin m\theta = 0 \qquad \rightarrow \theta = 0, \frac{\pi}{m}, \frac{2\pi}{m}, \ldots$ – uzlové priemery

2) $J_m(kr) + \lambda J_m(ikr) = 0$

pre kruhovú dosku s priemerom $\underline{a}$; uchyt. po obvode:

$$w_a = 0 \qquad \left(\frac{\partial w}{\partial r}\right)_a = 0 \qquad (9.75)$$

Formy, nemajúce uzlové priemery $(m = 0)$ v tom prípade dostaneme z (9.71) dosadením $w = R(r)$. [nezáv. od θ] $\rightarrow \frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr} \pm k^2 R = 0 \rightarrow$

Všeob. integrál $w(r) = A J_0(kr) + B J_0(ikr)$ [Bes. f. 1. druhu; 0 rádu]

Pre $r = a$, pevne uch. na okraji:

$$A J_0(ka) + B J_0(ika) = 0$$
$$A J_0'(ka) + B J_0'(ika) = 0 \quad , \quad \text{alebo, keď použ.:} \quad \left.\begin{aligned} J_0'(ka) &= -J_1(ka) \\ J_0'(ika) &= -i J_1(ika) \end{aligned}\right\} \quad (9.76)$$

$$\left.\begin{aligned} &A J_0(ka) + B J_0(ika) = 0 \\ &+A J_1(ka) + B i J_1(ika) = 0 \end{aligned}\right\} \rightarrow$$

$$J_0(ka) J_1(ika) i + J_0(ika) J_1(ka) = 0 \qquad (9.77)$$

Najmenší koreň (9.77) [Reley] $ka = 3{,}20$ $\rightarrow$

$$p_1 = \frac{10{,}24}{a^2}\sqrt{\frac{Dg}{\gamma h}} \Rightarrow$$ 

deriv. len podľa „r“ !

dá sa odvodiť z Angota (14.2.78)

V obecnom prípade okr. podmienky (9.75) vedú k rovniciam

$$\left.\begin{aligned} &A J_m(ka) + B J_m(ika) = 0 \\ &A J_m'(ka) + B i J_m'(ika) = 0 \end{aligned}\right| \quad \text{formuly:} \quad \begin{aligned} J_m'(kr) &= \tfrac{1}{2} k \left[J_{m-1}(kr) - J_{m+1}(kr) \right] \\ J_m(ikr) &= \tfrac{1}{2} k \left[J_{m-1}(ikr) + J_{m+1}(ikr) \right] \end{aligned}$$

$$I_m'(kr) = \frac{k}{2}\left[I_{m+1} + I_{m-1} \right]$$

$\rightarrow$

Frekvenčná rovnica:

$$\begin{vmatrix} J_m(ka) & J_m(ika) \\ J_{m-1}(ka) - J_{m+1}(ka) & J_{m-1}(ika) + J_{m+1}(ika) \end{vmatrix} = 0 \qquad (9.78)$$

Pre každé n, rovnica (9.78) dáva ∞ koreňov (súmerné kružnice - uzly)

Hodnoty ka pre prvé formy: (pre kruhovú, po obv. uch. dosku)

uzlové kružnice	uzlové priemery		
	$n = 0$	$n = 1$	$n = 2$
$m = 0$	3,19	4,61	5,90
$m = 1$	6,30	7,81	9,40
$m = 2$	9,43	10,98	12,60

Pre rovnorodý kolúč, pevne uch. na polomeri $b < a$:

$$(w)_{r=b} = 0 \qquad \left(\frac{\partial w}{\partial r}\right)_{r=b} = 0 \qquad (9.79)$$

Obecný integr. rovnice (9.73) $\left[\frac{d^2R}{dx^2} + \frac{1}{x}\frac{dR}{dx} + \left(1 - \frac{n^2}{x^2}\right) R = 0 \right]$ obsahuje f. Bess. 1. a 2. druhu. Číslo konštant bude 4 [a nie 2].

$$w(r, A) = \mathcal{L} \sin n\theta \left[J_m(kr) + A N_m(kr) + B J_m(ikr) + C N_m(ikr) \right]$$

/ N_m – Neumanova f. n. ~~druhu~~ rádu.

Konštanty sa vyp. zo 4 okr. podmienok (2 na vonk., 2 na vnút. (9.79) okraji)

Pr.: 1. fr. rovnorodej kruhovej, po obvode uch. dosky:

$$2\Pi_{max} = D\int_0^{2\pi}\int_0^{a}\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right)^2 r\, d\theta\, dr \qquad (9.80)$$

Ako minimalizujúcu formu s n uzlovými priemermi použ. výraz:

$$w(r,\theta) = \alpha\, r^s (a^2 - r^2)^2 \cos n\theta \qquad (9.81)$$

s – parameter, určí. z podmienky pre minimum kvadráta prvej frekvencie.

$(9.81) \to (9.80)$:

$$2\Pi_{max} = D\pi\int_0^{a}\Bigg(\underbrace{\frac{\partial^2 w_0}{\partial r^2} + \frac{1}{r}\frac{\partial w_0}{\partial r} - \frac{n^2}{r^2} w_0}_{=\alpha\left[a^4 r^{s-2}(s^2-n^2) - 2a^2 r^s\{(s+2)^2 - n^2\} + r^{s+2}\{(s+4)^2 - n^2\}\right]}\Bigg)^2 r\,.\,dr \qquad / \; w_0 = \alpha r^s (a^2 - r^2)^2$$

Pre $n = 0$ ~ súmerné kmity, bez uzl. priemerov:

$$\int_0^{a}\left(\frac{\partial^2 w_0}{\partial r^2} + \frac{1}{r}\frac{\partial w_0}{\partial r}\right)^2 r\,dr = \alpha^2 a^{2s+6}\left[\frac{s^4}{2s-2} + \frac{4(s+2)^4}{2s+2} + \frac{(s+4)^4}{2s+6} - \frac{4s^2(s+2)^2}{2s} + \frac{2s^2(s+4)^2}{2s+2} - \frac{4(s+2)(s+4)^2}{2s+4}\right].$$

$$2T_{max} = \frac{\pi \gamma h p^2}{g}\int_0^{a} w_0^2 r\,dr = \frac{\pi \gamma h p^2 \alpha^2}{g}\int_0^{a}(a^4 r^s - 2a^2 r^{s+2} + r^{s+4})^2 r\,dr =$$

$$= \frac{\pi \gamma p^2 \alpha^2 h}{g}\left[\frac{1}{2s+2} + \frac{6}{2s+6} + \frac{1}{2s+10} - \frac{4}{2s+4} - \frac{4}{2s+8}\right] a^{2s+10}.$$

$$p_1^2 = \frac{\Pi_{max}}{T'_{max}}$$; tu vystupuje s.

Z minima $p_1^2 \to s = 0$ $\to$

$$\left.\begin{aligned} 2\Pi_{max} &= \frac{32}{3} D\pi \alpha^2 a^6 \\ 2T_{max} &= \frac{\pi \gamma h p^2 \alpha^2 a^{10}}{10 g} \end{aligned}\right\} \to \; p_1 = \frac{10{,}33}{a^2}\sqrt{\frac{Dg}{h\gamma}}$$

Pre fr. s 1 uzl. priemerom, treba vziať $s = 1$:

$$\left.\begin{aligned} 2\Pi_{max} &= 8\alpha^2 D\pi a^8 \\ 2T_{max} &= 0{,}0167\,\frac{\alpha^2 \gamma h \pi p^2 a^{12}}{g} \end{aligned}\right\} \to \; p_1 = \frac{21{,}9}{a^2}\sqrt{\frac{Dg}{\gamma h}}$$

Pri 2 uzl. priemeroch: $n = 2$ pre rôzne hodnoty s:

$s =$	1,5	1,75	2,0	3,0
$p_1 a^2 \sqrt{\frac{\gamma h}{D g}} =$	36,2	35,5	36,5	45,8.

Najpresnejšie, pre $s = 1{,}7$: $p_1 = \dfrac{35{,}4}{a^2}\sqrt{\dfrac{Dg}{\gamma h}}$

Pr.: Formy vynútených kmitov rovnorodej kruhovej dosky s polomerom $\underline{a}$. pevne uch. okrajmi. Pôsobí po povrchu rovnomerne rozlož. sila:

$$f = A \sin \omega t$$

$$D\Delta^2 w - \frac{\gamma h \omega^2}{g} w = A$$

Ob. int. ľavej časti: $R_1(r) = C_1 J_0(kr) + C_2 J_0(ikr)$ (9.82)

Čiast. int. : $R_2 = -\frac{A}{Dk^4}$ / $k^4 = \frac{\gamma h \omega^2}{Dg}$

Všeob. int. : $w = R_1 + R_2 = C_1 J_0(kr) + C_2 J_0(ikr) - \frac{A}{Dk^4}$ (9.83)

Po vonk. obvode: $w(a) = 0$ $w'(a) = 0$

$$C_1 J_0(ka) + C_2 J_0(ika) = \frac{A}{Dk^4}$$
$$C_1 J_0'(ka) + C_2 i J_0'(ika) = 0$$

/ použ. (9.76)

$$\left.\begin{aligned} C_1 J_0(ka) + C_2 J_0(ika) &= \frac{A}{Dk^4} \\ -C_1 J_1(ka) + C_2 J_1(ika) &= 0 \end{aligned}\right\} \rightarrow$$

$$C_1 = \frac{A}{Dk^4} \frac{J_1(ika)}{J_0(ka) J_1(ika) + J_1(ka) J_0(ika)}$$

$$C_2 = \frac{A}{Dk^4} \frac{J_1(ka)}{J_0(ka) J_1(ika) + J_1(ka) J_0(ika)}$$

Forma vynútených kmitov:

$$w(r) = \frac{A}{Dk^4}\left[\frac{J_0(kr) J_1(ika) + J_0(ikr) J_1(ka)}{J_0(ka) J_1(ika) + J_0(ika) J_1(ka)} - 1\right].$$

Keď $J_0(ka) J_1(ika) + J_0(ika) J_1(ka) = 0$, ako vypl. z (9.77), fr. dosky = fr. vynuc. síl. Vznikne rezonancia.

Kmity membrán

Merhaut - Příručka elektroakustiky (Praha 1964)

Obdĺžnikové membrány : riešením vlnovej rovnice : $\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=\frac{1}{c_0^2}\frac{\partial^2\phi}{dt^2}$

v pol. súr. $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2\phi}{\partial\varphi^2}=\frac{1}{c_0^2}\frac{\partial^2\phi}{\partial t^2}$ ..... $\phi = X(x).Y(y).e^{j\omega t}$

Integrál vln. rovnice : $\phi = A\cos(k_x X-\varphi_x)\cos(k_y y-\varphi_y)\cos(k c_0 t-\varphi)$

$/\,k^2 = k_x^2+k_y^2 \qquad k=\frac{\omega}{c_0}$

Pre okraj. podmienku : $u=\emptyset$ ...

$$u = \varphi_{mn}(x,y)\cos(\omega_{mn}t-\varphi)$$
$$\varphi_{mn}(x,y) = \sin\left(\frac{\pi m x}{a}\right)\sin\left(\frac{\pi n y}{b}\right)$$
$$f_{mn} = \frac{c_0}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$$
$$c_0=\sqrt{\frac{\nu}{\sigma}}$$

$m, n = 1, 2, 3\ldots$ ~ rád vlň. kmitu.
c_0 ~ rýchl. šír. priečn. vlny.
ν ~ napätie na jednotku dĺžky membrány
σ ~ hmota membr. na jedn. plochy

Kruhová membrána :

$$\left.\begin{array}{r}\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial u}{\partial r}\right]+\frac{1}{r^2}\frac{\partial^2 u}{\partial\varphi^2}=\frac{1}{c_0^2}\frac{\partial^2 u}{\partial t^2}\\ u = R(r)\Phi(\varphi)e^{j\omega t}\end{array}\right\} u = \cos m(\varphi-\alpha)\,J_m(kr)\cos(\omega t-\Omega)$$

/ α – udáva polohu membr. voči súr. systému
/ Ω – časový začiatok. } väčšinou $=\emptyset$

Membrána s polomerom R, na okrajoch upevnená :

$J_{(m)}(kr)=\emptyset \quad\rightarrow\quad f_{mn} = \frac{c}{2R}\beta_{mn} \qquad c=\sqrt{\frac{\nu}{\sigma}}$

[m n – u, Morse : Vibration and Sound.]

uzl. kružn.

β_{ij}	1	2	3
0	0,7655	1,7571	2,7546
1	1,2197	2,2330	3,2383
2	1,6347	2,6793	3,6987

diamet.

Pre veľké n :

$\beta_{mn} = n+\frac{m}{2}-\frac{1}{4}$

n - kružn.
m - polomer } uzl. čiary

Dosky : obecne : $\nabla^4 u+\frac{3\rho(1-\sigma^2)}{Eh^2}\frac{\partial^2 u}{\partial t^2}=\emptyset$

ρ - merná hmota
σ ~ Poisson. konšt.
E ~ mod. pružn. v ť.
h ~ polov. hrúbka dosky.

Kruhová doska, votknutá na okraji : $f_{mn} = \frac{\pi h}{2R^2}\sqrt{\frac{E}{3\rho(1-\sigma^2)}}\,(\beta_{mn})^2$ ~ norm. frekv.

Voľná doska : $f_{02} = \frac{0{,}824h}{R^2}\sqrt{\frac{E}{\rho(1-\sigma^2)}}$ $\qquad f_{21} = 2{,}134\, f_{02}$

V strede upevn. doska : $f_{03} = \frac{0{,}344h}{R^2}\sqrt{\frac{E}{\rho(1-\sigma^2)}}$

Na okraji podoprené : $f_{02} = \frac{0{,}466h}{R^2}\sqrt{\frac{E}{\rho(1-\sigma^2)}}$

β_{ij}	1	2	3
0	1,015	2,007	3,000
1	1,468	2,483	3,490
2	1,879	2,992	4,000

$n\to\infty$: $\beta_{mn}\to n+\frac{m}{2}$

Theory of Vibration with Applications (William T. Thomson), str. 292.

Kmitanie membrán: $\rho\, dx\, dy \frac{\partial^2 w}{\partial t^2} = T\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) dx\, dy + p(x,y)\, dx\, dy$

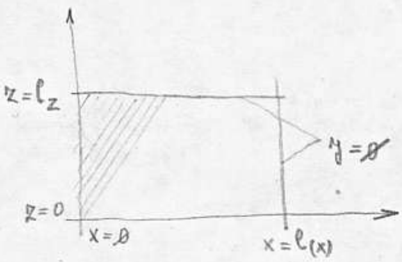
$$\frac{\partial^2 w}{\partial t^2} = c^2 \nabla^2 w + \frac{1}{\rho} p(x,y) \qquad \Big/ \; \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \quad c = \sqrt{\frac{T}{\rho}}$$

$$\omega = \frac{\alpha_{mn}}{a}\sqrt{\frac{T}{\rho}}$$

pri $y(a) = 0$ $\quad r = a$
$\quad r = 0$

$n \backslash m$	1	2
0	$\alpha = 2{,}404$	5,520
1	3,832	7,016
2	5,135	8,417

Kinsler . Frey : Fundamentals of acoustics :



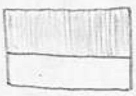
$$y = Y \sin k_x x \sin k_z z \cdot e^{j\omega t}$$

Y - max. ampl.

$k_x = \frac{m_x \pi}{l_x}$ $\qquad m_x = 1, 2, 3 \ldots$

$k_z = \frac{m_z \pi}{l_{z}}$ $\qquad m_z = 1, 2, 3 \ldots$

$m_z = 2$



$m_x = 1$

$$f = \frac{\omega}{2\pi} = \frac{c}{2}\sqrt{\left(\frac{m_x}{l_x}\right)^2 + \left(\frac{m_z}{l_z}\right)^2}$$

[Predp. rieš. rovnice $\frac{d^2\psi}{dr^2} + \frac{1}{r}\frac{d\psi}{dr} + k^2\psi = 0$ (pre kruh. membr., voľné súmerné kmity), kde $k^2 = \frac{\omega^2}{c^2} = \frac{\sigma\omega^2}{T}$
vo forme $y = \psi e^{j\omega t}$, $\psi = a_0 + a_1 r + a_2 r^2 + \ldots$, po der. a dosaď do rovn. získ. koeficienty a tým ψ.
$\psi = A J_0(kr)$. $\approx a_0\left[1 - \frac{(kr)^2}{2^2} + \frac{(kr)^4}{2^2 4^2} - \frac{(kr)^6}{2^2 4^2 6^2} + \ldots\right]$]

Vlastnosti Bess. funkcií:

$J_0(x) \to 1 - \frac{x^2}{4}$, keď $x \to 0$

$J_1(x) \to \frac{x}{2}\left(1 - \frac{x^2}{8}\right)$ $\quad x \to 0$

$J_0(x) \to \sqrt{\frac{2}{\pi x}} \cos\left(x - \frac{\pi}{4}\right)$ $\quad x \to \infty$

$J_1(x) \to \sqrt{\frac{2}{\pi x}} \sin\left(x - \frac{\pi}{4}\right)$. $\quad x \to \infty$

$\int x J_0(x)\, dx = x J_1(x)$

$\int x J_0^2(x)\, dx = \frac{x^2}{2}\left[J_0^2(x) + J_1^2(x)\right]$

$\int J_1(x)\, dx = -J_0(x)$

$J_0(x) + J_2(x) = \frac{2}{x} J_1(x)$

$J_0(x) = 0$ pre $x = 2{,}405$; $5{,}520$; $8{,}654$; $11{,}792$, ... aprox: $x = \left(n - \frac{1}{4}\right)\pi$

Okrajové podmienky pri hrane, pre kruhovú membránu: $y = 0$ pri $r = a$; tak $J_0(ka) = 0$

zákl. frekvencie sú preto: $f_1 = \frac{\omega_1}{2\pi} = \frac{k_1 c}{2\pi} = \frac{2{,}405}{2\pi a} c = \frac{2{,}405}{2\pi a}\sqrt{\frac{T}{\sigma}}$

$$f_2 = 2{,}295 f_1 \qquad f_3 = 3{,}598 f_1 \qquad f_4 = 4{,}90 f_1 \quad \ldots$$

Potom napr. $k_2 r = \frac{5{,}520}{a} r \; (= 2{,}405) \rightarrow r = 0{,}436 a$ / pre 1 vyššie harmonické (keď $J_0 = 0$.

Keď centrálna časť je vytlačená, susedný kruh je tlač. dole – a opačne.

Priemerná amplitúda premiestnenia povrchu membrány: $\bar{\Psi}_m = \frac{\int_S \Psi_m(r) \cdot dS}{\pi a^2} = \frac{\int_0^a A_m J_0(k_m r) \cdot 2\pi r \, dr}{\pi a^2}$

$$\bar{\Psi}_m = \frac{2A_m}{k_m a} J_1(k_m a)$$

$$\bar{\Psi}_1 = \frac{2A_1}{k_1 a} J_1(k_1 a) = \frac{2A_1}{2{,}405} J_1(2{,}405) = 0{,}432 A_1 \quad ; \quad \bar{\Psi}_2 = -0{,}123 A_2$$

Vynútené kmity membrán (kruh.)

Keď pôs. sila $p = P\cos\omega t$; $p = Pe^{j\omega t}$:

$$\frac{\partial^2 y}{\partial t^2} = c^2 \nabla_r^2 y + \frac{P}{\sigma} e^{j\omega t}$$

Keď predp. $y = \Psi e^{j\omega t}$ .... potom $\nabla_r^2 \Psi + k^2 \Psi = -\frac{P}{\sigma c^2} = -\frac{P}{T}$ / $k = \frac{\omega}{c}$

$$\Psi = A J_0(kr) - \frac{P}{k^2 T}$$

Pri okr. podm.: $\Psi = 0$ pri $r = 0$ ..... $A = \frac{P}{k^2 T} \frac{1}{J_0(ka)}$ →

$$y = \frac{P}{k^2 T}\left[\frac{J_0(kr)}{J_0(ka)} - 1\right] e^{j\omega t}$$

/ teda pre hociaké koordináty membrány $\Psi = \frac{P}{T}\left[\frac{J_0(kr) - J_0(ka)}{k^2 J_0(ka)}\right]$

Stredná ampl.: $\bar{y} = \ldots = \frac{P}{k^2 T} \frac{J_2(ka)}{J_0(ka)} e^{j\omega t}$

Pre ľubov. frekv., $ka \ll 1$: $J_0(ka) \approx 1 - \frac{(ka)^2}{4}$; $J_2(ka) \approx \frac{(ka)^2}{8}\left[1 - \frac{(ka)^2}{12}\right]$

$$\bar{y} \approx \frac{Pa^2}{8T}\left[1 + \frac{(ka)^2}{6}\right] e^{j\omega t}$$

Nesúmerné voľné kmity kruh. membrán ;

$f_1 = \frac{2{,}405}{2\pi a}\sqrt{\frac{T}{\sigma}}$ / $\sqrt{\frac{T}{\sigma}} = c$

Relatívne frekvencie: f_{mn} (n – uzlové kružnice, m – uzl. ~~polom~~ priemery)

$f_{01} = 1{,}0$	$f_{11} = 1{,}593$	$f_{21} = 2{,}135$
$f_{02} = 2{,}295$	$f_{12} = 2{,}917$	$f_{22} = 3{,}500$
$f_{03} = 3{,}598$	$f_{13} = 4{,}230$	$f_{23} = 4{,}832$

Kmity tenkých dosák

$$\frac{\partial^2 y}{dt^2} = -\frac{\kappa^2 Y}{\rho(1-\sigma^2)} \nabla_r^4 y$$

/ ρ ~ kg/m³
σ ~ Poiss. koeif.
Y – Youngov modul
κ ~ povrchový polomer zotrvačnosti
Pre rovnomernú hrúbku: $\kappa = \frac{t}{\sqrt{12}}$

→

Jednoduché harmonické kmity (tenkých dosák) — Znamienko?

Predp.: $y = \Psi(r) \cdot e^{j\omega t}$ $\rightarrow$ $\nabla_r^4 \Psi = \frac{\omega^2 \rho (1-\sigma^2)}{\kappa^2 Y} \Psi = K^4 \Psi$ $\rightarrow$

$$(\nabla_r^4 - K^4)\Psi = 0 \rightsquigarrow (\nabla_r^2 + K^2)\Psi = 0 \; ; \; (\nabla_r^2 - K^2)\Psi = 0$$

Prvá rovn. je ident. pre kruh. membr.: .... $\Psi = AJ_0(Kr)$

Z druhej .... $\Psi = BJ_0(jKr)$, obvykle píš.: $\Psi = BI_0(Kr)$

$$I_m(x) = j^{-m} J_m(jx) \qquad I_0(x) = J_0(jx) = 1 + \frac{x^2}{2^2} + \frac{x^4}{2^2 4^2} + \frac{x^6}{2^2 4^2 6^2} + \dots$$

Riešenie: $\Psi = AJ_0(Kr) + BI_0(Kr)$

$$\int x I_0(x)\,dx = x I_1(x)$$
$$\int I_1(x)\,dx = I_0(x)$$
$$I_0(x) - I_2(x) = \frac{2}{x} I_1(x)$$

Okrajové podmienky:

Keď pre $r = a$ $\quad \Psi = 0, \; \frac{\partial \Psi}{\partial r} = 0$:

$$0 = AJ_0(Ka) + BI_0(Ka), \quad 0 = -AKJ_1(Ka) + BKI_1(Ka)$$

$$\frac{dJ_0(Kr)}{dr} = -KJ_1(Kr) \qquad \frac{dI_0(Kr)}{dr} = KI_1(Kr)$$

$$\downarrow$$

$$\frac{J_0(Ka)}{J_1(Ka)} = -\frac{I_0(Ka)}{I_1(Ka)} \implies Ka = 3{,}20;\ 6{,}30;\ 9{,}44;\ 12{,}57;\ \dots$$

aprox.: $Ka = m\pi$ / $m = 1, 2, 3, \dots$

$$\omega = \kappa K^2 \sqrt{\frac{Y}{\rho(1-\sigma^2)}}$$

Potom zákl. frekvencia (keďže $\kappa = \frac{t}{\sqrt{12}}$; $K^2 = \frac{(Ka)^2}{a^2}$):

$$f_1 = \frac{3{,}2^2}{2\pi a^2} \frac{t}{\sqrt{12}} \sqrt{\frac{Y}{\rho(1-\sigma^2)}} = 0{,}47 \frac{t}{a^2} \sqrt{\frac{Y}{\rho(1-\sigma^2)}}$$ ~ normálne (niekruh.) fr.

$$f_2 = \left(\frac{6{,}3}{3{,}2}\right)^2 f_1 = 3{,}88 f_1 \qquad f_3 = 8{,}70 f_1 \qquad \dots$$

Pre kruhovú dosku; zákl. mód:

$$y_1 = \cos(\omega_1 t + \phi_1)\left[A_1 J_0\left(\frac{3{,}2}{a} r\right) + B_1 I_0\left(\frac{3{,}2}{a} r\right)\right]$$, z okr. podm....

$$B_1 = -A_1 \frac{J_0(K_1 a)}{I_0(K_1 a)} = -A_1 \frac{J_0(3{,}2)}{I_0(3{,}2)} = +0{,}0555 A_1$$, potom

$$y_1 = A_1 \cos(\omega_1 t + \phi_1)\left[J_0\left(\frac{3{,}2}{a} r\right) + 0{,}0555\, I_0\left(\frac{3{,}2}{a} r\right)\right]$$

„stredná" výchylka: $\bar{\Psi}_1 = 0{,}326 A_1 = 0{,}309 y_0 \quad (y_0 = 1{,}0555 A_1)$

Zaťažené a rozkmitávané dosky

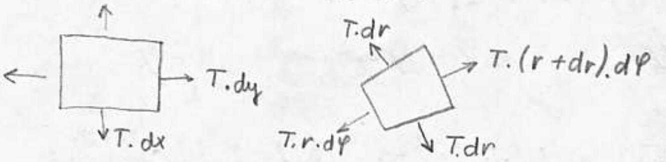
Kinsler. Frey : Fundamentals of acoustics :

str. 106. Pr : membrána kondenz. mikrofónu zo kruhovej Al fólie priemeru 3 cm , hrúbky 2.10^{-5} m môže byť rozťiahnutá do max . 2.10^{8} N/m^2.

Aké je max. nap T v N/m – na čo Al fól. môže byť rozťiahnutá ?
Aká môže byť jej zákl. frekv. – keď sa roztiahn. do tohoto napätia ?
Aká môže byť max. ampl. v centre zvuk. vlny 500 Hz s tlakovou amplitúdou 2 N/m^2 ?
Aká môže byť priemerná amplitúda ?

[4000 N/m] [6950 Hz] [$2{,}84 . 10^{-8}$ m] [$1{,}41 . 10^{-8}$ m]

Morse : Vibration and Sound. :

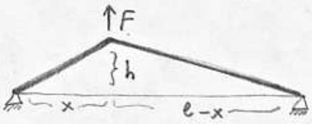


Vlnová rovnica pre membrány :

$$\nabla^2 \eta = \frac{1}{c^2}\frac{\partial^2 \eta}{\partial t^2} \qquad / \; c = \sqrt{\frac{T}{\sigma}}$$

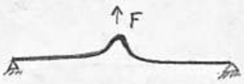
Laplaceov operátor pre pravouhl. súradn. : $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = \nabla^2$

pre polárne súr. : $\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2}$



vert. komponenty sily F : $T\frac{h}{x}$; $T\frac{h}{l-x}$

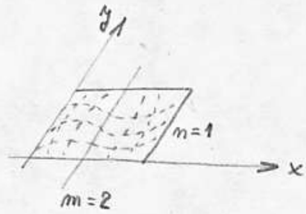
výchylka : $h = Fx\,\frac{l-x}{Tl}$



v strede kruh. membr. : $\eta = \frac{2F}{T}\ln\frac{a}{r}$

Na obvode uchyt. membrána , so stranami a, b :

$$\eta = A\,\Psi_{mn}(x,y)\cos(2\pi\nu_{mn}t - \phi)$$
$$\Psi_{mn}(x,y) = \sin\left(\frac{\pi m x}{a}\right)\sin\left(\frac{\pi n y}{b}\right)$$
$$\nu_{mn} = \frac{1}{2}\sqrt{\frac{T}{\sigma}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

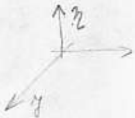


Kmity dosák :

$$\nabla^4\eta + \frac{3\rho(1-s^2)}{Qh^2}\frac{\partial^2\eta}{\partial t^2} = 0$$

bez vonkajš. zaťaž.

ρ ~ hustota materiálu
Q – modul pruž.
s – Poiss. č.
h – polov. hrúbka dosky.



$$(\nabla^2 - \gamma^2)(\nabla^2 + \gamma^2)\,\Psi = 0 \qquad \gamma^4 = \frac{12\pi^2\nu^2\rho(1-s^2)}{Qh^2} = \text{„}\omega^2 k^4\text{"}$$

$3.(2\pi\nu)^2$

ako na predošl. strane

Pre $\Psi(a,\varphi) = 0$; $\left(\frac{\partial\Psi}{\partial r}\right)_{r=a} = 0$: $\beta_{01} = 1{,}015$ $\beta_{02} = 2{,}007$

$\beta_{11} = 1{,}468$

$\beta_{21} = 1{,}879$

[.... o 3 strany dopredu]

$$\gamma_{mn} = \left(\frac{\pi}{a}\right)\beta_{mn}$$

$$\nu_{mn} = \frac{\pi h}{2a^2}\sqrt{\frac{Q}{3\rho(1-s^2)}}\,(\beta_{mn})^2$$

$$\nu_{01} = 0{,}9342\left(\frac{h}{a^2}\right)\sqrt{\frac{Q}{\rho(1-s^2)}}$$

$\nu_{11} = 2{,}091\,\nu_{01}$ $\nu_{21} = 3{,}426\,\nu_{01}$ $\nu_{02} = 3{,}909\,\nu_{01}$ $\nu_{12} = 5{,}983\,\nu_{01}$

Núlený pohyb (dosky) (Vyrišenie kmity dosky)

Namiesto pohybovej rovnice, pre kondenz. mikrofon s fól. membr. [20.6]

$$\sigma\frac{\partial^2\eta}{\partial t^2}=T\nabla^2\eta+\underbrace{i\omega\pi\bar{\eta}}_{-R\frac{\partial m}{\partial t}}+Fe^{-i\omega t}$$

/ str. ampl.: $\bar{\eta}=\frac{1}{\pi a^2}\int_0^{2\pi}d\varphi\int_0^a\eta\, r\,dr$

, píšeme: $$\frac{\partial^2\eta}{\partial t^2}=-\frac{4\pi^2\nu^2}{\gamma^4}\nabla^4\eta+\frac{F_0}{h\rho}e^{-2\pi i\nu t}-\frac{R}{\pi a^2 h\rho}\iint\left(\frac{\partial m}{\partial t}\right)r\,d\varphi dr \qquad \boxed{21.7}$$

/ $\gamma^4=[12\pi^2\nu^2\rho(1-s^2)/Qh^2]$

Zvol. si funkciu: $$\eta=\frac{Ae^{-2\pi i\nu t}}{I_1(\gamma a)}\left[I_1(\gamma a)J_0(\gamma r)+J_1(\gamma a)I_0(\gamma r)-I_1(\gamma a)J_0(\gamma a)-J_1(\gamma a)I(\gamma a)\right],$$

ktorá má 0 hodnotu v $r=a$, a ktorá má nasled. priemerné hodnoty:

$$\frac{1}{\pi a^2}\iint\eta r\,dr\,d\varphi=\frac{A}{I_1(\gamma a)}\left[I_1(\gamma a)J_2(\gamma a)-J_1(\gamma a)I_2(\gamma a)\right]e^{-2\pi i\nu t}$$

Postupne porovn. výsledky tohto výpočtu s jednotlivými – v predchádz. časti o membránach prebranými. Použ. tie isté premenné:

$$\mu=2{,}405\frac{\nu}{\nu_{01}} \quad a \quad \zeta=0{,}3828\frac{Z}{2h\rho\nu_{01}}$$; ako v [20.8]:

$$A=\frac{Fa^2/T\mu^2}{J_0(\mu)+(\zeta/i\mu)J_2(\mu)}$$; $$\left.\begin{array}{l}\mu=(\omega a/c)=\pi\beta_{01}\frac{\nu}{\nu_{01}}\\ \zeta=\frac{\pi a}{\sigma c}=\frac{\pi\beta_{01}}{2\sigma\nu_1}=\theta-i\chi=\frac{a}{\sigma c}(R-iX)\end{array}\right\}$$

A – ampl   T – napätie

F – sila ν_{0m} – rez. frekv.

ζ – imped. parameter

Jediný rozdiel: pre ν_{01}; namiesto hodnoty z [19.7]: $\nu_{01}=0{,}38274\frac{1}{a}\sqrt{\frac{T}{\sigma}}$ $\nu_{11}=1{,}5933\nu_{01}$ $\nu_{21}=2{,}1355\nu_{01}$

$\nu_{02}=2{,}2954\nu_{01}$ $\nu_{31}=2{,}6531\nu_{01}$ $\nu_{12}=2{,}9173\nu_{01}$

(kruhové membr.),

, použ. hodnoty z rovnice [21,6] – pre základ. frekvencie dosky:

$$\nu_{mn}=\frac{\pi h}{2a^2}\sqrt{\frac{Q}{3\rho(1-s^2)}}(\beta_{mn})^2 \qquad \beta_{01}=1{,}015\ ;\ \beta_{11}=1{,}468 \text{ atď. [v predchádz.]}$$

$$\nu_{01}=0{,}9342\left(\frac{h}{a^2}\right)\sqrt{\frac{Q}{\rho(1-s^2)}} \qquad \nu_{11}=2{,}091\,\nu_{01} \text{ atď. [v predch.]}$$

h – polov. hr.

„V" podmienkach pre tú neznámu: $(a^2\gamma^2)=4{,}231\mu$ a $(a\gamma)=2{,}057\sqrt{\mu}$

Dosaď. výraz pre μ do [21,7], a vymeňme nov. neznám.. Získame výraz podobný k [20.8]

$$A=\frac{F/\omega^2h\rho}{I_1(2{,}06\sqrt{\mu})}\,\frac{1}{L_0(\mu)+(\zeta/i\mu)L_2(\mu)}$$

$$L_0(\mu)=I_1(2{,}06\sqrt{\mu})J_0(2{,}06\sqrt{\mu})+I_0(2{,}06\sqrt{\mu})J_1(2{,}06\sqrt{\mu})$$

$$L_2(\mu)=I_1(2{,}06\sqrt{\mu})J_2(2{,}06\sqrt{\mu})-I_2(2{,}06\sqrt{\mu})J_1(2{,}06\sqrt{\mu})$$

Výpoč. kond. mikrof. s dosk. membr. ide priamo ~~cez~~ spôsobom výrazu pre membránu, a výraz pre produkciu napätia: E = ...

Stredová skrutka pre menič „$\phi 50$"

1.) Predp.: keram. $\phi 50, \phi 19 \Rightarrow F_k = \pi \cdot (2{,}5^2 - 0{,}95^2) \doteq \pi (6{,}25 - 0{,}9) = \pi \cdot 5{,}35 = 16{,}8\ cm^2$

2.) Predpätie: $\approx 350\ kp/cm^2$ } $P_{predpät.} = 16{,}8 \cdot 350 = 5880\ kp$

3.) Prípust.: $\sigma_{\check{t}\,ker\,dyn} = 350\ kp/cm^2$; predp. $E_{ker\ \check{t}} \approx 8 \cdot 10^5\ kp/cm^2$

$$\Delta l_{2\,ker} = \frac{2 l_{ker} \cdot P_{dyn}}{E_k \cdot F_k} = \frac{2 l_{ker}}{E_{ker}} \cdot \sigma_{\check{t}\,ker\,dyn} = \frac{2 \cdot 0{,}6\ cm^3}{8 \cdot 10^5\ kp} \cdot 350 \frac{kp}{cm^2} = \frac{420\ cm}{8 \cdot 10^5} = \frac{420\ cm}{80} = 5{,}25\ \mu m$$

4.) Materiál skrutky: 10K ..... $\sigma_{k\check{t}} = 90\ kp/mm^2$

Predp. vrub. a únav. účinn. ... zníž. 3x } $\sigma_\Delta = \frac{90}{3} \doteq 3000\ kp/cm^2$

Vol. M16 x 1,5 ČSN 02 1143.7 ($\sigma_k = 90$)

5.) Deformácie: Refl..: $\Delta l_r \doteq \dfrac{63\ mm \cdot P\ kp}{2{,}1 \cdot 10^4 \frac{kp}{mm^2} \cdot 17{,}33 \cdot 10^2\ mm^2} = \dfrac{P \cdot 1{,}73}{10^6} \dfrac{mm}{kp} \doteq P \cdot 1{,}73 \cdot 10^{-3}\ \mu m$

keram.: $\Delta l_k = \dfrac{12 \cdot P}{8 \cdot 10^3 \cdot 16{,}74 \cdot 10^2} = \dfrac{P \cdot 12}{8 \cdot 10^3 \cdot 1{,}674 \cdot 10^3} \doteq 0{,}895\ P \cdot 10^{-3}\ \mu m$

elektr.: $\Delta l_e = \dfrac{2 \cdot P}{1{,}25 \cdot 10^4 \cdot 1{,}674 \cdot 10^3} = \dfrac{2P}{2{,}093 \cdot 10^7} \doteq 0{,}995\ P \cdot 10^{-4}\ \mu m$

hliník: „$\Delta l''_{al} = \dfrac{20\ mm \cdot P}{7{,}22 \cdot 10^3 \cdot 1{,}674 \cdot 10^2} \doteq 1{,}65\ P \cdot 10^{-3}\ \mu m$

skrutka: „$\Delta l''_s \doteq \dfrac{95 \cdot P}{2{,}1 \cdot 10^4 \cdot 1{,}54 \cdot 10^2} \doteq 29{,}3\ P \cdot 10^{-3}\ \mu m$

$$\lambda_r = 63\ mm \cdot 12 \cdot 10^{-6}\ deg^{-1} \cdot \Delta t = 756 \cdot 10^{-6} \Delta t_r\ (mm) \doteq 0{,}756\, \Delta t_r\ [\mu]$$

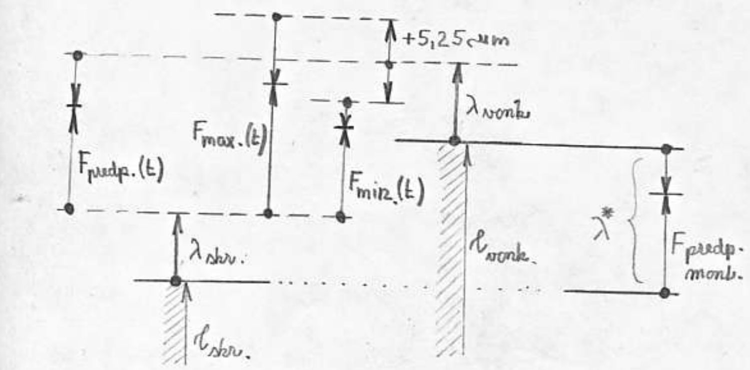
$$\lambda_k = 12 \cdot 2{,}5 \cdot 10^{-6} \Delta t = 30 \cdot 10^{-6} \Delta t\ (mm) \doteq 0{,}3 \cdot \Delta t_k\, 10^{-1}\ [\mu m]$$

$$\lambda_e \doteq 17{,}5 \cdot 10^{-6} \cdot 2 \Delta t = 35 \cdot 10^{-6} \Delta t\ (mm) \doteq 0{,}35\, \Delta t_e \cdot 10^{-1}\ [\mu m]$$

$$„\lambda''_{al} = 20 \cdot 23{,}8 \cdot 10^{-6} \Delta t = 476 \cdot 10^{-6} \Delta t_{al} \doteq 0{,}476\, \Delta t_{al}\ [\mu m]$$

$$„\lambda''_{skr} = 95 \cdot 12 \cdot 10^{-6} \Delta t = 1140 \cdot 10^{-6} \Delta t_{skr} \doteq 1{,}14 \cdot \Delta t_{skr}\ [\mu m]$$

6.) Funkčná predstava:



$\Delta l_{skr.}$ „P"$_{predp.}$ $= 5880 \cdot 29{,}3 \cdot 10^{-3} = 172\ \mu m$

$\Delta l_{vnút}$ „P"$_{predp.}$ $= 5880 \cdot 10^{-3} (1{,}73 + 0{,}895 + 0{,}1 + 1{,}65) =$

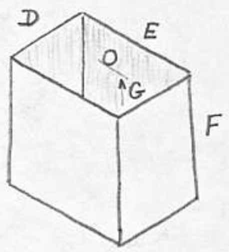
$= 5{,}88 \cdot 4{,}375 = 25{,}7\ \mu m$

pomer $\approx 6{,}7$

$\lambda^* = 172 + 25{,}7 = 197{,}7\ \mu m$

→

Uz vaňa, Branson



čl.	Mod.	D	E	F	G	W	menič
8	610-3	150	250	250	200	500	3
19	1012-6 -8	250	300	300	250	1000	6 8
42	1216-8 12	300	400	400	350	2000	8 12
70	1620-12 -18	400	500	400	350	3000	12 18
110	1824-18	450	600	450	400	4000	18
220	2630-B-24	750	650	500	450	8000	24

LTH - 25 kHz
ATH - 40 kHz

45 kHz 20 (max 40 W) vaňa ϕ 72 h = 60
35 kHz 600 (1200) W vaňa 505 x 300 - 200 (28 l)
25 kHz 150 (300) W vaňa 300 x 230 - 200 (11 l) 3PZT
320 x 150 - 250 (12 l) 3PZT (1,5 mm FeCr)

KLN

fr. kHz	W	Rozmery vaní	PZT
22	150	290 x 130 x 150	2
40			3
22	300	290 x 220 x 200	3
40			6
22	300	490 x 290 x 200	4
40			6
22	600	490 x 290 x 200	6
40			12
22	600	500 x 300 x 300	8
40			12
22	600	500 x 300 x 450	8
40			12
22	900	600 x 400 x 450	12
40			18

22,1 kHz 156 x 206 mm ~ 100 W

7, Sila v centr. skrutke:

$$\lambda^* + \lambda_{\text{vonk}} \pm \begin{bmatrix} 0 \\ 5{,}25 \end{bmatrix}_{\mu m} - \lambda_{\text{skr.}} = P\left(\frac{\Delta l_{\text{skr}}}{P} + \frac{\Delta l_{\text{vnút}}}{P} \right)$$

$$F = \frac{197{,}7 + (0{,}756\,\Delta t_{rad} + 0{,}03\,\Delta t_{ker} + 0{,}035\,\Delta t_{el} + 0{,}476\,\Delta t_{al}) \pm \begin{bmatrix} 0 \\ 5{,}25 \end{bmatrix} - 1{,}14\,\Delta t_{skr}}{(29{,}3 + 4{,}375)} \cdot 10^3 \ [kp]$$

Predp.:
$$\left.\begin{array}{l} \Delta t_r = 15°C\ (20 \to 35) \\ \Delta t_k = 25°C\ (20 \to 45) \\ \Delta t_e = 30°C\ (20 \to 50) \\ \Delta t_{al} = 15°C\ (20 \to 35) \\ \Delta t_{skr} = 35°C\ (20 \to 55) \end{array}\right\}$$

$$F_{predp.}(t) = \frac{197{,}7 + (11{,}3 + 0{,}75 + 1{,}05 + 7{,}13) + \emptyset - 39{,}9}{33{,}675} \cdot 10^3$$

$$F_{predp}(t) = \frac{217{,}93 - 39{,}9 + \emptyset}{33{,}675 \cdot 10^{-3}} \doteq \frac{178}{33{,}675} \cdot 10^3 \doteq 5280\ kp$$

$F_{predp.\,opt} \approx 5880$ $\Longrightarrow$ Zvýšiť krút. moment pri montáži $\approx$ 1,11-krát.

8.) $M_k \approx$

	závity	pod hlavou		
f = 0,1	673	588	→ 12,61 kpm	14 kpm
f = 0,15	955	882	18,37 kpm	20,4 kpm

9.) Napätie v skrutke: $\sigma_{t} \approx \frac{5880}{1{,}54} = 3820\ kp/cm^2$ $\quad \sigma_{Kt} \approx 9000\ kp/cm^2$

Gonda : Kmitanie pružných telies SAV, 1961

Membr. nepren. ohyb. moment.

S ~ vlastné napätie, prenáš. jednotkou prierezu membr-y

$q_v(x,y,t)$ ~ zaťaženie

$$dy\,S\left(\varphi_1+\frac{\partial\varphi_1}{\partial x}dx\right)-dy\,S\varphi_1+dx\,S\left(\varphi_2+\frac{\partial\varphi_2}{\partial y}dy\right)-dx\,S\varphi_2-\underbrace{\rho\,dx\,dy\,\frac{\partial^2 z}{\partial t^2}}_{\text{sila zo zrýchl. vlastnej hmoty membrány}}=-dx\,dy\,q_v(x,t,y) \qquad (62.1)$$

zo rovnováhy vertik. síl, keď rov. membr. je horizont.

ρ ~ mern. hm. membr-y

Pre malé priehyby sa zanedb. vyšš. mocn-y derivácií $\frac{\partial z}{\partial x}$; $\frac{\partial z}{\partial y}$.

Predp. : pri pohybe x, y, sa nemení, len z.

φ_1 ~ ∡ sklonu membr. v smere x. ..... $\varphi_1=\frac{\partial z}{\partial x}$ (62,2)

Z (62.1) :

$$\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}-\frac{\rho}{S}\frac{\partial^2 z}{dt^2}=-\frac{q_v(x,y,t)}{S} \qquad (62,3)$$

Vlastné kmity membrány :

$$q_v=0 \quad\rightarrow\quad \frac{S}{\rho}\left(\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}\right)=\frac{\partial^2 z}{\partial t^2} \qquad \Big/\ a^2=\frac{S}{\rho}$$

$$a^2\left(\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}\right)=\frac{\partial^2 z}{\partial t^2} \qquad (62,4)$$

Predp. : $z=Z(x,y).T(t) \rightarrow \frac{\partial^2 z}{\partial x^2}=\frac{\partial^2 Z}{\partial x^2}T \qquad \frac{\partial^2 z}{\partial y^2}=\frac{\partial^2 Z}{\partial y^2}T \qquad \frac{\partial^2 z}{\partial t^2}=Z\frac{d^2T}{dt^2}$

$$\dots\ \frac{a^2}{Z}\left(\frac{\partial^2 Z}{\partial y^2}+\frac{\partial^2 Z}{\partial y^2}\right)=\frac{1}{T}\frac{d^2T}{dt^2}=\cancel{\alpha}^2 \quad (62,6) \quad \dots \text{polož. :} \quad -a^2(k_1^2+k_2^2)=-\alpha^2$$

$$\Rightarrow \begin{cases} \dfrac{d^2T}{dt^2}+\alpha^2T=0 & (62,7) \\[2ex] \dfrac{\partial^2 Z}{\partial x^2}+\dfrac{\partial^2 Z}{\partial y^2}+(k_1^2+k_2^2)Z=0 & (62,8) \end{cases} \qquad Z(x,y)=X(x).Y(y)$$

$$\frac{\partial^2 Z}{\partial x^2}=Y\frac{d^2X}{dx^2} \qquad \frac{\partial^2 Z}{\partial y^2}=X\frac{d^2Y}{dy^2}$$

Potom (62,8) : $Y\left(\frac{d^2X}{dx^2}\right)+X\frac{d^2Y}{dy^2}+k_1^2XY+k_2^2XY=0$, úpr. :

$$\underbrace{Y\left(\frac{d^2X}{dx^2}+k_1^2X\right)}_{0}+\underbrace{X\left(\frac{d^2Y}{dy^2}+k_2^2Y\right)}_{0}=0 \qquad (62,10)$$

Teda (62,4) sa rozloží na $\underbrace{(62,7) \text{ a na } 2\ (62,10)}_{3\text{ obych. rovn.}}$.

Vlastné kmity obdĺžn. membr.-y / netlmené !

Partik. integr. rovn.: $X(x) = A_1 \sin k_1 x + B_1 \cos k_1 x$

$Y(y) = A_2 \sin k_2 y + B_2 \cos k_2 y$

$T(t) = C \sin \alpha t + D \cos \alpha t$ $\quad / \ \alpha^2 = a^2(k_1^2 + k_2^2)$ $\quad [\, a = \sqrt{\frac{S}{\varrho}} \,]$

umiestnenie súr. systému ⟺ Pre okraj. podm.: $Z(0,y) = Z(p,y) = Z(x,0) = Z(x,q) = 0$ .... po obvode $u = 0$ .. $Z = 0$

Vyhov.: $Z(x,y) = \sin k_1 x \sin k_2 y$ $\quad / \ \sin k_1 p = 0 \ \ldots \ k_1 p = 0, \pi, 2\pi, \ldots$

$\sin k_2 q = 0 \ \ldots \ k_2 q = 0, \pi, \ldots$

$$Z(x,y) = \sin \frac{i\pi x}{p} \sin \frac{j\pi y}{q} \rightarrow$$

„priehyb ij": $u(x,y,t) = Z(x,y)\,T(t) = \sin \frac{i\pi x}{p} \sin \frac{j\pi y}{q} (C \sin \alpha t + D \cos \alpha t)$ (63,7)

„celk. prieh.": $Z = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} (C_{ij} \sin \alpha_{ij} t + D_{ij} \cos \alpha_{ij} t) \sin \frac{i\pi x}{p} \sin \frac{j\pi y}{q}$ (63,8)

Kruhové fr.: $\alpha = a\sqrt{k_1^2 + k_2^2} = \sqrt{\frac{S}{\varrho}} \sqrt{k_1^2 + k_2^2}$ ... $\alpha_{ij} = \pi \sqrt{\frac{i^2}{p^2} + \frac{j^2}{q^2}} \sqrt{\frac{S}{\varrho}}$ ~ uchyt. po obvode !

„a"

C_{ij}, D_{ij} ... z poč. podm.

Pr: $t = 0$: $u(x,y,0) = f(x,y)$ deform.

$\frac{\partial u(x,y,0)}{\partial t} = g(x,y)$ rýchl. def.

$$C_{ij} = \frac{4}{pq} \int_0^p \int_0^q f(x,y) \sin \frac{i\pi x}{p} \sin \frac{j\pi x}{q} \, dx\, dy$$

$$D_{ij} = \frac{4}{\alpha_{ij} pq} \int_0^p \int_0^q g(x,y) \sin \frac{i\pi x}{p} \sin \frac{j\pi y}{q} \, dx\, dy \quad (63.10)$$

Vlastné kmity kruhovej membr.-y.

$\frac{dx}{dx} = 1 = \frac{\partial r}{\partial x} \cos\varphi + r(-\sin\varphi) \frac{d\varphi}{dx}$

Pol. súr.: $x = r\cos\varphi \quad y = r\sin\varphi$

$\partial x = \partial r \cos\varphi - r \sin\varphi\, \partial\varphi$

$\frac{dy}{dx}$ $\partial y = \partial r \sin\varphi + r\cos\varphi\, \partial\varphi$

$\frac{\partial r}{\partial x} = \cos\varphi \quad \frac{\partial \varphi}{\partial x} = -\frac{\sin\varphi}{r}$

$$\frac{\partial Z}{\partial x} = \frac{\partial Z}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial Z}{\partial \varphi}\cdot\frac{\partial \varphi}{\partial x} = \frac{\partial Z}{\partial r}\cos\varphi - \frac{\partial Z}{\partial \varphi}\cdot\frac{\sin\varphi}{r}$$

Potom: $$\frac{\partial^2 Z}{\partial x^2} = \frac{\partial}{\partial r}\left(\frac{\partial Z}{\partial r}\cos\varphi - \frac{\partial Z}{\partial\varphi}\frac{\sin\varphi}{r}\right)\frac{\partial r}{\partial x} + \frac{\partial}{\partial\varphi}\left(\frac{\partial Z}{\partial r}\cos\varphi - \frac{\partial Z}{\partial\varphi}\frac{\sin\varphi}{r}\right)\frac{\partial\varphi}{\partial x} =$$

$$= \cos^2\varphi \frac{\partial^2 Z}{\partial r^2} \boxed{-2\frac{\sin\varphi\cos\varphi}{r}\frac{\partial^2 Z}{\partial r\partial\varphi}} + \frac{\sin^2\varphi}{r^2}\frac{\partial^2 Z}{\partial\varphi^2} + \frac{\sin^2\varphi}{r}\frac{\partial Z}{\partial r} \boxed{+2\frac{\sin\varphi\cos\varphi}{r^2}\frac{\partial Z}{\partial\varphi}}$$

Analog: $\frac{\partial r}{\partial y}\sin\varphi + r\cos\varphi\frac{\partial\varphi}{\partial y} = 1 = \frac{dy}{dy}$

$\frac{dx}{dy} = \frac{\partial r}{\partial y}\cos\varphi - r\sin\varphi\frac{\partial\varphi}{\partial y} = 0$

$\frac{\partial r}{\partial y} = \sin\varphi \quad \frac{\partial\varphi}{\partial y} = \frac{\cos\varphi}{r}$

$$\frac{\partial Z}{\partial y} = \frac{\partial Z}{\partial r}\frac{\partial r}{\partial y} + \frac{\partial Z}{\partial\varphi}\cdot\frac{\partial\varphi}{\partial y} = \frac{\partial Z}{\partial r}\sin\varphi + \frac{\partial Z}{\partial\varphi}\frac{\cos\varphi}{r}$$

$$\frac{\partial^2 Z}{\partial y^2} = \frac{\partial^2 Z}{\partial r^2}\sin^2\varphi \boxed{+2\frac{\partial^2 Z}{\partial r\partial\varphi}\frac{\sin\varphi\cos\varphi}{r}} \boxed{-2\frac{\partial Z}{\partial\varphi}\frac{\sin\varphi\cos\varphi}{r^2}} + \frac{\partial^2 Z}{\partial\varphi^2}\frac{\cos^2\varphi}{r^2} + \frac{\partial Z}{\partial r}\frac{\cos^2\varphi}{r}.$$

→ .... Rovn. (62,8) v pol. súr.:

(vlastné! netlmené): $$\frac{\partial^2 Z}{\partial r^2} + \frac{\partial^2 Z}{\partial\varphi^2}\frac{1}{r^2} + \frac{1}{r}\frac{\partial Z}{\partial r} + k^2 Z = 0 \quad / \ k^2 = \frac{\alpha^2}{a^2}$$

(64,1)

/ Polož: $Z(r,\varphi) = P(r)\,\Phi(\varphi)$ → 2 rovnice:

$$\frac{\partial^2 Z}{\partial r^2} = \frac{d^2P}{dr^2}\phi \qquad \frac{\partial^2 Z}{\partial\varphi^2} = \frac{d^2\phi}{d\varphi^2}P \qquad \frac{\partial Z}{\partial r} = \frac{dP}{dr}\phi \qquad +(64,1) \rightarrow$$

$$\frac{d^2P}{dr^2}\phi + \frac{1}{r^2}\frac{d^2\phi}{d\varphi^2}P + \frac{1}{r}\frac{dP}{dr}\phi + k^2P\phi = 0$$, separ. prem. : $\frac{r^2}{P}\frac{d^2P}{dr^2} + \frac{r}{P}\frac{dP}{dr} + r^2k^2 = -\frac{1}{\phi}\frac{d^2\phi}{d\varphi^2}$ položí: $= m^2$

$\Downarrow$ $$\frac{d^2\phi}{d\varphi^2} + m^2\phi = 0 \qquad (64,3) \qquad \rightarrow \phi(\varphi) = C\sin m\varphi + D\cos m\varphi \quad (64,5)$$

$$\frac{d^2P}{dr^2} + \frac{1}{r}\frac{dP}{dr} + \left(k^2 - \frac{m^2}{r^2}\right)P = 0 \quad (64,4) \quad \rightarrow P(r) = J_m(kr)$$ / $N(kr) = 0$ [rozmierne pri $r = 0$ ... konečn. ~~chyb~~ priehyb.]

$k^2 = \alpha^2/a^2$, $k = \alpha/a$

všeob.: $P(r) = C_1 J_m(kr) + C_2 \mathcal{J}_m(kr)$

$$P = r^m(a_0 + a_1 r + a_2 r^2 + \dots) = \dots = a_0 r^m\left[1 - \frac{k^2r^2}{2(m+2)} + \frac{k^4r^4}{2(m+2)4(2m+4)} - \dots\right]$$

$a_0 \sim$ ľubov. $\Rightarrow a_0 = h \cdot k^m$ $\Rightarrow P = J_m(kr)$

$$Z(r,\varphi) = (C\sin m\varphi + D\cos m\varphi)\cdot J_m(kr) \qquad (64,8)$$

Parcih. rieš. rovn.: $$\frac{\partial^2 z}{\partial r^2} + \frac{1}{r}\frac{\partial z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 z}{\partial\varphi^2} = \frac{1}{a^2}\frac{\partial^2 z}{\partial t^2} \rightarrow$$

$$z = TP\phi = (C_1\sin\alpha t + D_1\cos\alpha t)(C\sin m\varphi + D\cos m\varphi)\, J_m(kr) \qquad (64,10)$$

Na okr.: $r = R \dots z = 0 \Rightarrow J_m(kR) = 0$

$\frac{\alpha}{a}$

Ritzova metóda:

Kinet. energ. membr.: $$T = \frac{\rho h}{2}\iint \dot z^2\,dx\,dy \qquad (65,1)$$ / $\dot z_{max} = \alpha Z(x,y)$
ρh – hm. jednotk. plochy m-y.

$$T_{max} = \frac{\rho h\alpha^2}{2}\iint Z^2_{(x,y)}\,dx\,dy \qquad (65,2)$$

Pot. en. kmit. m-y: $$V_{max} = \frac{S}{2}\iint\left[\left(\frac{\partial Z}{\partial x}\right)^2 + \left(\frac{\partial Z}{\partial y}\right)^2\right]dx\,dy \qquad (65,3)$$ / predp.: $z(x,y,t) = Z(x,y)\cos\alpha t$

Kruh. fr.: $$\alpha^2 = \frac{S}{\rho h}\,\frac{\iint\left[\left(\frac{\partial Z}{\partial x}\right)^2 + \left(\frac{\partial Z}{\partial y}\right)^2\right]dx\,dy}{\iint Z^2\,dx\,dy} \qquad (65,5)$$

Zv.: $Z = a_1\varphi_1(x,y) + a_2\varphi_2(x,y) + \dots + a_n\varphi_n(x,y)$.

... každý člen vyhov. okr. podm. a podm.: minimalnosť $\} \Rightarrow$

$$\frac{\partial}{\partial a_n}\,\frac{\iint\left[\left(\frac{\partial Z}{\partial x}\right)^2 + \left(\frac{\partial Z}{\partial y}\right)^2\right]dx\,dy}{\iint Z^2\,dx\,dy} = 0 \qquad (65,7) \rightarrow$$

$$\frac{\partial}{\partial a_m}\iint\left[\left(\frac{\partial Z}{\partial x}\right)^2 + \left(\frac{\partial Z}{\partial y}\right)^2 - \frac{\rho h\alpha^2}{S}Z^2\right]dx\,dy = 0$$ ... sústava rovn. pre a_m

$D_{\text{"}(a_m=?)\text{"}} = 0$ ~ fr. rovn. membrány.

Pre mnohostranné membr.: $Z = (b^2 - x^2)(b^2 - y^2)(a_1 + a_2x^2 + a_3y^2 + a_4x^2y^2 + \dots)$ / štvorec

$$Z = (b_0x + b_1y + b_2)(c_0x + c_1y + c_2)\dots(m_0x + m_1y + m_2)\sum_{i,j}^{m} a_{ij}x^iy^i.$$

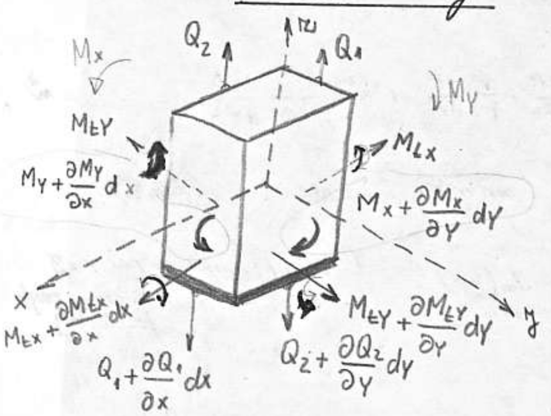
Kruhová m.: $$Z = a_1\cos\frac{\pi r}{2R} + a_2\cos\frac{3\pi r}{2R} + \dots a_n\cos\frac{(n+1)\pi r}{2R}$$ /

$$T_{max} = \frac{\rho h\alpha^2}{2}\int_0^R Z^2(r)\,2\pi r\,dr \quad , \quad V_{max} = \frac{S}{2}\int_0^R\left(\frac{\partial Z}{\partial r}\right)^2 2\pi r\,dr \quad , \quad \frac{\partial}{\partial a_m}\int_0^R\left[\left(\frac{\partial Z}{\partial r}\right)^2 - \frac{\rho h\alpha^2}{S}Z^2\right]2\pi r\,dr = 0$$

Pre rôzne tvary: kruh. fr. $\alpha = \beta\sqrt{\frac{S}{\rho h F}} = \beta\sqrt{\frac{S}{m}}$ — pre najnižš. fr.

kruh: $\beta = 4{,}261$ rovnostr. trojuh.: $\beta = 4{,}774$
štvorec: $\beta = 4{,}443$ polkruh: $4{,}803$
kvadrant: $4{,}551$

Kmitanie dosky.



Súčet síl v smere w : $dm.\ \frac{\partial^2 w}{\partial t^2}$

Q ~ pos. sily v w, na jednotk. prier. plochu

M ~ oh. mom. k osiam x, y – vzť. na jedn. prier. plochu

M_t ~ krutné mom. okolo x, y —"—

P ~ osam. vonk. zať. v sm. w.

q_v ~ $\frac{kg}{m^2}$ —"—

vlastn. hmota

$$- h\,dx\,dy\,\rho \frac{\partial^2 w}{\partial t^2} + Q_1 h\,dy - \left(Q_1 + \frac{\partial Q_1}{\partial x}dx\right) h\,dy + Q_2 h\,dy - \left(Q_2 + \frac{\partial Q_2}{\partial y}dy\right) h\,dx = q_v(x,y,t)\,dx\,dy$$

pos. sily — vonk. zať. "+" v smere osi w !! — sily v smere w.

$$\rho \frac{\partial^2 w}{\partial t^2} = \frac{\partial Q_1}{\partial x} + \frac{\partial Q_2}{\partial y} + \frac{q_v(x,y,t)}{h} \qquad (66,1)$$

Rovnov. mom., pri zanedb. zotrv. : (k x)

$$-M_x h\,dx + \left(M_x + \frac{\partial M_x}{\partial y}dy\right) h\,dx + Q_2 h\,dx\,dy \pm \left(M_{tx} + \frac{\partial M_{tx}}{\partial x}dx\right) h\,dy \mp M_{tx} h\,dy + q_v \frac{dx\,dy^2}{2} = 0$$

na plo. jedn. plocha — rameno — plocha

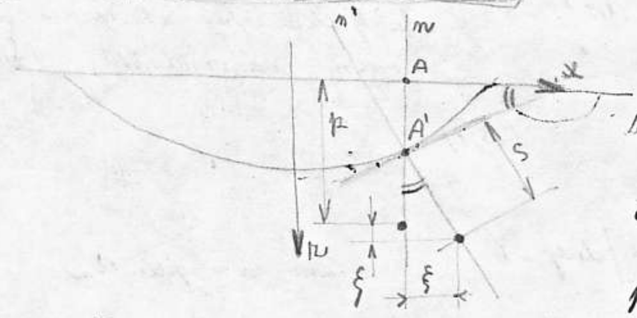
$$+\frac{\partial M_x}{\partial y} + \frac{\partial M_{tx}}{\partial x} + Q_2 = 0 \left(= -\frac{q_v\,dy}{2h}\right) \qquad (66,2)$$

zanedb.

Obdobne pre y : $$\frac{\partial M_y}{\partial x} + \frac{\partial M_{ty}}{\partial y} + Q_1 = 0 \left(= -\frac{q_v\,dx}{2h}\right) \qquad (66,3)$$

der. (66,2) ... y , der. (66,3) ... x → dosaď do (66,1) →

$$-\rho \frac{\partial^2 w}{\partial t^2} + \boxed{\frac{\partial^2 M_x}{\partial y^2} + \frac{\partial^2 M_{ty}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial x^2} + \frac{\partial^2 M_{tx}}{\partial x \partial y}} = \frac{q_v(x,y,t)}{h} \qquad (66,4)$$



$tg\,\measuredangle = -\frac{dw}{dx} \approx \sin\measuredangle = \frac{\xi}{s}$ ⇒

Pre malé zmeny: $\xi = -s\frac{\partial w}{\partial x}$; $\eta = -s\frac{\partial z}{\partial y}$

$\varepsilon_x = \frac{\partial \xi}{\partial x} = -s\frac{\partial^2 w}{\partial x^2}$ $\quad \varepsilon_y = \frac{\partial \eta}{\partial y} = -s\frac{\partial^2 w}{\partial y^2}$ $\quad \varepsilon_w = 0$

$\gamma_{xy} = \frac{\partial \xi}{\partial y} + \frac{\partial \eta}{\partial x} = -2s\frac{\partial^2 w}{\partial x \partial y}$ $\quad \gamma_{xz} = \gamma_{yz} = 0$

Pre malé def. ... $\sigma_w = 0$ $\quad \tau_{xy} = \tau_{xz} = 0$ (66,6)

$$\sigma_x = \frac{E}{1-\mu^2}(\varepsilon_x + \mu\varepsilon_y)$$

$$\sigma_x = \cancel{\frac{Em}{m^2-1}(m\varepsilon_x - \varepsilon_y)} = +\frac{E\cancel{m}}{m^2-1}s\left(m\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)$$

$$\sigma_y = +\frac{E\cancel{m}}{m^2-1}s\left(m\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial x^2}\right)$$

$$\tau_{xy} = G\gamma_{xy} = -2Gs\frac{\partial^2 w}{\partial x \partial y} \qquad (66,8)$$

[či nie je mysl. na Poiss. konštantu ? "$m = \frac{1}{\mu}$" ...
pozn.: 26. VII. 1976.]

Momenty pom. týchto výrazov:

$$G = \frac{E}{2(1+\mu)}$$

$$\left|\left(M_x h\right) dx\right|'' = \left(+\int_{-\frac{h}{2}}^{+\frac{h}{2}} s\,\sigma_y\, ds\right) dx = \left[\frac{Em}{m^2-1}\int_{-\frac{h}{2}}^{+\frac{h}{2}} s^2 \left(m\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial x^2}\right) ds\right] dx = \frac{Em}{m^2-1}\,\frac{2h^3}{24}\left(m\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)dx \qquad (66{,}9)$$

$$\left|\left(M_y h\right) dy\right|'' = \left(\int_{-\frac{h}{2}}^{+\frac{h}{2}} s\,\sigma_x\, ds\right) dy = +\frac{Em\,2h^3}{(m^2-1)\,24}\left(m\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial x^2}\right)dy \qquad (66{,}10)$$

$$\left|M_{tx} h\, dy\right|'' = \left(\int_{-\frac{h}{2}}^{+\frac{h}{2}} s\,\tau_{xy}\, ds\right) dy = -\frac{4}{24}\,G h^3 \frac{\partial^2 w}{\partial x \partial y}\,dy \qquad (66{,}11)$$

$$M_{(x)} = \frac{Eh^2}{12(m^2-1)}\left[\frac{\partial^2 z}{\partial y^2} + m\frac{\partial^2 z}{\partial x^2}\right]$$

$$\left|M_{ty} h\, dx\right|'' = -\frac{4}{24}\,G h^3 \frac{\partial^2 w}{\partial x \partial y}\,dx$$

$$M_{tx} = \frac{Eh^2}{12(m^2-1)}\,\frac{\partial^4 w}{\partial x^2 \partial y^2}\,(1-m)$$

Dos. do (66,4) 2. der. w pod. 4 výr. $\rightarrow$ zákl. parc. dif. rovn. pre vlastné km. dosák:

$$-\frac{\partial^2 w}{\partial t^2} + \frac{Em h^2}{12(m^2-1)\rho}\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right) = \frac{q_v(x,y,t)}{\rho h} \qquad (66{,}13)$$

~ pre malé def.!

ozn.: $\begin{cases} D = \dfrac{Eh^3 m}{12(m^2-1)} \quad \dots \quad \dfrac{1}{1-m^2} \\ k^4 = \dfrac{\rho h}{D} \end{cases}$

$$k^4 \frac{\partial^2 z}{\partial t^2} + \Delta^2 w = \frac{q_v(x,y,t)}{D} \qquad (66{,}14)$$

$/ \ \Delta^2 w \equiv \Delta\Delta w$

Potenc. energ.: $dV = \left(\frac{\varepsilon_x \sigma_x}{2} + \frac{\varepsilon_y \sigma_y}{2} + \frac{\tau_{xy}\gamma_{xy}}{2}\right) dw\, dx\, dy$

sem dos. z (66,6), (66,8) a integr. podľa s $\left(+\frac{h}{2} \rightarrow -\frac{h}{2}\right)$:

$$V = \frac{Eh^3 m}{24(m^2-1)} \iint\left[\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \left(\frac{\partial^2 w}{\partial y^2}\right)^2 + \frac{2}{m}\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + \frac{2(m-1)}{m}\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right] dx\, dy \quad ??? \qquad (66{,}17)$$

Kin. energ. $T = \frac{\rho h}{2}\iint \dot{w}^2\, dx\, dy \qquad (66{,}18)$

správny, kontrolov. vzorec pre V vpredu, od Babakova!

Vlastné kmity obdĺžn. dosky:

Rieš. sa (66,14) bez pravej str.-y.

Pre harmon. km.:

$$\frac{\partial^4 Z(x,y)}{\partial x^4} + 2\frac{\partial^4 Z(x,y)}{\partial x^2 \partial y^2} + \frac{\partial^4 Z(x,y)}{\partial y^4} - \lambda^4 Z(x,y) = 0 \qquad (67{,}1)$$

Predp.: $Z(x,y) = \sum_{i=1}^{\infty} F_i(y) \sin\frac{i\pi x}{p} \qquad (67{,}1a)$ $\qquad \lambda^4 = \frac{\rho h \omega^2}{D} = k^4\omega^2$ (ω ~ kruh. fr.)

Keď: strany: p, q ... podopr. okraje, $\rightarrow$ $x=0$ $x=p$: $Z(x,y)=0$, $\frac{\partial^2 Z(x,y)}{\partial x^2} = 0$

$$\frac{\partial^4 Z}{\partial x^4} = \frac{i^4\pi^4}{p^4}\sum_{i=1}^{\infty} F_i(y) \sin\frac{i\pi x}{p}; \quad \frac{\partial^4 Z}{\partial x^2 \partial y^2} = -\frac{i^2\pi^2}{p^2}\sum_{i=1}^{\infty}\frac{\partial^2 F_i(y)}{\partial y^2}\sin\frac{i\pi x}{p}; \quad \frac{\partial^4 Z}{\partial y^4} = \sum_{i=1}^{\infty}\frac{\partial^4 F_i(y)}{\partial y^4}\sin\frac{i\pi x}{p}$$

Po dos. do (67,1): $\sum_{i=1}^{\infty}\left[\frac{i^4\pi^4}{p^4}F_i(y) - 2\frac{i^2\pi^2}{p^2}\frac{\partial^2 F_i(y)}{\partial y^2} + \frac{\partial^4 F_i(y)}{\partial y^4} - \lambda^4 F_i(y)\right] = 0$, $\rightarrow$

$$F_i^{(4)}(y) - \frac{2i^2\pi^2}{p^2}F_i''(y) + \left(\frac{i^4\pi^4}{p^4} - \lambda^4\right)F_i(y) = 0 \qquad (67{,}2)$$

↳ char. rovn., korene, ...

Všeob. int.: $F_i(y) = A_i \operatorname{sh} r_1 y + B_i \operatorname{ch} r_1 y + C_i \operatorname{sh} r_2 y + D_i \operatorname{ch} r_2 y \qquad (67{,}5)$

Všeob. rieš. (67,1) = $Z(x,y) =$

$\rightarrow$

$$Z(x,y) = \sum_{i=1}^{\infty} \left(A_i \,\mathrm{sh}\, w_1 y + B_i \,\mathrm{ch}\, w_1 y + C_i \,\mathrm{sh}\, w_2 y + D_i \,\mathrm{ch}\, w_2 y\right) \sin \frac{i\pi x}{p} \qquad (67,6)$$

Pre 4 podopr. okraje:

$$Z(x,0) = 0 \qquad \frac{\partial^2 Z(x,0)}{\partial y^2} = 0$$

$$Z(x,q) = 0 \qquad \frac{\partial^2 Z(x,q)}{\partial y^2} = 0$$

podom pre im. korene char. rovn.;

(67,5): $F_i(y) = A_1 \sin \lambda_1 y + B_i \cos \lambda_1 y + C_i \,\mathrm{sh}\, w_2 y + D_i \,\mathrm{ch}\, w_2 y$

$$\lambda_1 = \sqrt{\lambda^2 - \frac{i^2\pi^2}{p^2}}$$

Z okr. podm.: $B_i = C_i = D_i = 0$ $\qquad \sin \lambda_1 q = 0 \ldots$ fr. rovn. $\rightarrow \lambda_1 q = j\pi \quad (j = 1, 2, 3, \ldots)$

$$\lambda^4 = k^4 \alpha^2 = \frac{\rho h \alpha^2}{D} = \left(\lambda_1 + \frac{i^2\pi^2}{p^2}\right)^2 = \pi^4 \left(\frac{i^2}{p^2} + \frac{j^2}{q^2}\right)^2$$

Z toho kruh. fr. vlastn. kmit.:

$$\alpha_{ij} = \pi^2 \left(\frac{i^2}{p^2} + \frac{j^2}{q^2}\right) \sqrt{\frac{D}{\rho h}} \qquad (67,8)$$

Tvary kmitania:

$$Z(x,y) = \sum_{ij=1}^{\infty} a_{ij} \sin \lambda_1 y \sin \frac{i\pi x}{p} = \sum_{ij=1}^{\infty} a_{ij} \sin \frac{i\pi x}{p} \sin \frac{j\pi y}{q} \qquad (67,9)$$

Výsl. rieš.:

$$w(x,y,t) = \sum_{ij=1}^{\infty} T_{ij}(t) \sin \frac{i\pi x}{p} \sin \frac{j\pi y}{q} \qquad (67,10)$$

Toto a 2 der. dos. do pot. a kin en.:

$$V = \frac{Eh^3 m^2}{12(m^2-1)} \cdot \frac{\pi^4 pq}{8} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} T_{ij}^2(t) \left(\frac{i^2}{p^2} + \frac{j^2}{q^2}\right)^2 \qquad (67,11)$$

$$T = \frac{\rho h}{2} \frac{pq}{4} \sum_{ij=1}^{\infty} \dot{T}_{ij}^2(t).$$

$$V = \frac{1}{2} \sum c_{kk} p_k^2$$

$$T = \frac{1}{2} \sum m_{kk} \dot{p}_k^2 \qquad m_{kk} \ddot{p}_k + c_{kk} p_k = 0 \quad \text{, analog.:}$$

$$\rho h pq \ddot{T}_{ij}(t) + \frac{Eh^3 m^2}{12(m^2-1)} \pi^4 pq \left(\frac{i^2}{p^2} + \frac{j^2}{q^2}\right) T_{ij}(t) = 0 \quad \text{, po úpr.:}$$

$$\ddot{T}_{ij}(t) + \frac{\pi^4}{\rho h} \left(\frac{i^2}{p^2} + \frac{j^2}{q^2}\right) \frac{Eh^3 m^2}{12(m^2-1)} T_{ij}(t) = 0$$

$$\ddot{T}_{ij}(t) + \alpha_{ij}^2 T_{ij}(t) = 0 \qquad \rightarrow \qquad T_{ij}(t) = C_1 \cos \alpha_{ij} t + D_1 \sin \alpha_{ij} t \qquad (67,14)$$

kruh. fr.: $$\alpha_{ij} = \pi^2 \sqrt{\frac{1}{\rho h} \left(\frac{i^2}{p^2} + \frac{j^2}{q^2}\right) \frac{Eh^3 m^2}{12(m^2-1)}} \qquad (67,15)$$

$$\alpha_{ij} = \pi^2 \left(\frac{i^2}{p^2} + \frac{j^2}{q^2}\right) \sqrt{\frac{D}{\rho h}} \qquad (67,17)$$

Vlastné kmitanie kruhovej dosky.

Prepíš. (66,14) pre pol. súr. pouš.: $\Delta(r,\varphi) = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2}$ (68,1)

Rovn. bez prav. str-y, vl. kmit.: $k^4 \frac{\partial^2 z}{\partial t^2} + \Delta^2 z = 0$ (68,2)

$T(t) = E\sin\alpha t + F\cos\alpha t$; po krátení $T(t)$:

$-k^4\alpha^2 Z(r,\varphi) + \Delta^2 Z(r,\varphi) = 0$

$\Delta^2 Z(r,\varphi) - \underbrace{\frac{\rho h \alpha^2}{D}}_{\lambda^4} Z(r,\varphi) = 0$

$$\frac{\partial^2 Z(r,\varphi)}{\partial r^2} + \frac{1}{r}\frac{\partial Z(r,\varphi)}{\partial r} + \frac{1}{r^2}\frac{\partial^2 Z(r,\varphi)}{\partial \varphi^2} \pm \lambda^2 Z(r,\varphi) = 0 \qquad (68,5)$$

$Z(r,\varphi) = P(r)\cdot\phi(\varphi)$

$$\frac{d^2P(r)}{dr^2} + \frac{1}{r}\frac{dP(r)}{dr} + \frac{1}{r^2}\frac{d^2\phi(\varphi)}{d\varphi^2}P(r)\frac{1}{\phi(\varphi)} \pm \lambda^2 P(r) = 0 \quad \Big| \frac{r^2}{P(r)} \qquad (68,6)$$

6.III.78.

Vzn. 2 rovn.: $\frac{d^2\phi(\varphi)}{d\varphi^2} + m^2\phi(\varphi) = 0$ (68,8) nech $= -m^2$ $\to$ $\phi(\varphi) = A\sin m\varphi$ ~ pri sh. okr. podm.

$$\frac{d^2P(r)}{dr^2} + \frac{1}{r}\frac{dP(r)}{dr} + \left(\pm\lambda^2 - \frac{m^2}{r^2}\right)P(r) = 0 \qquad (68,9)$$ po dos.: $r \to \lambda r$:

$r^2u'' + ru' + (k^2r^2 - m^2)u = 0$

Riešenie: (pri $N_n(\lambda r) = 0$) Bess. f. prvého rodu, n-tého rádu.

Keď v (68,9) bude $(-\lambda^2)$ ... $\eta = i\lambda r$ ... riešenie $J_n(ikr)$ $\to$

Všeob. integr.: $P(r) = BJ_m(\lambda r) + CJ_m(i\lambda r)$ (68,11)

Riešenie (68,5): $Z(r,\varphi) = A\sin m\varphi\,[B_m J_m(\lambda r) + C_m J_m(i\lambda r)]$

$w(r,\varphi,t) = A_m \sin m\varphi\,[B_m J_m(kr\sqrt{\alpha}) + C_m J_m(ikr\sqrt{\alpha})](E\sin\alpha t + F\cos\alpha t)$

$/\ \lambda = k\sqrt{\alpha}$

Uzl. čiary vyplýv. z nulovosti: $A_m \sin m\varphi = 0 \to \varphi = 0;\ \frac{\pi}{m};\ \frac{2\pi}{m}$

$B_m J_m(kr\sqrt{\alpha}) + C_m J_m(ikr\sqrt{\alpha}) = 0$

Výpočet kruhovej dosky Ritzovou metódou

$$V = \frac{D}{2}\int_0^{2\pi}\int_0^R \left\{\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial\varphi^2}\right)^2 - \frac{2(m-1)}{m}\frac{\partial^2 w}{\partial r^2}\left(\frac{1}{r}\frac{\partial z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \varphi^2}\right) + \frac{2(m-1)}{m}\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial z}{\partial \varphi}\right)\right]^2\right\} r\,d\varphi\,dr$$

Pri stredove symetr. prehnutí: $V = \pi D\int_0^R \left[\left(\frac{\partial^2 z}{\partial r^2} + \frac{1}{r}\frac{\partial z}{\partial r}\right)^2 - \frac{2(m-1)}{m}\frac{\partial^2 z}{\partial r^2}\frac{1}{r}\frac{\partial z}{\partial r}\right] r\,dr$ (69,1)

kin. en.: $T = \frac{\rho h}{2}\int_0^{2\pi}\int_0^R \dot z^2\, r\,dr\,d\varphi$ pre sym.: $T = \pi\rho h\int_0^R \dot z^2\, r\,dr$

Pre voľn. okraj; $w = Z(r)\cos\alpha t$ (69,3)

Predp.: $Z(r) = a_1\left(1 - \frac{r^2}{R^2}\right)^2 + a_2\left(1 - \frac{r^2}{R^3}\right)^3 + \dots$ (69,4) .... $r = R \to Z = 0$

Potom: $\dot z = -Z(r)(\sin\alpha t)\cdot\alpha$; $T_{max} = \pi\rho h\alpha^2\int_0^R Z^2(r)\, r\,dr$ (69,5)

Dos (69,3) → (69,1) ....

$$V_{max} = \pi D \int_0^R \left[\left(\frac{d^2Z}{dr^2} + \frac{1}{r}\frac{dZ}{dr}\right)^2 - \frac{2(m-1)}{m}\frac{d^2Z}{dr^2}\frac{1}{r}\frac{dZ}{dr}\right] r\,dr \qquad (69,6)$$

Porovn. (69,5) (69,6) →

kruh. fr. : $$\alpha^2 = \frac{1}{\pi \rho h}\frac{V_{max}}{\int_0^R Z^2 r\,dr} \qquad (69,7)$$

Koef. a_1, a_2 atď., aby:

$$\frac{\partial}{\partial a_m}\frac{\int_0^R \left[\left(\frac{d^2Z}{dr^2} + \frac{1}{r}\frac{dZ}{dr}\right)^2 - \frac{2(m-1)}{m}\frac{d^2Z}{dr^2}\frac{1}{r}\frac{dZ}{dr}\right] r\,dr}{\int_0^R Z^2 r\,dr} = 0$$

Z toho po dos (69,7), po upr.:

$$\frac{\partial}{\partial a_m}\int_0^R \left[\left(\frac{d^2Z}{dr^2} + \frac{1}{r}\frac{dZ}{dr}\right)^2 - \frac{\alpha^2 \rho h}{D} Z^2\right] r\,dr = 0$$

Keď ponech. z (69,7) len prvé dva členy, podľa (69,8) dostaneme:

$$\left.\begin{aligned}\int_0^R \left(\frac{d^2Z}{dr^2} + \frac{1}{r}\frac{dZ}{dr}\right)^2 r\,dr &= \frac{96}{9R^2}\left(a_1^2 + \frac{3}{2}a_1a_2 + \frac{9}{10}a_2^2\right) \\ \int_0^R Z^2 r\,dr &= \frac{R^2}{10}\left(a_1^2 + \frac{5}{3}a_1a_2 + \frac{5}{7}a_2^2\right)\end{aligned}\right\} \quad \begin{aligned} a_1\left(\frac{192}{9} - \frac{x}{5}\right) + a_2\left(\frac{144}{9} - \frac{x}{6}\right) = 0 \\ a_1\left(\frac{144}{9} - \frac{x}{6}\right) + a_2\left(\frac{96}{5} - \frac{x}{7}\right) = 0\end{aligned}$$

$$\Big/ x = R^4 \alpha^2 \frac{\rho h}{D} \qquad (69,10)$$

Z determ. ľahko rovn. → fr. rovn. 2°, $x_1 = 104{,}3$; $x_2 = 1854$.

Vlastné kr. fr. z (69,10): $\alpha^2 = \frac{Dx}{R^4 \rho h}$ $= \frac{1}{R^2}\sqrt{\frac{Dx}{\rho h}}$ →

$$\alpha_1 = \frac{10{,}21}{R^2}\sqrt{\frac{D}{\rho h}} \qquad \alpha_2 = \frac{43{,}04}{R^2}\sqrt{\frac{D}{\rho h}} \qquad (69,11)$$

Тимошенко: Theorie des Vibrations. Béranger, Paris/Liège 1954.

Vynútené kmitanie kruhovej dosky

Pr: Luho voľn. okr., rovnomerne rozlož. bremeno: $q_v(x,y,t) = q_0 \sin\omega t$

Pís. (66,14) aj s pravou str.: $k^4 \frac{\partial^2 z}{\partial t^2} + \Delta^2 z = \frac{q_v(x,y,t)}{D}$, v pol. súr.: $\Big/ k^4 = \frac{\rho h}{D}$, $D = \frac{Eh^3}{12(m^2-1)}$

$$\rho h \frac{\partial^2 z}{\partial t^2} + D\Delta^2 z = q_v(r,\varphi,t) = q_0 \sin\omega t \qquad (70,2)$$

predp.: $z = P(r)\cdot\phi(\varphi)\sin\omega t = Z(r,\varphi)\sin\omega t$ (70,2a)

↓

$$D\Delta^2 Z(r,\varphi) - \rho h \omega^2 Z(r,\varphi) = q_0 \qquad (70,3)$$

Bez uzlových polomerov: $n = 0$ ... $\frac{d^2P(r)}{dr^2} + \frac{1}{r}\frac{dP(r)}{dr} \pm \lambda^2 P(r) = 0$ (68,9) $\Big/ Z(r,\varphi) = P(r)$

Všeob. int. (70,3) podľa (68,11) pri $n = 0$:

$$Z(r,\varphi) = P_1(r) = B_0 J_0(kr\sqrt{\alpha}) + C_0 J_0(ikr\sqrt{\alpha})$$

Part. int.: $P_2(r) = -\frac{q_0}{Dk^4}$ $\Big/ k^4 = \frac{\rho h \omega^2}{D}$

Všeob. int. (70,3): $P(r) = P_1(r) + P_2(r)$ (70,4)

Pri okr. podm.: $Z(R) = 0 \quad Z'(R) = 0$, v (70,4):

$$B_0 J_0(kR\sqrt{\alpha}) + C_0 J_0(ikR\sqrt{\alpha}) = \frac{q_0}{Dk^4}$$

resp.: $$B_0 J_0'(kR\sqrt{\alpha}) + iC_0 J_0'(ikR\sqrt{\alpha}) = 0 \tag{70,5}$$

$$\downarrow$$

$$-B_0 J_1(kR\sqrt{\alpha}) + C_0 J_1(ikR\sqrt{\alpha}) = 0 \tag{70,6}$$

Z rovn. (70,5), (70,6) int. konšt.:

$$B_0 = \frac{q_0}{Dk^4} \frac{J_1(ikR\sqrt{\alpha})}{J_0(kR\sqrt{\alpha})\,J_1(ikR\sqrt{\alpha}) + J_1(kR\sqrt{\alpha})\,J_0(ikR\sqrt{\alpha})}$$

$$C_0 = \frac{q_0}{Dk^4} \frac{J_1(kR\sqrt{\alpha})}{J_0(kR\sqrt{\alpha})\,J_1(ikR\sqrt{\alpha}) + J_1(kR\sqrt{\alpha})\,J_0(ikR\sqrt{\alpha})}$$

Všeob. int. potom: (podľa 70,4)

$$Z(r) = \left[\frac{J_1(ikR\sqrt{\alpha})\,J_0(kr\sqrt{\alpha}) + J_1(kr\sqrt{\alpha})\,J_0(ikr\sqrt{\alpha})}{J_0(kr\sqrt{\alpha})\,J_1(ikR\sqrt{\alpha}) + J_1(kR\sqrt{\alpha})\,J_0(ikR\sqrt{\alpha})} - 1\right]\frac{q_0}{Dk^4}$$

Rezonancia kmitania nastane pre nulový menovateľ.

Kmenité pre kruhové dosky:

votknutý okraj: $$f_{ekv.} = \frac{\alpha_i}{2\pi R^2}\sqrt{\frac{D}{h\rho}}$$ h - celá hrúbka

$\alpha_i = 10{,}21 \quad 39{,}78 \quad 88{,}90$

$ka = 3{,}19 \quad 6{,}30 \quad 9{,}43 \quad 12{,}57$

oprętý stred: $\alpha_{01} = 3{,}75 \quad \alpha_{02} = 20{,}91 \quad \alpha_{03} = 119{,}7$

pevný stred: $$f_{03} = \frac{0{,}344\left(\frac{h}{2}\right)}{R^2}\sqrt{\frac{E}{\rho(1-\sigma^2)}} = \frac{3{,}74}{2\pi R^2}\sqrt{\frac{D}{h\rho}}$$

voľná doska: $$f_{02} = \frac{0{,}824\left(\frac{h}{2}\right)}{R^2}\sqrt{\frac{E}{\rho(1-\sigma^2)}} = \frac{8{,}96}{2\pi R^2}\sqrt{\frac{D}{h\rho}}$$

Výpočet tenkej kruhovej dosky, opretej v jej strede;

$$f = \frac{\alpha_i}{2\pi r^2}\sqrt{\frac{D}{\rho h}} \longrightarrow r = \sqrt{\frac{\alpha_i}{2\pi f}\sqrt{\frac{1}{\rho h}\frac{Eh^3}{12(1-\mu^2)}}}$$ / Dobrovolný

$$r = \sqrt{\alpha_i}\cdot\frac{1}{\sqrt{2\pi f}}\cdot\sqrt[4]{\frac{Eh^2}{12.\rho(1-\mu^2)}} = \sqrt{\alpha_i}\,\frac{\sqrt{h}}{\sqrt{2\pi f}}\sqrt[4]{\frac{E}{12.\rho.(1-\mu^2)}}$$

$$E_{oceľ} = 2{,}1.10^6\frac{kp}{cm^2} = \frac{2{,}1.10^6}{cm^2}.\,1kg\,\frac{981cm}{s^2} = 20{,}6.10^8\frac{kg}{cm\,s^2}$$

$$\rho = 7{,}87.10^{-3}\,kg/cm^3$$

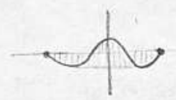
$$\left.\begin{array}{l}\mu^2 = 0{,}28^2 = 0{,}0783\\ 1-\mu^2 \approx 0{,}92\end{array}\right\} \underbrace{12.\,7{,}87.0{,}92.10^{-3}}_{menovateľ} = 86{,}7.10^{-3}\ [kg/cm^3]$$

$$\sqrt[4]{\frac{20{,}6.10^8\,kg/cm\,s^2}{0{,}0867\ kg/cm^3}} = 10^2.\sqrt[4]{238} = 10^2\,3{,}93 = 393\sqrt{cm\,s}$$

$$\sqrt{2\pi f} = \sqrt{2\pi\ 21000} = 10^2\sqrt{13{,}2} = 363\sqrt{s^{-1}}$$

$$r = \sqrt{\alpha_i}\sqrt{h}\cdot\frac{393}{363}\quad [cm;\ cm]$$

	0	1	2
$\alpha_i =$	3,75	20,91	119,7
$\sqrt{\alpha_i} =$		4,57	10,92



$$r = 11{,}82.\sqrt{h}$$

$h = 0{,}1cm$ .... $r_{0,1} = 11{,}82.0{,}316 = 3{,}74\ cm$

$h = 0{,}3cm$ .... $r_{0,3} = 11{,}82.0{,}548 = 6{,}5\ cm$

$h = 0{,}5\ cm$ ...... $r_{0,5} = 8{,}35\ cm$

$h = 0{,}7\ cm$ ...... $r_{0,7} = 9{,}9\ cm$

$h = 1\ cm$ .... $r_1 = 11{,}82\ cm$

Voľná doska:

celá hrúbka!

$$f_{02} = \frac{0{,}412h}{R^2}\sqrt{\frac{E}{\rho(1-\sigma^2)}} \longrightarrow R = \sqrt{\frac{0{,}412h}{f_{02}}\sqrt{\frac{E}{\rho(1-\sigma^2)}}}$$ / Merhaut

$$R = \sqrt{h}\cdot\sqrt{\frac{0{,}412}{2{,}1.10^4}\sqrt{\frac{20{,}6.10^8\,kg/cm\,s^2}{7{,}87.10^{-3}(kg/cm^3).0{,}92}}} = \sqrt{h\cdot\frac{0{,}196}{10^4}\cdot\frac{10^4}{10^{-1}}\sqrt{\frac{20{,}6}{0{,}724}}} = \sqrt{h}\cdot\sqrt{1{,}96\cdot5{,}33} = \sqrt{h}\cdot3{,}236$$

oceľ:

$h = 0{,}1cm$ ..... $R_1 = 1{,}022\ cm$

$h =$

$h = 0{,}5cm$ ...... $R_5 = 2{,}285\ cm$

$h = 1\ cm$ ..... $R_{10} = 3{,}236cm$

Volná Al doska (Merhaut:)

$$R = \sqrt{h}\sqrt{\frac{0{,}412}{2{,}1\cdot 10^4}\sqrt{\frac{7{,}0828\cdot 10^8}{2{,}69\cdot 10^{-3}\cdot 0{,}891}}} = \sqrt{h}\sqrt{\frac{19{,}619}{10^6}\,10^4\sqrt{\frac{7{,}0828\cdot 10^3}{2{,}3968}}} = \frac{\sqrt{h}}{10}\,4{,}429\sqrt[4]{2955{,}1} = \sqrt{h}\cdot 0{,}443\cdot 7{,}37$$

$$R = 3{,}265\sqrt{h}$$

$E_{al} = 7{,}22\cdot 10^5\cdot 981 = 7082{,}82\cdot 10^5\ \frac{kg}{cm\,s^2} = 7{,}0828\cdot 10^8$

$\rho = 2{,}69\cdot 10^{-3}\ kg/cm^3$

$1-\sigma^2 = 1-0{,}1089 = 0{,}8911$

$\sigma_{al} = 0{,}33 \div 0{,}34$

$h = 0{,}1$ cm	$R = 1{,}033$ cm	„f_{02}"
0,2		
0,3	1,461	
0,4	1,79	
0,5	2,062 cm	
	2,31 cm	
0,6	2,53 cm	

V strede opretá Al doska (Dobrovolný):

$$R = \frac{\sqrt{\alpha_i}\sqrt{h}}{\sqrt{2\pi f}}\sqrt[4]{\frac{E}{12\cdot\rho(1-\mu^2)}} = \frac{\sqrt{\alpha_i}\sqrt{h}}{363}\sqrt[4]{\frac{7{,}0828\cdot 10^8}{12\cdot 2{,}69\cdot 10^{-3}\cdot 0{,}89}} = \frac{\sqrt{\alpha_i}\sqrt{h}\cdot 100}{363}\sqrt[4]{\frac{7082{,}8}{28{,}729}} = \sqrt{\alpha_i}\sqrt{h}\ 0{,}276\cdot 3{,}96$$

2465   1,091

$\sqrt{\alpha_{02}} = 10{,}92 \rightarrow R = \sqrt{h}\cdot 12{,}02$ (11,914) $R_{02(?)} \doteq 5\cdot\sqrt{h}$

$h = 0{,}1$ cm	1,58 cm	3,79 cm
0,2	2,236	5,3664 (5,33)
0,3	2,7385	6,5724
0,4	3,163	7,59 (7,56)
0,5	3,5355	8,4852
0,6	3,87	9,295 (9,23)

Kruhová, na okraji votknutá Al doska: (Merhaut)

$$f_{mn} = \frac{\pi h}{2R^2}\sqrt{\frac{E}{3\rho(1-\sigma^2)}}\,(\beta_{mn})^2 \rightarrow R = \sqrt{\frac{\pi h}{2f}}\,\beta_{mn}\sqrt[4]{\frac{E}{3\rho(1-\sigma^2)}}$$

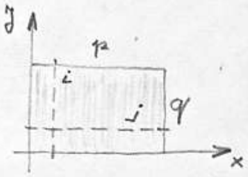
$\beta_{02} = 2{,}007$; $\beta_{03} = 3{,}000$;

$$R_{02} = \sqrt{h_{(cm)}}\sqrt{\frac{\pi\ sec}{2\cdot 2{,}1\cdot 10^4}}\cdot 2{,}007\sqrt[4]{\frac{7{,}0828\cdot 10^8\ \frac{kg}{cm\,s^2}}{3\cdot 2{,}69\cdot 10^{-3}\frac{kg}{cm^3}\cdot 0{,}8911}} = \sqrt{h}\sqrt{\frac{12{,}648}{4{,}2}}\sqrt[4]{\frac{7082{,}8}{7}} = \sqrt{h_{cm}}\ 1{,}735\cdot 5{,}64 = 9{,}7854\sqrt{h}$$

1011,83   31,81

$h = 0{,}1$ cm	$R_{02} = 3{,}094$ cm
$h = 0{,}2$	4,376
0,3	5,36
0,4	6,19 cm
0,5	6,92
0,6	7,58

Tenká, obdĺžniková doska $p\times q - h$, s nepárnymi počtami uzl. priamok, – s 1 žiaričom v strede, okraje votknuté.



Predpokl.: $w(x,y) = A\sin\pi\frac{i}{p}x \cdot \sin\pi\frac{j}{q}y$ | i – počet vnút. uzl. priam + 1 okraj

$$V_{max} = T_{max}$$

Max. kinet. energ.: $\frac{1}{2}\cdot\rho h\iint\limits_{XY}\dot{w}^2\,dx\,dy + \frac{1}{2}M\dot{w}^2$

$$T_{max} = \frac{1}{2}\rho h\alpha^2\iint\limits_{XY} w^2\,dx\,dy + \frac{1}{2}M\alpha^2 \overset{1.A^2}{w^2} = \frac{1}{2}\alpha^2\left[\rho h\iint\limits_{XY} w^2\,dx\,dy + A^2 M\right]$$

Max pot. energ.:

$$V_{max} = \frac{D}{2}\iint\limits_{XY}\left[\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)^2 - 2(1-\mu)\left\{\frac{\partial^2 w}{\partial x^2}\cdot\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x\partial y}\right)^2\right\}\right]dx\,dy \qquad D = \frac{Eh^3}{12(1-\mu^2)}$$

$$\frac{\partial w}{\partial x} = \frac{i\pi}{p}\cdot\sin\frac{j\pi}{q}y\cdot\cos\frac{i\pi}{p}x \cdot A \qquad \frac{\partial^2 z}{\partial x^2} = -\left(\frac{i\pi}{p}\right)^2\sin\frac{j\pi}{q}y\cdot\sin\frac{i\pi}{p}x \cdot A$$

$$\frac{\partial w}{\partial y} = \frac{j\pi}{q}\cdot\sin\frac{i\pi}{p}x\cdot\cos\frac{j\pi}{q}y \cdot A \qquad \frac{\partial^2 z}{\partial y^2} = -\left(\frac{j\pi}{q}\right)^2\sin\frac{i\pi}{p}x\cdot\sin\frac{j\pi}{q}y \cdot A$$

$$\frac{\partial^2 w}{\partial x\partial y} = \frac{i\pi}{p}\cdot\frac{j\pi}{q}\cos\frac{i\pi}{p}x\cdot\cos\frac{j\pi}{q}y \cdot A$$

$$V_{max} = \frac{D}{2}\iint\limits_{XY}\left[\left\{-\left(\frac{i\pi}{p}\right)^2\sin\frac{j\pi}{q}y\cdot\sin\frac{i\pi}{p}x - \left(\frac{j\pi}{q}\right)^2\sin\frac{i\pi}{p}x\cdot\sin\frac{j\pi}{q}y\right\}^2 - 2(1-\mu)\left\{\left(\frac{i\pi}{p}\right)^2\cdot\left(\frac{j\pi}{q}\right)^2\sin^2\frac{i\pi}{p}x\cdot\sin^2\frac{j\pi}{q}y - \right.\right.$$
$$\left.\left. - \left(\frac{i\pi}{p}\right)^2\left(\frac{j\pi}{q}\right)^2\cos^2\left(\frac{i\pi}{p}x\right)\cdot\cos^2\left(\frac{j\pi}{q}y\right)\right\}\right]dx\,dy$$

$$V_{max} = \frac{D}{2}\iint\limits_{XY}\left[\left\{\left(\frac{i\pi}{p}\right)^4\sin^2\frac{j\pi}{q}y\cdot\sin^2\frac{i\pi}{p}x + \left(\frac{j\pi}{q}\right)^4\sin^2\frac{i\pi}{p}x\cdot\sin^2\frac{j\pi}{q}y + 2\left(\frac{i\pi}{p}\right)^2\left(\frac{j\pi}{q}\right)^2\sin^2\frac{i\pi}{p}x\cdot\sin^2\frac{j\pi}{q}y\right\} - \right.$$
$$\left. - 2(1-\mu)\left\{\left(\frac{i\pi}{p}\right)^2\left(\frac{j\pi}{q}\right)^2\sin^2\frac{i\pi}{p}x\cdot\sin^2\frac{j\pi}{q}y - \left(\frac{i\pi}{p}\right)^2\left(\frac{j\pi}{q}\right)^2\cos^2\left(\frac{i\pi}{p}x\right)\cdot\cos^2\left(\frac{j\pi}{q}y\right)\right\}\right]dx\,dy$$

$$V_{max} = \frac{D}{2}\left[\left\{\left(\frac{i\pi}{p}\right)^4\cdot\int_0^q\sin^2\frac{j\pi}{q}y\,dy\cdot\int_0^p\sin^2\frac{i\pi}{p}x\,dx + \left(\frac{j\pi}{q}\right)^4\int_0^p\sin^2\frac{i\pi}{p}x\,dx\cdot\int_0^q\sin^2\frac{j\pi}{q}y\,dy + 2\left(\frac{i\pi}{p}\right)^2\left(\frac{j\pi}{q}\right)^2\int_0^p\sin^2\frac{i\pi}{p}x\,dx\cdot\int_0^q\sin^2\frac{j\pi}{q}y\,dy\right\} - \right.$$
$$\left. - 2(1-\mu)\left\{\left(\frac{i\pi}{p}\right)^2\left(\frac{j\pi}{q}\right)^2\int_0^p\sin^2\frac{i\pi}{p}x\,dx\cdot\int_0^q\sin^2\frac{j\pi}{q}y\,dy - \left(\frac{i\pi}{p}\right)^2\left(\frac{j\pi}{q}\right)^2\int_0^p\cos^2\left(\frac{i\pi}{p}x\right)dx\cdot\int_0^q\cos^2\frac{j\pi}{q}y\cdot dy\right\}\right]$$

$$\int_0^p\sin^2\frac{i\pi}{p}x\,dx = 2i\int_0^{\frac{p}{2i}}\sin^2\frac{i\pi}{p}x\,dx = 2i\left[\frac{p}{4i} - \frac{i\pi}{4p}\cdot\sin 2\cdot\frac{i\pi}{p}\cdot\frac{p}{2i} - 0 + 0\right] = \frac{p}{2}$$

$$\int_0^q\sin^2\frac{j\pi}{q}y\,dy = \frac{q}{2}$$

$$\int_0^p\cos^2\frac{i\pi}{p}x\,dx = \frac{p}{2}$$

$$\int_0^q\cos^2\frac{j\pi}{q}y\,dy = \frac{q}{2}$$

$$V_{max} = \frac{D}{2}\left[\left(\frac{i\pi}{p}\right)^4 \cdot \frac{q}{2}\cdot\frac{p}{2} + \left(\frac{j\pi}{q}\right)^4 \frac{p}{2}\cdot\frac{q}{2} + 2\left(\frac{i\pi}{p}\right)^2\left(\frac{j\pi}{q}\right)^2 \frac{p}{2}\cdot\frac{q}{2}\right\} - 2(1-\mu)\left\{\left(\frac{i\pi}{p}\right)^2\left(\frac{j\pi}{q}\right)^2 \frac{p}{2}\cdot\frac{q}{2} - \left(\frac{i\pi}{p}\right)^2\left(\frac{j\pi}{q}\right)^2 \frac{p}{2}\cdot\frac{q}{2}\right\}\right]$$

$$V_{max} = A^2\frac{D}{2}\,\frac{\pi^4}{4}pq\left[\left(\frac{i}{p}\right)^4 + \left(\frac{j}{q}\right)^4 + 2\left(\frac{i}{p}\right)^2\left(\frac{j}{q}\right)^2 - 2(1-\mu)\left\{\left(\frac{i}{p}\right)^2\left(\frac{j}{q}\right)^2 - \left(\frac{i}{p}\right)^2\left(\frac{j}{q}\right)^2\right\}\right]$$

$$V_{max} = A^2\frac{D}{2}\pi^4\frac{pq}{4}\left[\left(\frac{i}{p}\right)^2 + \left(\frac{j}{q}\right)^2\right]^2$$

$$A^2\iint_{XY} \sin^2\frac{i\pi}{p}x \cdot \sin^2\frac{j\pi}{q}y\,dx\,dy = A^2\int_0^p \sin^2\frac{i\pi}{p}x\,dx \cdot \int_0^q \sin^2\frac{j\pi}{q}y\,dy = A^2\frac{p}{2}\cdot\frac{q}{2}$$

$$T_{max} = \frac{1}{2}\omega^2\left[\rho h\frac{pq}{4} + M\right]\cdot A^2$$

$$\omega^2 = \frac{A^2\frac{D}{2}\pi^4\frac{pq}{4}\left[\left(\frac{i}{p}\right)^2 + \left(\frac{j}{q}\right)^2\right]^2}{\frac{1}{2}\left[\rho h\frac{pq}{4} + M\right]A^2} = \frac{D\left(\frac{\pi^2}{2}\right)^2\cdot pq\left[\left(\frac{i}{p}\right)^2 + \left(\frac{j}{q}\right)^2\right]^2}{\left[\rho h\frac{pq}{4} + M\right]} = (2\pi)^2\cdot f^2 \quad / \sqrt{}$$

neplatí pre $hpq \approx 300\,cm^3$

$$\left(\frac{\pi}{2}\right)\left[\left(\frac{i}{p}\right)^2 + \left(\frac{j}{q}\right)^2\right]\cdot\sqrt{Dpq} = f\cdot\sqrt{\rho h pq + M.4}$$

$$\frac{\pi}{2f}\left[\left(\frac{i}{p}\right)^2 + \left(\frac{j}{q}\right)^2\right] = \sqrt{\frac{\rho h}{D} + \frac{4M}{Dpq}} = \frac{1}{\sqrt{D}}\sqrt{\rho h + \frac{4M}{pq}}$$

(pre nepárne i, j)

Keď: $p = q = s$; $i = j = u$:

$$\frac{\pi\sqrt{D}}{2f}\left[2\left(\frac{u}{s}\right)^2\right] = \sqrt{\rho h + \frac{4M}{s^2}}$$

$$\frac{\pi^2 D}{f^2}\frac{u^4}{s^4} = \frac{s^2\rho h + 4M}{s^2} \rightarrow \frac{\pi^2 D u^4}{f^2} = s^4\rho h + s^2 4M \rightarrow s^4 + s^2\frac{4M}{\rho h} - \frac{\pi^2 D u^4}{f^2\rho h} = 0$$

$$S^2 + S\frac{4M}{\rho h} - \frac{\pi^2 D u^4}{f^2\rho h} = 0$$ ... (pre nepárny počet i, j !)

$$S_{1,2} = -\frac{2M}{\rho h} \pm \sqrt{\left(\frac{2M}{\rho h}\right)^2 + \frac{\pi^2 D u^4}{f^2\rho h}}$$

$$s = \sqrt{\sqrt{\left(\frac{2M}{\rho h}\right)^2 + \left(\frac{\pi u}{f}\right)^2\frac{u^2 D}{\rho h}} - \frac{2M}{\rho h}}$$

štvorcová doska : $s^2 - h'$; počet uzlových priamok $u + w$ (w ~ nepárne) ; v strede 1 žiarič.
doska z hliníka , voľnuté okraje.

$$s = \sqrt{\sqrt{\left(\frac{2M}{\rho h}\right)^2 + \left(\frac{\pi u}{f}\right)^2 \frac{u^2 D}{\rho h}} - \frac{2M}{\rho h}}$$

Hmotnosť meniča : $M = 0{,}8\,kg$

špec. hm. dosky : $\rho = 2{,}69 . 10^{-3}\,kg/cm^3$

hrúbka dosky : $h\,[cm]$

frekvencia : $f\,[\frac{1}{s}]$

modul pružn. Al : $E \approx 7{,}083 . 10^8 \frac{kg}{cm\,s^2}$

Poisson. konšt. Al : $\mu \approx 0{,}33$

$$1 - \mu^2 \doteq 0{,}89$$

$$D = \frac{Eh^3}{12(1-\mu^2)} = h^3 . \frac{70{,}83 . 10^7 \frac{kg}{cm\,s^2}}{12 . 0{,}89} = h^3 . 6{,}64 . 10^7 \frac{kg\,cm^2}{s^2}$$

$h = 0{,}1\,cm$; $u = 3$:

$$s_{[cm]} = \sqrt{\sqrt{\left(\frac{1{,}6\,kg}{2{,}69 . 10^{-3}\frac{kg}{cm^3} . 0{,}1\,cm}\right)^2 + \left(\frac{\pi . 3}{2 . 10^4 \frac{1}{s}}\right)^2 . \frac{3^2 . 0{,}1^3 . 6{,}64 . 10^7 \frac{kg\,cm^2}{s^2}}{2{,}69 . 10^{-3}\frac{kg}{cm^3} . 0{,}1\,cm}} - \frac{1{,}6\,kg}{2{,}69 . 10^{-3}\frac{kg}{cm^3} . 0{,}1\,cm}} =$$

$$= \sqrt{\sqrt{\left(\frac{1{,}6 . 10^3}{0{,}269}\right)^2 + \left(\frac{3\pi}{2 . 10^4}\right)^2 \frac{9 . 6{,}64 . 10^7}{0{,}269}} - \frac{1{,}6 . 10^3}{0{,}269}} = \sqrt{\sqrt{10^6 . 35{,}4 + \frac{1}{10^8} . 22{,}2 . 10^7 . 223} - 10^3 . 5{,}95} =$$

$$= \sqrt{\sqrt{35{,}4 . 10^6 + 496} - 5{,}95 . 10^3}$$

. . . . .

$h = 1\,cm$; $u = 7$:

$$s_{[cm]} = \sqrt{\sqrt{\left(\frac{1{,}6 . 10^3}{2{,}69}\right)^2 + \left(\frac{\pi . 7}{2 . 10^4}\right)^2 \frac{49 . 6{,}64 . 10^7 . 10^3}{2{,}69}} - \frac{1{,}6 . 10^3}{2{,}69}} = \sqrt{\sqrt{10^6 . 0{,}354 + 10^2 . 121 . 121} - 10^3 . 0{,}595}$$

$$s_{[cm]} = \sqrt{\sqrt{0{,}354 . 10^6 + 1{,}464 . 10^6} - 595} = \sqrt{10^3 \sqrt{1{,}818} - 595} = \sqrt{1347 - 595} = \sqrt{752} = 27{,}4$$

$h = 0{,}3\,cm$; $u = 5$:

$$s_{[cm]} = \sqrt{\sqrt{\left(\frac{1{,}6 . 10^3}{2{,}69 . 0{,}3}\right)^2 + \left(\frac{\pi . 5}{2 . 10^4}\right)^2 \frac{5^2 . 0{,}3^3 . 6{,}64 . 10^{10}}{2{,}69 . 0{,}3}} - \frac{1{,}6 . 10^3}{2{,}69 . 0{,}3}} = \sqrt{\sqrt{3{,}92 . 10^6 + \frac{61{,}8}{10^8} . \frac{10^{10} 4{,}480}{0{,}807}} - 1980} =$$

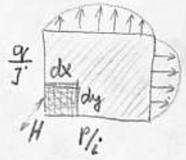
$$= \sqrt{\sqrt{3{,}92 . 10^6 + 34400} - 1980} = \sqrt{\sqrt{3954400} - 1980} = \sqrt{1990 - 1980} = \sqrt{10} \approx 3{,}16$$

– s meničom $\phi 38$ príliš nereálne.

Nepresnosť – s uvažovaním bodového zaťaženia ; " $\frac{2M}{\rho W}$ "

1.) bodové zať. : $T_b = \frac{1}{2}(\alpha^2 \cdot 1^2) \cdot M$

2.) plošné zaťaž. : P kg/cm² ; $r = \sin\frac{\pi i}{p}x \cdot \sin\frac{\pi j}{q}y$



$$r'^2 = \alpha^2 \cdot r^2$$

$$T_p = 4\int_0^{\frac{p}{2i}}\int_0^{\frac{q}{2j}} \frac{1}{2} H\,dx\,dy \cdot \frac{\rho}{p} \cdot \alpha^2 \cdot \sin^2\frac{\pi i}{p}x \cdot \sin^2\frac{\pi j}{q}y = \frac{4}{2} H\rho_p \alpha^2 \int_0^{\frac{p}{2i}} \sin^2\frac{\pi i}{p}x\,dx \int_0^{\frac{q}{2j}} \sin\frac{\pi j}{q}y\,dy$$

$$T_p = \frac{4}{2} H\rho_p\alpha^2 \cdot \frac{p}{4i} \cdot \frac{q}{4j} = \frac{\alpha^2}{2} \cdot \frac{H.p.q.\rho_p}{4.i.j} = \frac{\alpha^2}{2} \cdot \frac{1}{4ij} \cdot M_p = \frac{T_b}{2.i.j}$$

~ bez uvaž. strát (trenie!)

Kruhová doska, hrúbka h, opretý stred, hliník, len uzl. kruž. $R = 1{,}0909\sqrt{h}\sqrt{\alpha_j}$ [cm]

h	1	2	3	4	5	6	7	10
$\sqrt{\alpha_i}$ = 1,936	6,7	9,4	11,6	13,4	14,9	16,4		
4,573	15,8	22,3	27,3	31,6	35,2	38,6		
10,941	37,7	53,4	65,4	75,5	84,2	92,5	99	118,2

[mm]

Votknutý okraj :

h						
$\sqrt{\alpha_i}$ = 3,195	11,02	15,56	19,1	22,0	24,6	27,0
6,31	21,8	30,8	37,7	43,5	48,7	53,3
9,43	32,5	46,0	56,3	65,1	72,74	79,7
12,57	43,4	61,3	75,1	86,7	97,0	106,2

Pevný stred. $R_{03} = 2{,}11\sqrt{h}$ cm

6,7 9,4 11,6 13,3 14,9 16,4

Voľná doska $R_{02} = 3{,}27\sqrt{h}$ cm

10,3 14,6 17,9 20,6 23,1 25,3 31,9 (10)

Merhaut : kruh. dosky:

$\beta_{01} = 1{,}015$ 2,007 $3{,}000 = \beta_{03}$

$\beta_{11} = 1{,}468$ 2,483 3,49

$\beta_{21} = 1{,}879$ 2,992 $4{,}000 = \beta_{23}$

volkn. okr. : $f_{mn} = \frac{\pi h}{2R^2}\sqrt{\frac{E}{3\rho(1-\sigma^2)}}\,(\beta_{mn})^2$ (m – diay, n – kruž. + okr.)

voľná d. : $f_{02} = \frac{0{,}824h}{R^2}\sqrt{\frac{E}{\rho(1-\sigma^2)}}$

$f_{21} = 2{,}134 f_{02}$

v strede upevn. : $f_{03} = \frac{0{,}344h}{R^2}\sqrt{\frac{E}{\rho(1-\sigma^2)}}$

okraj podopr. : $f_{02} = \frac{0{,}466h}{R^2}\sqrt{\frac{E}{\rho(1-\sigma^2)}}$

UZ – mechanicko akustická energia – pasívna
aktívna ($\uparrow$ampl.)

pl., kv. – len pozdĺ. vlnenie
Pozdĺžne vlny – neohr. prostr. $\gg \lambda$.
Priečne –"–
Ohybové – ohr. prostr. 1x, 2x $\ll \lambda$
Pozdĺ., rad. –"– $\emptyset$
Torzné
Povrchové

Akust. tlak $p = p_0 + \omega A \rho c \cos \omega (t - \frac{x}{c})$; max: $P = \omega A \rho c = V . Z_0$

Energ. $E = \frac{1}{2} \rho \omega^2 A^2$

Intenz. $I = c . E$

Rýchlosť šírenia: $c = \sqrt{\frac{1-\sigma}{(1+\sigma)(1-2\sigma)} \cdot \frac{E}{\rho}}$ / σ – Poisson. k. / pre priečne vlny: $c_p = \sqrt{\frac{G}{\rho}}$

$c = \sqrt{\frac{1}{\rho . \beta_{ad}}}$ / β – stlačiteľnosť / plyny + kvap.

$\downarrow$ampl. so vzd. $A_x = A_0 e^{-\alpha x}$ / α – koef. tlmenia $= \frac{\alpha'}{frekv^2}$ / $\alpha' = \pi \frac{f}{c} \cdot \varepsilon$

koef. absorbcie — stratový činiteľ

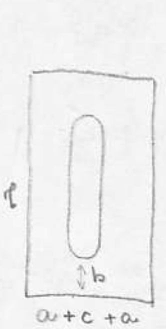
Mech. úč.: koagulácia, disperzia
Tep. –"–
Chem.
Kavitácia → UZ čistenie
mgnstr.

Pre úč. UZ čistenie: mn. predm. < 50% kvap.
~ rovný akust. odpor
tenké, dlhé otvory: $\uparrow$fr. [$\Rightarrow \uparrow$výkon]
ožiar. z boku – len u Al. a skla
nečistoty nepolárn. char.-u sa rozp. v nepol. kv. (org. rozpúšť.)
mäkké mat.-y nevhodné

Vzd. medzi meničmi na dne $\uparrow$ vaní < $1/2 \lambda$

Čist. vane oceľ 18Cr 8Ni ; hrúbka $0{,}5 \div 1{,}5$ mm. Vzd. čisteného predm. od steny < 60 cm.
Keď hladina nad meničom < $20 \div 30$ cm ; $10 \div 20$ W/l.
int. klesá štv. vzd.

Výška hladiny má byť $n \cdot \frac{1}{2} \lambda$ – pre meničov na dne.
(2 meniče na protiľahl. stenách vylv. sp. podm. pre stoj. vlnenie)



$f_0 = f_t \sqrt{\frac{1}{1 + \frac{2c}{l}}}$ $\quad b = a . \frac{B_0 + B_m}{B_s}$

$f_t = \frac{c_t}{2l}$ $\quad B_s$ ~ nasýt. ind.

$c_t = \sqrt{\frac{E}{\rho}}$

Exponenciálny nadstavec $S_x = S_0 \cdot e^{-2\lambda x}$

$\alpha = \frac{1}{l} \cdot \ln \frac{D_0}{D_1} = \frac{1}{l} \ln N$ | $\frac{\xi_1}{\xi_0} = \frac{D_0}{D_1}$

$l_{rez.} = \frac{\lambda}{2} \sqrt{\frac{(m\pi)^2 + \ln^2 N}{\pi^2}} = \frac{c_1}{\omega} \sqrt{(m\pi)^2 + \ln^2 N}$

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$c_1 = \frac{c_t}{1 - \frac{\alpha^2 c_t^2}{4\omega^2}}$ | $X_{uzl} (\text{od } D_0) = \frac{c_e}{\omega} . \operatorname{arctg} \frac{2\omega}{c_e}$

Stupňovitý nadstavec (najväčšie zosilnenie)

$$A = \left(\frac{D_0}{D_1}\right)^2 = N^2 \quad , \text{keď} \quad l_1 = l_2 = \frac{c_t}{4f}$$

Skrátenie nadstavca (– s nástrojom!) $\Delta l = \dfrac{m}{S_1 \varrho_1}$

~ pre rovnaké prierezy S_1 a nástroja!

m – hmota nástroja
S_1 – plocha úzk. konca nadstavca
ϱ_1 – šp. hm. nadst.

Pre valcový nástroj $\Delta l = \dfrac{S_m \cdot \varrho_m}{S_1 \cdot \varrho_1} \cdot h$ $\quad h$ ~ dĺžka nástroja