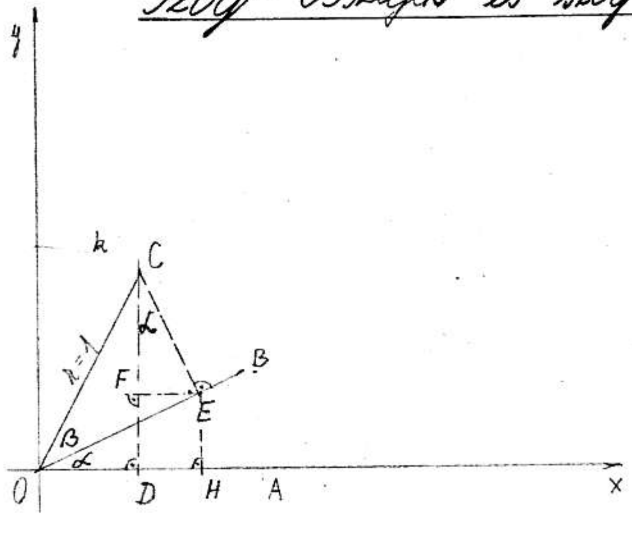
24. óra. Összef. különb. szögek ∢ között

Szög-összegek és szög-különbségek szögfüggvényei



$k = (O;\ r = 1)$

$r = OA = OB = OC = 1$

$\alpha = AOB\sphericalangle$

$\beta = BOC\sphericalangle$

$\alpha + \beta = DOC\sphericalangle$

$AOB\sphericalangle = FCE\sphericalangle = \alpha$

$\sin(\alpha+\beta) = \frac{CD}{OC} = CD$, mert $r = 1$ [ODC△]

$CD = CF + FD$, de $FD = EH$

$CD = CF + EH = CE \cdot \cos\alpha + EH =$

$= CE \cdot \cos\alpha + OE \cdot \sin\alpha =$

$= \sin\beta \cdot \cos\alpha + \cos\beta \cdot \sin\alpha$

$\cos\alpha = \frac{CF}{CE}$

$\sin\alpha = \frac{EH}{OE}$

$\sin\beta = \frac{CE}{OC} = CE$

$\cos\beta = \frac{OE}{OC} = OE$

$CD = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta$

$$\underline{\underline{\sin(\alpha+\beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta}}$$

$\cos(\alpha+\beta)$

$\cos(\alpha+\beta) = \frac{OD}{OC} = OD$

$OD = OH - DH$

$OD = OH - FE$

$\cos(\alpha+\beta) = OH - FE = OE \cdot \cos\alpha - \sin\alpha \cdot CE =$

$= \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$

$\cos\alpha = \frac{OH}{OE}$

$\sin\alpha = \frac{FE}{CE}$

$\cos\beta = \frac{OE}{OC} = OE$

$\sin\beta = \frac{EC}{OC} = EC$

$$\underline{\underline{\cos(\alpha+\beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta}}$$

$$\operatorname{tg}(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{\sin\alpha\cdot\cos\beta + \cos\alpha\cdot\sin\beta}{\cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta} \Big/ \cdot \frac{1}{\cos\alpha\cdot\cos\beta}$$

$$= \frac{\dfrac{\sin\alpha\cdot\cos\beta}{\cos\alpha\cdot\cos\beta} + \dfrac{\cos\alpha\cdot\sin\beta}{\cos\alpha\cdot\cos\beta}}{\dfrac{\cos\alpha\cdot\cos\beta}{\cos\alpha\cdot\cos\beta} - \dfrac{\sin\alpha\cdot\sin\beta}{\cos\alpha\cdot\cos\beta}} = \frac{\dfrac{\sin\alpha}{\cos\alpha} + \dfrac{\sin\beta}{\cos\beta}}{1 - \dfrac{\sin\alpha}{\cos\alpha}\cdot\dfrac{\sin\beta}{\cos\beta}} =$$

$$= \frac{\operatorname{tg}\alpha + \operatorname{tg}\beta}{1 - \operatorname{tg}\alpha\cdot\operatorname{tg}\beta}$$

$$\operatorname{tg}(\alpha+\beta) = \frac{\operatorname{tg}\alpha + \operatorname{tg}\beta}{1 - \operatorname{tg}\alpha\cdot\operatorname{tg}\beta}$$

$$\operatorname{cotg}(\alpha+\beta) = \frac{\cos(\alpha+\beta)}{\sin(\alpha+\beta)} = \frac{\cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta}{\sin\alpha\cdot\cos\beta + \cos\alpha\cdot\sin\beta} =$$

$$= \frac{\dfrac{\cos\alpha\cdot\cos\beta}{\sin\alpha\cdot\sin\beta} - \dfrac{\sin\alpha\cdot\sin\beta}{\sin\alpha\cdot\sin\beta}}{\dfrac{\sin\alpha\cdot\cos\beta}{\sin\alpha\cdot\sin\beta} + \dfrac{\cos\alpha\cdot\sin\beta}{\sin\alpha\cdot\sin\beta}} = \frac{\operatorname{cotg}\alpha\cdot\operatorname{cotg}\beta - 1}{\operatorname{cotg}\beta + \operatorname{cotg}\alpha} =$$

$$\operatorname{cotg}(\alpha+\beta) = \frac{\operatorname{cotg}\alpha\cdot\operatorname{cotg}\beta - 1}{\operatorname{cotg}\alpha + \operatorname{cotg}\beta}$$

$$\sin(\alpha-\beta) = \sin\alpha\cdot\cos(-\beta) + \cos\alpha\cdot\sin(-\beta) =$$
$$= \sin\alpha\cdot\cos\beta - \cos\alpha\cdot\sin\beta$$

$$\cos(\alpha-\beta) = \cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta =$$
$$= \cos\alpha\cdot\cos(-\beta) - \sin\alpha\cdot\sin(-\beta) =$$
$$= \cos\alpha\cdot\cos\beta + \sin\alpha\cdot\sin\beta$$

$$\operatorname{tg}(\alpha-\beta) = \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)} = \frac{\sin\alpha\cdot\cos\beta - \cos\alpha\cdot\sin\beta}{\cos\alpha\cdot\cos\beta + \sin\alpha\cdot\sin\beta}$$

$$\frac{\dfrac{\sin\alpha\cdot\cos\beta}{\cos\alpha\cdot\cos\beta} - \dfrac{\cos\alpha\cdot\sin\beta}{\cos\alpha\cdot\cos\beta}}{\dfrac{\cos\alpha\cdot\cos\beta}{\cos\alpha\cdot\cos\beta} + \dfrac{\sin\alpha\cdot\sin\beta}{\cos\alpha\cdot\cos\beta}} = \frac{\dfrac{\sin\alpha}{\cos\alpha} - \dfrac{\sin\beta}{\cos\beta}}{1 + \dfrac{\sin\alpha}{\cos\alpha}\cdot\dfrac{\sin\beta}{\cos\beta}} =$$

$$\operatorname{tg}(\alpha-\beta) = \frac{\operatorname{tg}\alpha - \operatorname{tg}\beta}{1 + \operatorname{tg}\alpha\cdot\operatorname{tg}\beta}$$

$$\operatorname{ctg}(\alpha-\beta) = \frac{\cos(\alpha-\beta)}{\sin(\alpha-\beta)} = \frac{\cos\alpha\cdot\cos\beta + \sin\alpha\cdot\sin\beta}{\sin\alpha\cdot\cos\beta - \sin\beta\cdot\cos\alpha} =$$

$$\operatorname{ctg}(\alpha-\beta) = \frac{\operatorname{ctg}\alpha\cdot\operatorname{ctg}\beta + 1}{\operatorname{ctg}\beta - \operatorname{ctg}\alpha}$$

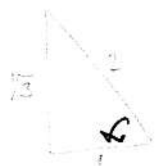
1967. III. 1.

25. óra

Ismerjük a 60°-os és a 45°-os szög szögfüggvényeit. Határozzuk meg a 105° és a 15° összes szögfüggvényeit.

$\sin(\alpha+\beta) = \sin 105°$

$\sin(\alpha-\beta) = \sin 15°$



$$\sin(\alpha+\beta) = \sin\alpha\cdot\cos\beta + \cos\alpha\cdot\sin\beta = \frac{\sqrt3}{2}\cdot\frac{\sqrt2}{2} + \frac12\cdot\frac{\sqrt2}{2} = \frac{\sqrt6}{4} + \frac{\sqrt2}{4} = \frac{\sqrt6+\sqrt2}{4}$$

$$\cos(\alpha+\beta) = \cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta = \frac12\cdot\frac{\sqrt2}{2} - \frac{\sqrt3}{2}\cdot\frac{\sqrt2}{2} = \frac{\sqrt2}{4} - \frac{\sqrt6}{4} = \frac{\sqrt2-\sqrt6}{4}$$

$$\operatorname{tg}(\alpha+\beta) = \frac{\operatorname{tg}\alpha + \operatorname{tg}\beta}{1 - \operatorname{tg}\alpha \cdot \operatorname{tg}\beta} = \frac{\sqrt{3} + 1}{1 - \sqrt{3} \cdot 1} = \frac{\sqrt{3}+1}{1-\sqrt{3}} = \frac{(1+\sqrt{3})^2}{1-3} = -\frac{(\sqrt{3}+1)^2}{2}$$

$$-2-\sqrt{3}$$

$$\operatorname{cotg}(\alpha+\beta) = \frac{\operatorname{cotg}\alpha \cdot \operatorname{cotg}\beta - 1}{\operatorname{cotg}\alpha + \operatorname{cotg}\beta} = \frac{\frac{1}{\sqrt{3}} \cdot \frac{1}{1} - 1}{\frac{1}{\sqrt{3}} + \frac{1}{1}} = \frac{\frac{1-\sqrt{3}}{\sqrt{3}}}{\frac{1+\sqrt{3}}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}\,(1-\sqrt{3})}{\sqrt{3}\,(1+\sqrt{3})} = \frac{1-\sqrt{3}}{1+\sqrt{3}} = \frac{(1-\sqrt{3})^2}{1-3} = -\frac{(1-\sqrt{3})^2}{2}$$

$$\sqrt{3}-2-$$

$$\sin(60^\circ - 45^\circ) = \sin 15^\circ$$

$$\sin(\alpha-\beta) = \sin\alpha \cdot \cos\beta - \cos\alpha \cdot \sin\beta = \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} =$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

$$\cos(\alpha-\beta) = \cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta = \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} =$$

$$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\operatorname{tg}(\alpha-\beta) = \frac{\operatorname{tg}\alpha - \operatorname{tg}\beta}{1 + \operatorname{tg}\alpha \cdot \operatorname{tg}\beta} = \frac{\sqrt{3} - 1}{1 + \sqrt{3} \cdot 1} = \frac{\sqrt{3}-1}{1+\sqrt{3}} = \frac{(\sqrt{3}-1)(1+\sqrt{3})}{3-1} = \frac{(\sqrt{3}-1)^2}{2}$$

$$2-\sqrt{3}$$

$$\operatorname{cotg}(\alpha-\beta) = \frac{\operatorname{cotg}\alpha \cdot \operatorname{cotg}\beta + 1}{\operatorname{cotg}\beta - \operatorname{cotg}\alpha} = \frac{\frac{1}{\sqrt{3}} \cdot \frac{1}{1} + 1}{\frac{1}{1} - \frac{1}{\sqrt{3}}} = \frac{\frac{1+\sqrt{3}}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}} =$$

$$= \frac{\sqrt{3}\,(1+\sqrt{3})}{\sqrt{3}\,(\sqrt{3}-1)} = \frac{1+\sqrt{3}}{\sqrt{3}-1} = \frac{(\sqrt{3}+1)^2}{2} \; ; \; 2+\sqrt{3}$$

Kétszeres szögek sz. fg. értékei

$\sin 2\alpha = \sin(\alpha+\alpha) = \sin\alpha\cdot\cos\alpha + \sin\alpha\cdot\cos\alpha = \underline{2\sin\alpha\cdot\cos\alpha}$

$\cos 2\alpha = \cos(\alpha+\alpha) = \cos\alpha\cdot\cos\alpha - \sin\alpha\cdot\sin\alpha = (2\cos) = \underline{\underline{\cos^2\alpha - \sin^2\alpha}}$

$\operatorname{tg} 2\alpha = \dfrac{\operatorname{tg}\alpha + \operatorname{tg}\alpha}{1 - \operatorname{tg}\alpha\cdot\operatorname{tg}\alpha} = \underline{\underline{\dfrac{2\operatorname{tg}\alpha}{1-\operatorname{tg}^2\alpha}}}$

$\operatorname{ctg} 2\alpha = \dfrac{\operatorname{ctg}\alpha\cdot\operatorname{ctg}\alpha - 1}{\operatorname{ctg}\alpha + \operatorname{ctg}\alpha} = \underline{\underline{\dfrac{\operatorname{ctg}^2\alpha - 1}{2\operatorname{ctg}\alpha}}}$

1967. III. 2.

26. óra.

$\sin(\alpha\pm\beta) \quad \cos(\alpha\pm\beta) \qquad \sin\alpha = \frac{3}{5} \quad \sin\beta = \frac{5}{13}$

$$\sin(\alpha+\beta) = \sin\alpha\cdot\cos\beta + \cos\alpha\cdot\sin\beta =$$
$$= \sin\alpha\cdot\sqrt{1-\sin^2\beta} + \sqrt{1-\sin^2\alpha}\cdot\sin\beta =$$
$$= \frac{3}{5}\cdot\sqrt{1-\frac{25}{169}} + \sqrt{1-\frac{9}{25}}\cdot\frac{5}{13} =$$
$$= \frac{3}{5}\cdot\sqrt{\frac{144}{169}} + \frac{5}{13}\cdot\sqrt{\frac{16}{25}} =$$
$$= \frac{3}{5}\cdot\frac{12}{13} + \frac{5}{13}\cdot\frac{4}{5} = \left(\frac{20}{13} + \frac{4}{13} = \underline{\underline{\frac{24}{13}}}\right) \frac{36}{65} + \frac{20}{65} = \underline{\underline{\frac{56}{65}}}$$

$$\sin(\alpha-\beta) = \sin\alpha\cdot\cos\beta - \cos\alpha\cdot\sin\beta =$$
$$= \sin\alpha\cdot\sqrt{1-\sin^2\beta} - \sqrt{1-\sin^2\alpha}\cdot\sin\beta =$$
$$= \frac{3}{5}\cdot\sqrt{1-\frac{25}{169}} - \frac{5}{13}\sqrt{1-\frac{9}{25}} =$$
$$= \frac{3}{5}\cdot\sqrt{\frac{144}{169}} - \frac{5}{13}\cdot\sqrt{\frac{16}{25}} =$$
$$= \frac{3}{5}\cdot\frac{12}{13} - \frac{5}{13}\cdot\frac{4}{5} = \frac{36}{65} - \frac{20}{65} = \underline{\underline{\frac{16}{65}}}$$

$$\cos(\alpha+\beta) = \cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta =$$
$$= \sqrt{1-\sin^2\alpha}\cdot\sqrt{1-\sin^2\beta} - \frac{3}{5}\cdot\frac{5}{13} =$$
$$= \sqrt{1-\frac{9}{25}}\cdot\sqrt{1-\frac{25}{169}} - \frac{15}{65} =$$
$$= \sqrt{\frac{16}{25}}\cdot\sqrt{\frac{144}{169}} - \frac{15}{65} = \frac{4}{5}\cdot\frac{12}{13} - \frac{15}{65} = \frac{48}{65} - \frac{15}{65} = \underline{\underline{\frac{33}{65}}}$$

$$\cos(\alpha-\beta) = \underline{\underline{\frac{63}{65}}}$$

$\sin 90° =$ $\sin 75° =$

$\cos 90° =$ $\cos 75° =$

$\operatorname{tg} 90° =$ $\operatorname{tg} 75° =$

$\operatorname{ctg} 90° =$ $\operatorname{ctg} 75° =$

$90° = 60° + 30°$

$\alpha = 60°$

$\sin 2\alpha =$

$\cos 2\alpha =$

$$\sin(60° + 30°) = \sin 60° \cdot \cos 30° + \cos 60° \cdot \sin 30° =$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{2} + \frac{1}{2} = \frac{2}{2} = 1$$

$$\cos(90°) = \cos(60° + 30°) = \cos 60° \cdot \cos 30° - \sin 60° \cdot \sin 30° =$$

$$= \frac{1}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{1}{2} = 0$$

$$\operatorname{tg}(60° + 30°) = \frac{\operatorname{tg} 30° + \operatorname{tg} 60°}{1 - \operatorname{tg} 60° \cdot \operatorname{tg} 30°} = \frac{\frac{1}{\sqrt{3}} + \frac{\sqrt{3}}{1}}{1 - \frac{\sqrt{3}}{1} \cdot \frac{1}{\sqrt{3}}} = \frac{\frac{1+3}{\sqrt{3}}}{1-1} =$$

$$= {}^{4}\!/_{0} = \infty$$

$$\operatorname{ctg} 90° = \frac{1}{\operatorname{tg} 90°} = 0$$

$$\sin 2\alpha = 2 \sin\alpha \cdot \cos\alpha = 2 \cdot \frac{\sqrt{3}}{2} \cdot \frac{1}{2} = \frac{\sqrt{3}}{2}$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} - \frac{3}{4} = -\frac{1}{2}$$

$$\sin 75° = \sin(45° + 30°) = \sin 45° \cdot \cos 30° + \cos 45° \cdot \sin 30° =$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

$$\cos 75° = \cos(45° + 30°) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta =$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \left(\frac{\sqrt{3}}{3}\right) \cdot \frac{1}{2} = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6} - \sqrt{2}}{4};$$

$$\operatorname{tg} 75° = \frac{\sin 75°}{\cos 75°} = \frac{\sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}} = \frac{(\sqrt{6} + \sqrt{2})^2}{4} = \frac{6 + 2 + 2\sqrt{12}}{4} = \frac{8 + 2\sqrt{12}}{4}$$

$$\operatorname{ctg} 75° = \frac{1}{\operatorname{tg} 75°} = \frac{4}{8 + 2\sqrt{12}} = \frac{\sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}} = \frac{(\sqrt{6} - \sqrt{2})^2}{6 - 2} = \frac{6 - 2 - 2\sqrt{12}}{4} = \frac{4 - 2\sqrt{12}}{4}$$

1967. III. 4.

27-28 ó

$$\sin 2\alpha = 2\sin\alpha\cdot\cos\alpha \qquad \sin\varphi = 2\sin\frac{\varphi}{2}\cdot\cos\frac{\varphi}{2}$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha \qquad \cos\varphi = \cos^2\frac{\varphi}{2} - \sin^2\frac{\varphi}{2}$$

Ha $2\alpha = \varphi$:

$$\mathrm{tg}\, 2\alpha = \frac{2\,\mathrm{tg}\,\alpha}{1-\mathrm{tg}^2\alpha} \qquad \mathrm{tg}\,\varphi = \frac{2\,\mathrm{tg}\,\frac{\varphi}{2}}{1-\mathrm{tg}^2\frac{\varphi}{2}}$$

$$\mathrm{ctg}\, 2\alpha = \frac{\mathrm{ctg}^2\alpha - 1}{2\,\mathrm{ctg}\,\alpha} \qquad \mathrm{ctg}\,\varphi = \frac{\mathrm{ctg}^2\frac{\varphi}{2} - 1}{2\,\mathrm{ctg}\,\frac{\varphi}{2}}$$

Pl: $\sin 120° = 2\cdot\sin 60°\cdot\cos 60° = \frac{\sqrt{3}}{2}$ (ismert a 60° szögf.)

Félszögek szögfüggvényei

$\sin\frac{\alpha}{2} = ?$

$\cos\frac{\alpha}{2} = ?$

$\mathrm{tg}\,\frac{\alpha}{2} = ?$

$\mathrm{ctg}\,\frac{\alpha}{2} = ?$

$$\cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2} = 1 \qquad (I.)$$

$$\cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2} = \cos\alpha \qquad (II.)$$

$\cos^2\alpha - \sin^2\alpha = \cos 2\alpha$

1.) (I.) − (II.)

$$\cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2} - \cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2} = 1 - \cos\alpha$$

$$2\sin^2\frac{\alpha}{2} = 1 - \cos\alpha$$

$$\sin\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{2}}$$

2.) (I.) + (II.)

$$\cos^2\frac{\alpha}{2} + \sin^2\frac{\alpha}{2} + \cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2} = 1 + \cos\alpha$$

$$2\cos^2\frac{\alpha}{2} = 1 + \cos\alpha$$

$$\cos\frac{\alpha}{2} = \sqrt{\frac{1+\cos\alpha}{2}}$$

$$\operatorname{tg}\frac{\alpha}{2} = \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} = \frac{\sqrt{\frac{1-\cos\alpha}{2}}}{\sqrt{\frac{1+\cos\alpha}{2}}} = \underline{\underline{\operatorname{tg}\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}}}$$

$$\operatorname{cotg}\frac{\alpha}{2} = \frac{\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}} = \frac{\sqrt{\frac{1+\cos\alpha}{2}}}{\sqrt{\frac{1-\cos\alpha}{2}}} = \underline{\underline{\operatorname{cotg}\frac{\alpha}{2} = \sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}}}$$

Pl: $\cos 15^\circ = ?$

$\cos 30^\circ = \frac{\sqrt{3}}{2}$

$$\cos\frac{\alpha}{2} = \sqrt{\frac{1+\cos\alpha}{2}} = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}};$$

$\sin\frac{\alpha}{2} = ?$

$\cos\alpha = \frac{4}{5}$

$$\sin\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{2}} = \sqrt{\frac{1}{10}};$$

$$\cos\frac{\alpha}{2} = \sqrt{\frac{1+\cos\alpha}{2}} = \frac{3}{\sqrt{10}};$$

$$\operatorname{tg}\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} = \frac{1}{3};$$

$$\operatorname{cotg}\frac{\alpha}{2} = \frac{1}{\operatorname{tg}\frac{\alpha}{2}} = 3;$$

$\sin\alpha = \sqrt{\frac{3}{7}}$

$$\sin\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{2}} = \sqrt{\frac{1-\sqrt{1-\sin^2\alpha}}{2}} =$$

$$= \sqrt{\frac{1-\frac{4}{7}}{2}} = \sqrt{\frac{3}{14}};$$

$$\cos\frac{\alpha}{2} = \sqrt{\frac{1+\cos\alpha}{2}} = \sqrt{\frac{1+\sqrt{1-\sin^2\alpha}}{2}} =$$

$$= \sqrt{\frac{\sqrt{7}+2}{2\cdot\sqrt{7}}} = \sqrt{\frac{14+4\cdot\sqrt{7}}{28}};$$

$$\frac{\sin(\alpha+\beta)-2\sin\alpha\cdot\cos\beta}{2\sin\alpha\cdot\sin\beta+\cos(\alpha+\beta)}=-\operatorname{tg}(\alpha-\beta)$$

1967. III. 7.

Szögfüggvények összege és különbsége

$\sin\alpha+\sin\beta=\ ?$

$\sin\alpha-\sin\beta=\ ?$

$\cos\alpha+\cos\beta=\ ?$

$\cos\alpha-\cos\beta=\ ?$

$$\frac{\alpha+\beta}{2}+\frac{\alpha-\beta}{2}=\alpha$$

$$\frac{\alpha+\beta}{2}-\frac{\alpha-\beta}{2}=\beta$$

$$\sin\alpha+\sin\beta=\sin\left(\frac{\alpha+\beta}{2}+\frac{\alpha-\beta}{2}\right)+\sin\left(\frac{\alpha+\beta}{2}-\frac{\alpha-\beta}{2}\right)=$$
$$=\sin\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2}+\sin\frac{\alpha-\beta}{2}\cdot\cos\frac{\alpha+\beta}{2}+$$
$$+\sin\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2}-\sin\frac{\alpha-\beta}{2}\cdot\cos\frac{\alpha+\beta}{2}=$$
$$=2\sin\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha-\sin\beta=2\cdot\sin\frac{\alpha-\beta}{2}\cdot\cos\frac{\alpha+\beta}{2}$$

$$\cos\alpha+\cos\beta=\cos\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2}-\sin\frac{\alpha+\beta}{2}\cdot\sin\frac{\alpha-\beta}{2}+$$
$$+\cos\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2}+\sin\frac{\alpha-\beta}{2}\cdot\sin\frac{\alpha+\beta}{2}=$$
$$=2\cos\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha-\cos\beta=-2\sin\frac{\alpha+\beta}{2}\cdot\sin\frac{\alpha-\beta}{2}$$

$$\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha-\sin\beta=2\sin\frac{\alpha-\beta}{2}\cdot\cos\frac{\alpha+\beta}{2}$$

$$\cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha-\cos\beta=-2\sin\frac{\alpha+\beta}{2}\cdot\sin\frac{\alpha-\beta}{2}$$

29. Hf. 1967. III. 7.

$$\frac{\sin(\alpha+\beta) - 2\sin\alpha\cdot\cos\beta}{2\sin\alpha\cdot\sin\beta + \cos(\alpha+\beta)} = -\mathrm{tg}(\alpha-\beta)$$

$$\frac{\sin\alpha\cdot\cos\beta + \sin\beta\cdot\cos\alpha - 2\sin\alpha\cdot\cos\beta}{2\sin\alpha\cdot\sin\beta + \cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta} = -\mathrm{tg}(\alpha-\beta)$$

$$\frac{\sin\beta\cdot\cos\alpha - \sin\alpha\cdot\cos\beta}{\cos\alpha\cdot\cos\beta + \sin\alpha\cdot\sin\beta} = -\mathrm{tg}(\alpha-\beta)$$

$$\frac{\sin\alpha\cdot\cos\beta - \sin\beta\cdot\cos\alpha}{\cos\alpha\cdot\cos\beta + \sin\alpha\cdot\sin\beta} = \mathrm{tg}(\alpha-\beta)$$

$$\frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)} = \mathrm{tg}(\alpha-\beta) = \frac{\mathrm{tg}\,\alpha - \mathrm{tg}\,\beta}{1+\mathrm{tg}\,\alpha\cdot\mathrm{tg}\,\beta}$$

30. óra

$$\mathrm{tg}\,\alpha + \mathrm{tg}\,\beta = \frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\beta} = \frac{\sin\alpha\cdot\cos\beta + \sin\beta\cdot\cos\alpha}{\cos\alpha\cdot\cos\beta} = \frac{\sin(\alpha+\beta)}{\cos\alpha\cdot\cos\beta}$$

$$\mathrm{tg}\,\alpha + \mathrm{tg}\,\beta = \frac{\sin(\alpha+\beta)}{\cos\alpha\cdot\cos\beta}$$

$$\mathrm{tg}\,\alpha - \mathrm{tg}\,\beta = \frac{\sin\alpha}{\cos\alpha} - \frac{\sin\beta}{\cos\beta} = \frac{\sin\alpha\cdot\cos\beta - \sin\beta\cdot\cos\alpha}{\cos\alpha\cdot\cos\beta} = \frac{\sin(\alpha-\beta)}{\cos\alpha\cdot\cos\beta}$$

$$\mathrm{tg}\,\alpha - \mathrm{tg}\,\beta = \frac{\sin(\alpha-\beta)}{\cos\alpha\cdot\cos\beta}$$

$$\mathrm{ctg}\,\alpha + \mathrm{ctg}\,\beta = \frac{\cos\alpha}{\sin\alpha} + \frac{\cos\beta}{\sin\beta} = \frac{\cos\alpha\cdot\sin\beta + \cos\beta\cdot\sin\alpha}{\sin\alpha\cdot\sin\beta} = \frac{\sin(\alpha+\beta)}{\sin\alpha\cdot\sin\beta}$$

$$\mathrm{ctg}\,\alpha + \mathrm{ctg}\,\beta = \frac{\sin(\alpha+\beta)}{\sin\alpha\cdot\sin\beta}$$

$$\mathrm{ctg}\,\alpha - \mathrm{ctg}\,\beta = \frac{\cos\alpha}{\sin\alpha} - \frac{\mathrm{ctg}\,\beta}{\sin\beta} = \frac{\cos\alpha\cdot\sin\beta - \cos\beta\cdot\sin\alpha}{\sin\alpha\cdot\sin\beta} = \frac{\sin(\beta-\alpha)}{\sin\alpha\cdot\sin\beta}$$

$$\mathrm{ctg}\,\alpha - \mathrm{ctg}\,\beta = \frac{\sin(\beta-\alpha)}{\sin\alpha\cdot\sin\beta}$$

$\sin 45° + \sin 30° = 2 \cdot \sin \frac{75}{2} \cdot \cos \frac{15}{2} = 2 \cdot \sin 37{,}5° \cdot \cos 7{,}5°$

$\sin 45° + \sin 15° = 2 \cdot \sin \frac{30°}{4} \cdot \cos 15° = 2 \cdot \frac{1}{2} \cdot \cos 15° = \cos 15°$

$\frac{\sin 75° - \sin 15°}{\cos 75° + \cos 15°} = \frac{2 \sin \frac{60}{2} \cdot \cos \frac{90}{2}}{2 \cdot \cos \frac{90}{2} \cdot \cos \frac{60}{2}} = \frac{2 \cdot \sin 30° \cdot \cos 45°}{2 \cdot \cos 45° \cdot \cos 30°} =$

$= \left(2 \cdot \frac{1}{2} \cdot \frac{\sqrt{2}}{2}\right) : \left(2 \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}\right) = \frac{\sin 30°}{\cos 30°} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{2}{2\sqrt{3}} = \frac{2 \cdot \sqrt{3}}{6}$

$= \frac{\sqrt{3}}{3} = tg\, 30°$

1967.III.8.

30.

251/28

$* \sin 70° - \sin 20° = 2 \cdot \sin \frac{\alpha + \beta}{2} \cdot \cos \frac{\alpha + \beta}{2} = 2 \cdot \sin 25° \cdot \cos 45° = \sqrt{2} \cdot \sin 25°$

$* \cos 75° + \cos 15° = 2 \cdot \cos \frac{\alpha + \beta}{2} \cdot \cos \frac{\alpha - \beta}{2} = 2 \cdot \cos 45° \cdot \cos 30° = \sqrt{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{6}}{2}$

$* \cos \frac{\pi}{18} - \cos \frac{\pi}{12} = -2 \sin \frac{10° + 15°}{2} \cdot \sin \frac{10° - 15°}{2} = -2 \cdot \sin 12{,}5° \cdot \sin -2{,}5°$

$\sin 25° + \cos 55° = \cancel{\left(\frac{tg\, 25°}{cotg\, 25°} + co\right)} = \cos 25° \cdot tg\, 25° + cotg\, 55° \cdot \sin 55°$

$\sin 25° + \cos 55° = \sin 25° + \sqrt{1 - \sin^2 55°} = 2 \cdot \sin \frac{25° + \sqrt{1-\sin^2 25}}{2} \cdot \cos \frac{25 + \sqrt{1-\sin^2 25}}{2}$

$* \sin(30-5) + \cos(60-5) = \sin 30 \cdot \cos 5 - \cos 30 \cdot \sin 5 + \cos 60 \cdot \cos 5 + \sin 60 \cdot \sin 5$

$\frac{1}{2} \cdot \cos 5° - \frac{\sqrt{3}}{2} \cdot \sin 5° + \frac{1}{2} \cdot \cos 5° + \frac{\sqrt{3}}{2} \cdot \sin 5° = \cos 5°$

$* \sin 3x - \sin x = 2 \cdot \sin x \cdot \cos 2x$

$* 1 + \sin \omega = \sin 90° + \sin \omega = 2 \cdot \sin \frac{90 + \omega}{2} \cdot \cos \frac{90 - \omega}{2}$

$\sin \frac{\pi}{5} - \cos \frac{3\pi}{5} = \sin 36° - \cos 108° = \sin 36° - \cos 3 \cdot 36° =$

$* \sin 200 + \sin 300 = 2 \sin 250 \cdot \cos -50$

 1967. III. 9.

Alakítsuk szorzattá:

$$\sin 50^\circ + \sin 40^\circ = 2\cdot\sin 45^\circ\cdot\cos 10^\circ/_2 = \sqrt{2}\cdot\cos 10^\circ/_2 = \underline{\sqrt{2}\cdot\cos 5^\circ}$$

$$\sin 50^\circ - \sin 40^\circ = 2\sin\frac{10^\circ}{2}\cdot\cos 45^\circ = \underline{\sqrt{2}\cdot\sin 5^\circ}$$

$$\sin 50^\circ - \sin 70^\circ + \sin 10^\circ = 0$$

$$\sin 50^\circ - \sin 70^\circ = -\sin 10^\circ$$

$$2\cdot(\sin -10^\circ)\cdot\cos 60^\circ = -\sin 10^\circ$$

$$2\cdot\frac{1}{2}\cdot -\sin 10^\circ = -\sin 10^\circ$$

$$\underline{\sin 10^\circ = \sin 10^\circ}$$

$$\frac{\sin 5^\circ - \sin 15^\circ + \sin 25^\circ}{\cos 5^\circ - \cos 15^\circ + \cos 25^\circ} = \frac{(2\cdot\sin -5^\circ\cdot\cos 10^\circ) + \sin 25^\circ}{(-2\cdot\sin 10^\circ\cdot\sin -5^\circ) + \cos 25^\circ} =$$

$$= \frac{-\sin 5^\circ\cdot\cos 10^\circ - \sin 5^\circ\cdot\cos 10^\circ + \sin 25^\circ}{\sin 10^\circ\cdot\sin 5^\circ + \sin 10^\circ\cdot\sin 5^\circ + \cos 25^\circ} =$$

$$= \frac{-\sin 5^\circ\cdot\cos 10^\circ - \sin 5^\circ\cdot\cos 10^\circ + \sin(30-5)^\circ}{\sin 10^\circ\cdot\sin 5^\circ + \sin 10^\circ\cdot\sin 5^\circ + \cos(30-5)^\circ} =$$

$$= \frac{-\sin 5^\circ\cdot\cos 10^\circ - \sin 5^\circ\cdot\cos 10^\circ + \frac{1}{2}\cdot\cos 5^\circ - \frac{\sqrt{3}}{2}\cdot\sin 5^\circ}{\sin 10^\circ\cdot\sin 10^\circ + \sin 10^\circ\cdot\sin 5^\circ + \frac{\sqrt{3}}{2}\cdot\cos 5^\circ + \frac{1}{2}\cdot\sin 5^\circ} =$$

$$(2\cdot\sin 15^\circ)$$

$$\frac{\sin 5^\circ - \sin 15^\circ + \sin 25^\circ}{\cos 5^\circ - \cos 15^\circ + \cos 25^\circ} = \frac{2\cdot\sin\frac{5+25}{2}\cdot\cos\frac{5-25}{2} - \sin 15^\circ}{2\cdot\cos\frac{5+25}{2}\cdot\cos\frac{5-25}{2} - \cos 15^\circ} =$$

$$= \frac{2\cdot\sin 15^\circ\cdot\cos -10^\circ - \sin 15^\circ}{2\cdot\cos 15^\circ\cdot\cos -10^\circ - \cos 15^\circ} = \frac{\sin 15^\circ(2\cdot\cos 10^\circ - 1)}{\cos 15^\circ(2\cdot\cos 10^\circ - 1)} =$$

$$= \operatorname{tg} 15^\circ = \sqrt{\frac{1-\cos 30^\circ}{1+\cos 30^\circ}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{1+\frac{\sqrt{3}}{2}}} = \sqrt{\frac{\frac{2-\sqrt{3}}{2}}{\frac{2+\sqrt{3}}{2}}} =$$

$$= \sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}} = \sqrt{\frac{(2-\sqrt{3})^2}{7}}$$

1967. VII. 11. úa.

$$\frac{\sin\alpha\cdot\cos\beta-\sin(\alpha+\beta)}{\cos\alpha\cdot\cos\beta-\cos(\alpha+\beta)}=-\operatorname{cotg}\alpha$$

$$\frac{\sin\alpha\cdot\cos\beta-\sin\alpha\cdot\cos\beta-\cos\alpha\cdot\sin\beta}{\cos\alpha\cdot\cos\beta-\cos\alpha\cdot\cos\beta+\sin\alpha\cdot\sin\beta}=-\operatorname{cotg}\alpha$$

$$\frac{-\cos\alpha\cdot\sin\beta}{\sin\alpha\cdot\sin\beta}=-\operatorname{tg}\alpha$$

$$-\frac{\cos\alpha}{\sin\alpha}=-\operatorname{cotg}\alpha$$

$$\underline{\operatorname{cotg}\alpha=\operatorname{cotg}\alpha}$$

$$\frac{1+\sin2\alpha}{\cos2\alpha}=\operatorname{tg}(45^\circ+\alpha)$$

$$\frac{1+2\sin\alpha\cdot\cos\alpha}{\cos^2\alpha-\sin^2\alpha}=\frac{\operatorname{tg}45^\circ+\operatorname{tg}\alpha}{1-\operatorname{tg}45^\circ\cdot\operatorname{tg}\alpha}$$

$$\frac{1+2\sin\alpha\cdot\cos\alpha}{\cos^2\alpha-\sin^2\alpha}=\frac{1+\operatorname{tg}\alpha}{1-\operatorname{tg}\alpha}=\frac{\frac{1+2\sin\alpha\cdot\cos\alpha}{\cos^2\alpha}}{\frac{\cos^2\alpha-\sin^2\alpha}{\cos^2\alpha}}=\frac{\frac{1+2\sin\alpha\cdot\cos\alpha}{\cos^2\alpha}}{1-\operatorname{tg}\alpha}$$

$$\frac{1+\operatorname{tg}\alpha}{1-\operatorname{tg}\alpha}=\frac{1+2\sin\alpha\cdot\cos\alpha}{\cos^2\alpha(1-\operatorname{tg}\alpha)}$$

$$\frac{1+\sin2\alpha}{\cos2\alpha}=\operatorname{tg}(45^\circ+\alpha)$$

$$\frac{\sin^2\alpha+\cos^2\alpha+2\cdot\sin\alpha\cdot\cos\alpha}{(\cos\alpha+\sin\alpha)(\cos\alpha-\sin\alpha)}=\frac{1+\operatorname{tg}\alpha}{1-\operatorname{tg}\alpha}$$

$$\frac{(\sin\alpha+\cos\alpha)(\sin\alpha+\cos\alpha)}{(\cos\alpha+\sin\alpha)(\cos\alpha-\sin\alpha)}=\frac{1+\operatorname{tg}\alpha}{1-\operatorname{tg}\alpha}$$

$$\frac{\sin\alpha+\cos\alpha}{\cos\alpha-\sin\alpha}=\frac{1+\frac{\sin\alpha}{\cos\alpha}}{1-\frac{\sin\alpha}{\cos\alpha}}$$

$$\frac{\sin\alpha + \cos\alpha}{\cos\alpha - \sin\alpha} = \frac{\dfrac{\cos\alpha + \sin\alpha}{\cos\alpha}}{\dfrac{\cos\alpha - \sin\alpha}{\cos\alpha}}$$

$$\underbrace{\frac{\sin\alpha + \cos\alpha}{\cos\alpha - \sin\alpha} = \frac{\cos\alpha + \sin\alpha}{\cos\alpha - \sin\alpha}}$$

$$\sin(60^\circ+\alpha) + \sin(60^\circ-\alpha) = 2\sin\frac{(60^\circ+\alpha)+(60^\circ-\alpha)}{2}\cdot\cos\frac{(60^\circ+\alpha)-(60^\circ-\alpha)}{2} =$$

$$= 2\sin\frac{120^\circ}{2}\cdot\cos\frac{2\alpha}{2} = 2.\sin 60^\circ.\cos\alpha = 2.\frac{\sqrt{3}}{2}\cdot\cos\alpha.$$

$$= \sqrt{3}.\cos\alpha$$

$$\frac{\cos 2\alpha}{1+\cos 2\alpha} = \frac{tg\alpha}{tg 2\alpha}$$

$$\frac{\cos^2\alpha - \sin^2\alpha}{\sin^2\alpha + \cos^2\alpha + \cos^2\alpha - \sin^2\alpha} = \frac{tg\alpha}{\dfrac{2tg\alpha}{1-tg^2\alpha}}$$

$$\frac{\cos^2\alpha - \sin^2\alpha}{2\cos^2\alpha} = \frac{tg\alpha\,(\sin^2\alpha + \cos^2\alpha - tg^2\alpha)}{2tg\alpha}$$

$$\frac{\cos^2\alpha - \sin^2\alpha}{2\cos^2\alpha} = \frac{\sin^2\alpha + \cos^2\alpha - tg^2\alpha}{2}$$

$$\cos^2\alpha - \sin^2\alpha = \cos^2\alpha\,(\underbrace{\cos^2\alpha + \sin^2\alpha} - tg^2\alpha)$$

$$\cos^2\alpha - \sin^2\alpha = \cos^2\alpha - tg^2\alpha.\cos^2\alpha$$

$$\underbrace{\cos^2\alpha - \sin^2\alpha = \cos^2\alpha - \sin^2\alpha}$$

$\sin(270°+\alpha) = -\cos\alpha$
$\cos(270°+\alpha) = \sin\alpha$

$\sin 270 \cdot \cos\alpha + \cos 270° \cdot \sin\alpha = -\cos\alpha$
$-\cos\alpha + 0 = -\cos\alpha$
$\cos\alpha = \cos\alpha$

$\cos 270° \cdot \cos\alpha - \sin 270° \cdot \sin\alpha = \sin\alpha$
$0 \cdot \cos\alpha - (-1) \cdot \sin\alpha = \sin\alpha$
$\sin\alpha = \sin\alpha$

$tg(270°+\alpha) = -ctg\,\alpha$

$$\frac{\sin(270°+\alpha)}{\cos(270°+\alpha)} = -ctg\,\alpha$$

$$\frac{\sin 270° \cdot \cos\alpha + \cos 270 \cdot \sin\alpha}{\cos 270 \cdot \cos\alpha - \sin 270 \cdot \sin\alpha} = -ctg\,\alpha$$

$$\frac{-\cos\alpha}{\sin\alpha} = -ctg\,\alpha$$

$-ctg\,\alpha = -ctg\,\alpha$

$ctg(270°+\alpha) = -tg\,\alpha$

$$\frac{\cos(270°+\alpha)}{\sin(270°+\alpha)} = -tg\,\alpha$$

$$\frac{\cos 270 \cdot \cos\alpha - \sin 270 \cdot \sin\alpha}{\sin 270 \cdot \cos\alpha + \cos 270 \cdot \sin\alpha} = -tg\,\alpha$$

$$\frac{0 + \sin\alpha}{-\cos\alpha + 0} = -tg\,\alpha$$

$-tg\,\alpha = -tg\,\alpha$

1967. III. 11.

$\sin(180°+\alpha) = -\sin\alpha$

$\sin 180 \cdot \cos\alpha + \sin\alpha \cdot \cos 180 = -\sin\alpha$

$0 \quad -\sin\alpha = -\sin\alpha$

$\cos(180°+\alpha) = -\cos\alpha$

$\cos 180° \cdot \cos\alpha - \sin 180° \cdot \sin\alpha = -\cos\alpha$

$-\cos\alpha - 0 = -\cos\alpha$

$\mathrm{tg}(180+\alpha) = \mathrm{tg}\,\alpha$

$\sin(180+\alpha) : \cos(180°+\alpha) = \sin\alpha : \cos\alpha$

$-\sin\alpha : -\cos\alpha = \sin\alpha : \cos\alpha$

$\mathrm{tg}\,\alpha = \mathrm{tg}\,\alpha$

$\sin(60°+\alpha) \pm \sin(60°-\alpha)$

$\sin 60° \cdot \cos\alpha + \cos 60° \cdot \sin\alpha \pm (\sin 60° \cdot \cos(-\alpha) + \cos 60° \cdot \sin(-\alpha)) =$

$= \frac{\sqrt{3}}{2} \cdot \cos\alpha + \frac{1}{2} \cdot \sin\alpha \pm \left(\frac{\sqrt{3}}{2} \cdot \cos\alpha - \frac{1}{2} \cdot \sin\alpha\right)$

$\sin(60°+\alpha) + \sin(60°-\alpha) = \underline{\sqrt{3} \cdot \cos\alpha}$

$\sin(60°+\alpha) - \sin(60°-\alpha) = \underline{\sin\alpha}$

$\sin 2\alpha \pm \sin\alpha = 2\sin\alpha \cdot \cos\alpha \pm \sin\alpha$

$= \sin 2\alpha \pm \sin\alpha = 2 \cdot \sin\frac{3\alpha}{2} \cdot \cos\frac{\alpha}{2} \quad (+)$

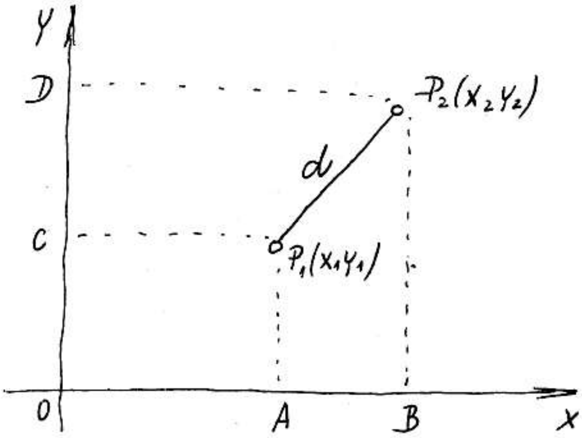
$= 2 \cdot \sin\frac{\alpha}{2} \cdot \cos\frac{3\alpha}{2} \quad (-)$

8. óra

Analitikus geometria

A szakasz hossza.

Az analitikus geometria mértani feladatot algebrailag old meg.



koordináták $\begin{cases} \text{abszcissza } x \\ \text{ordináta } y \end{cases}$

$P_1P_2 = d$

$AB = P_1Q$

$P_1Q = CQ - CP_1$

$P_1Q = x_2 - x_1$

$P_2Q = P_2B - BQ$.

$P_2Q = y_2 - y_1$

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

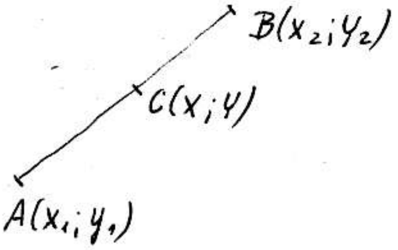
— Példák —

1967. 9. 21.

9. óra.

$$x = \frac{x_1 + x_2}{2}$$

$$y = \frac{y_1 + y_2}{2}$$

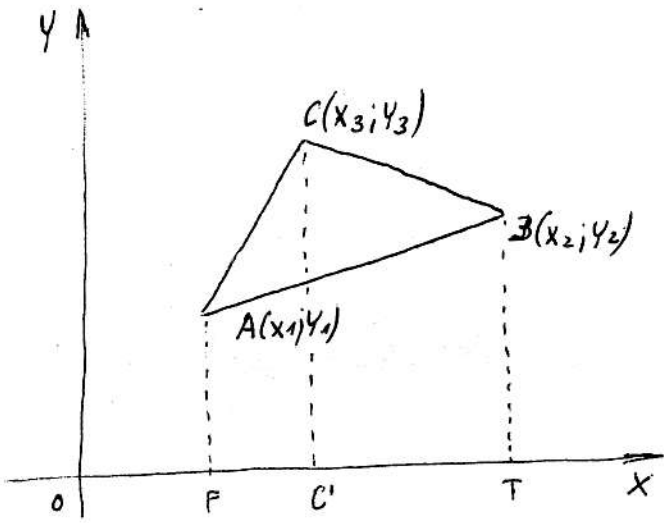


– Példák –

1967. 9. 23.

10. óra.

A háromszög területe



FC'CA trapéz területe $\frac{y_3 + y_1}{2}(x_3 - x_1)$

C'TBC –"– $\frac{y_2 + y_3}{2}(x_2 - x_3)$

$$\frac{x_3y_3 + x_3y_1 - x_1y_3 - x_1y_1 + x_2y_2 + x_2y_3 - y_2x_3 - x_3y_3}{2}$$

FTBA –"– $\frac{y_1 + y_2}{2}(x_2 - x_1)$

$$\frac{x_2y_1 + x_2y_2 - x_1y_1 - x_1y_2}{2}$$

$$T_{\triangle} = \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$

Pl. $A(3; 2)$ $T = 13$

$B(-3; 3)$

$C(1; -2)$

11. óra 1967. IX. 26.

$A(3;4)$
$B(7;8)$
$C(9;5)$

$$T_\Delta = \frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)]$$

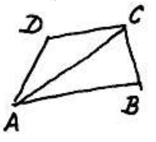
$$T_\Delta = \frac{1}{2}[3(8-5) + 7(5-4) + 9(4-8)]$$

$$T_\Delta = \frac{1}{2}[9+7-36]$$

$$|T_\Delta| = \left|\frac{1}{2}\cdot(-20)\right|$$

$$\underline{\underline{T = 10}}$$

$A(4;3)$
$B(-1;5)$
$C(-2;-1)$
$D(2;-2)$



$$T = T_{ABC} + T_{ACD}$$

$$T = \frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)] +$$
$$+\frac{1}{2}[x_1(y_3-y_4) + x_3(y_4-y_1) + x_4(y_1-y_3)].$$

$$T = \frac{1}{2}[4(5+1) + -1(-1-3) + -2(3-5)] +$$
$$+\frac{1}{2}[4(-1+2) + -2(-2-3) + 2(3+1)] = \frac{1}{2}[24+4+4] +$$
$$+\frac{1}{2}[4+10+8] = \frac{1}{2}(32+22) = \frac{1}{2}54 = \underline{\underline{27}}$$

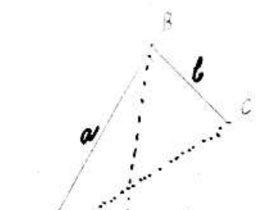
$A(0;0)$
$B(7;0)$
$C(5;4)$
$D(2;4)$

$$T = \frac{1}{2}[0.(0-4) + 7(4-0) + 5(0-0) + 0(4-4) + 5(4-0) + 2(0-4)]$$

$$T = \frac{1}{2}(28+20-8) = \frac{1}{2}40 = 20$$

$A(2;3)$
$B(4;7)$
$C(6;5)$

Keressük meg D-t úgy, hogy az ABCD idom parallelogramma legyen.
[4;1]



$\cancel{a = \sqrt{}}$

$$y_2 - y_3 = 7-5 = 2$$
$$x_3 - x_2 = 6-4 = 2$$

$$\left.\begin{array}{l} y_4 = y_1 - 2 = 3-2 = 1 \\ x_4 = x_1 + 2 = 2+2 = 4 \end{array}\right\} A(4;1)$$

1967. IX. 27. 12. ó.

1.) Számítsuk ki a köv. szakaszok felezőpontjának koordinátáit. NN MO MP NP OP NP NO

ha M(7)
N(4)
O(0)
P(-2)

• $MN = \frac{7+4}{2} = \frac{11}{2} = 5,5$ • $NM = 5,5$ • $OM = 3,5$ • $PM = 2,5$

• $MO = \frac{0+7}{2} = 3,5$ • $NO = \frac{0+4}{2} = 2$ • $ON = 2$ • $PN = 1$

• $MP = \frac{7-2}{2} = \frac{5}{2} = 2,5$ • $NP = \frac{4-2}{2} = 1$ • $OP = \frac{0-2}{2} = -1$ • $PO = -1$

2.) A(3) $AB \dots \frac{3+5}{2} = 4$
B(5) $AC \dots \frac{3-3}{2} = 0$
C(-3) $BC \dots \frac{5-3}{2} = 1$

3.) A(4;1) B(-2;3) S(?;?)

$$\left.\begin{aligned} x_s &= \frac{x_a + x_b}{2} = \frac{4-2}{2} = 1 \\ y_s &= \frac{y_a + y_b}{2} = \frac{1+3}{2} = 2 \end{aligned}\right\} \rightarrow S(1;2)$$

C(6;5)

M szimm. x szerint C-vel

M = (6;-5)

4.) △ A(10;8) K = ? Hat. meg az oldalak hosszúsága szerint, hogy milyen tulajdonságú.
B(7;4)
C(6;5)

A(2;11)
B(-1;6)
C(-2;10)

Szám. ki az ABC△ súlyvonalainak hosszát.

A(15;7)
B(-2;3)
C(10;13)

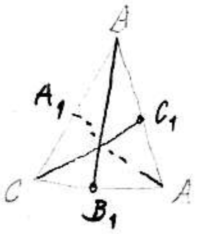
A(-9;-10)
B(8;-6)
C(-4;-16)

1, $AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{9+16} = 5$
$BC = \sqrt{1+1} = \sqrt{2}$
$AC = \sqrt{16+9} = 5$
→ egyenlőszárú

$K \doteq \underline{\underline{11,4}}$

$T = \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)] = \frac{1}{2}[10\cdot-1 + 7\cdot-3 + 6\cdot4]$

$|T| = |\frac{1}{2}[-10-21+24]| = |\frac{1}{2}\cdot(-7)| = \underline{\underline{3,5}}$



$C_1 = (8,5;6)$
$B_1 = (8;6,5)$
$A_1 = (6,5;4,5)$

$\cancel{s_a} = s_c = \sqrt{6,25+1} = \sqrt{7,25} \doteq \underline{\underline{2,7}}$
$s_b = \sqrt{1+6,25} \doteq \underline{\underline{2,7}}$
$s_a = \sqrt{12,25+12,25} \doteq \underline{\underline{<5}}$

2, $AB = \sqrt{9+25} = \sqrt{34} \doteq 6$
$AC = \sqrt{16+1} \doteq \sqrt{17} \doteq 4,1$
$BC = \sqrt{1+16} \doteq \sqrt{17} \doteq 4,1$
egyenlőszárú

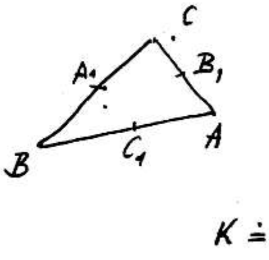
$K \doteq 14$

$|T| = \frac{1}{2}[2\cdot(-4)+(-1)\cdot4+(-2)1] = \frac{1}{2}\cdot(-8-4-2) = \underline{\underline{7.}}$

~~$C_1 = (6,5;8,5)$~~
$A_1 = (-1,5;8)$
$C_1 = (0,5;8,5)$

$s_a = \sqrt{12,25+9} \doteq \underline{\underline{4,6}} = s_b$
~~$s_c = \sqrt{72+2,25} \doteq$~~
$s_c = \sqrt{6,25+12,5} \doteq \underline{\underline{3}}$

3, $AB = \sqrt{289+16} \doteq \sqrt{305} \doteq 17,3$
$BC = \sqrt{144+100} = \sqrt{244} \doteq 15,6$
$AC = \sqrt{25+36} \doteq 7,8$
derékszögű △



$A_1 = (4;8)$
$B_1 = (6,5;5)$
$C_1 = (12,5;10)$

$s_a = \sqrt{121+1} \doteq \underline{11}$
$s_b = \sqrt{72+4} \doteq \underline{8,7}$
$s_c = \sqrt{6,25+9} \doteq \underline{\underline{3,9}}$

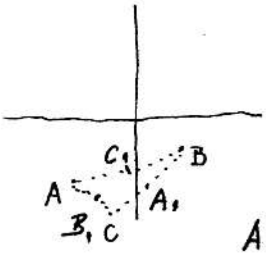
$K \doteq \underline{40,7}$

$T = \frac{1}{2}[15(4) - 2\cdot6 + 10\cdot4] = \frac{1}{2}\, 88 = \underline{44}$

$A(-9;-10)$ $\quad AB = \sqrt{290+16} \doteq 17{,}3$

$B(8;-6)$ $\quad BC = \sqrt{144+100} \doteq 15{,}6$ $\Big\}$ desiekos. Δ

$C(-4;-16)$ $\quad AC = \sqrt{25+36} \doteq 7{,}8$



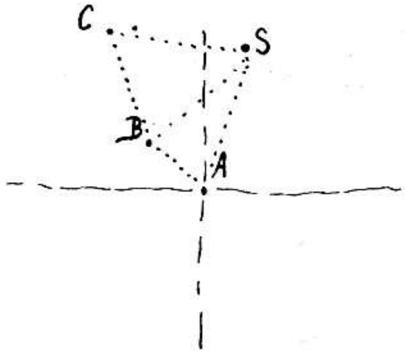
$K \doteq \underline{40{,}7}$

$T = \frac{1}{2}\left[-9.10 + 8.-6 - 4.-4\right] = \frac{1}{2}\left[-90 - 48 + 16\right] = \underline{61}$

$A_1 = (2;-11)$ $\quad S_a = \sqrt{121+1} \doteq \underline{11}$

$B_1 = (-6{,}5;-13)$ $\quad S_b = \left(\sqrt{2{,}25 + 38}\right) = \sqrt{210+49} \doteq \underline{16{,}1}$

$C_1 = (-0{,}5;-8)$ $\quad S_c = \sqrt{12{,}25+64} \doteq \underline{8{,}7}$



$$(x-m)^2 + (y-n)^2 = r^2$$

$$(x_1-m)^2 + (y_1-n)^2 = (x_2-m)^2 + (y_2-n)^2$$

$$(x_1-m)^2 + (y_1-n)^2 = (x_3-m)^2 + (y_3-n)^2$$

$$(0-m)^2 + (0-n)^2 = (-5-m)^2 + (5-n)^2$$

$$m^2 + n^2 = 25 + 10m + m^2 + 25 - 10n + n^2$$

$$10n - 10m = 50$$

$$(0-m)^2 + (0-n)^2 = (-9-m)^2 + (13-n)^2$$

$$m^2 + n^2 = 81 + 18m + m^2 + 169 - 13n + n^2$$

$$13n - 18m = 250$$

$n - m = 5 \rightarrow n = m + 5$ $\qquad n = -37 + 5 = -32$

$13n - 18m = 250$

$13(m+5) - 18m = 250$

$13m - 18m = 250 - 65$ $\qquad S|(-37;-32)|$

$5m = -185$

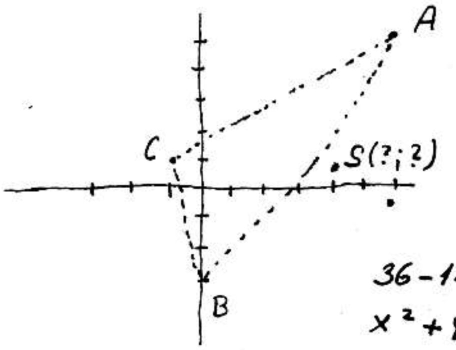
$m = -37$ $\qquad \underline{S(37;32)}$

Hat. meg az $S(x;y)$ pont koordinátáit, mely az ABC pontoktól egyenlő távolságra van.

$A(0;0)$ $\quad$ $A(6;5)$

$B(-5;5)$ $\quad$ $B(0;-3)$

$C(-9;13)$ $\quad$ $C(-2;1)$



$$AS = \sqrt{(6-x)^2+(5-y)^2} \qquad (6-x)^2+(5-y)^2$$

$$BS = \sqrt{(x+\cancel{0})^2+(y+3)^2} \qquad (x-0)^2+(y+3)^2$$

$$CS = \sqrt{(-2-x)^2+(1-y)^2} \qquad (-2-x)^2+(1-y)^2$$

$$36-12x+x^2+25-10y+y^2$$

$$x^2+y^2+6y+9$$

$$4+4x+x^2+1-2y+y^2$$

$$36-12x+x^2+25-10y+y^2 = x^2+y^2+6y+9$$

$$-12x - 16y = \cancel{70} \; -52$$

$$12x+16y = 52$$

$$3x+4y = 13$$

$$x^2+y^2+6y+9 = 4+4x+x^2+1-2y+y^2$$

$$8y - 4x = -4$$

$$2y - x = -1 \quad \rightarrow \quad x = 2y+1$$

$$3(2y+1) + 4y = 13$$

$$12y+3+4y = 13$$

$$16y = 16$$

$$\underline{y=1}$$

$$\underline{\underline{S(3;1)}}$$

$$x = 2+1$$

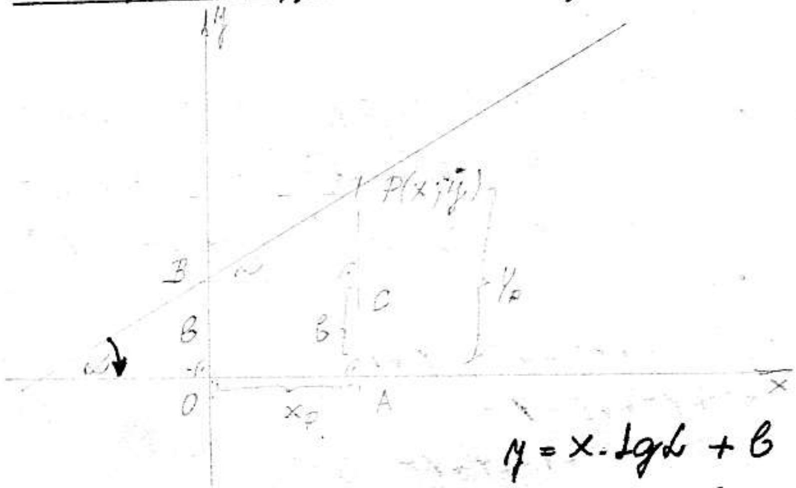
$$\underline{x=3}$$

1967. IX. 28.

13. óra.

Az egyenes egyenlete

I. Az egyenes egyenletének iránytényezős ~~alakja~~ egyenlete.



α - irányszög

$OB = b$

$P(x;y)$ tetszőleges pont.

$tg\alpha$ - iránytényező

$BCP\triangle \dots\; tg\alpha = \frac{y-b}{x}$

$x \cdot tg\alpha = y - b$

$x \cdot tg\alpha + b = y$

$$y = x \cdot tg\alpha + b$$

$tg\alpha = m \;\dots\dots\;$ $\underline{\underline{y = mx + b}}$ - iránytényezős alak

Ha az egyenes a kezdőponton megy keresztül, egyenlete: $y = mx$

Ha $\alpha = 45^\circ$ és az egyenes átmegy az O ponton, : $y = x$

II. Egy adott ponton átmenő egyenes egyenlete

$y = m\underbrace{x + b}$ $\quad P_1 = (x_1; y_1)$

$y_1 = m x_1 + b$

$y - y_1 = mx + b - mx_1 + (-b)$

$y - y_1 = mx - mx_1$

$$\underline{\underline{y - y_1 = m(x - x_1)}}$$

III. Két adott ponton átmenő egyenes egyenlete

$y = mx + b$ $\quad P_1 = (x_1; y_2)$

$P_2 = (x_2; y_2)$

$y_2 - y_1 = mx_2 - mx_1$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

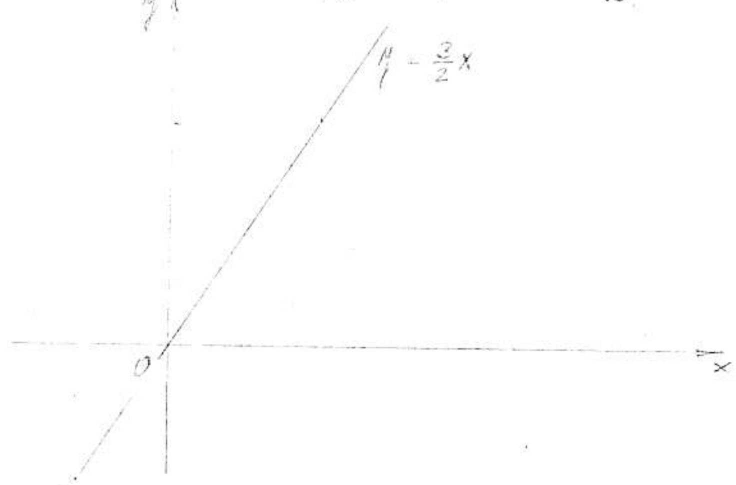
$$\underline{\underline{y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)}}$$

Írjuk föl annak az egyenesnek az egyenletét, mely az x tengellyel 60°-os szöget zár be és keresztül halad a kezdőponton:

$$y = mx + b \quad \left| \begin{array}{l} b = 0 \\ \alpha = 60° \rightarrow m = \sqrt{3} \end{array} \right.$$

$$y = \sqrt{3} \cdot x$$

Szerk. meg azt az egyenest, mely a kezdőponton megy keresztül és iránytangense $\frac{3}{2}$.



Y-t 5-ben metszi. X-szel 30°.

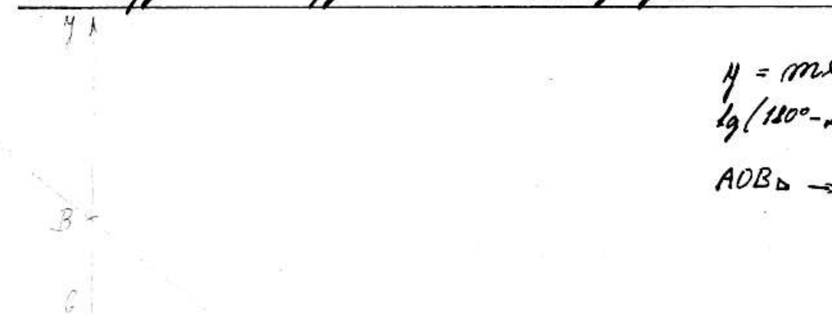
$b = 5$

$\alpha = 30° \quad tg\,\alpha = \frac{\sqrt{3}}{3} \qquad y = \frac{\sqrt{3}}{3}x + 5$

14. ó.

19. IX. 1967.

1.) Az egyenes egyenletének tengelymetszetes alakja.



$y = mx + b$

$tg(180° - \alpha) = -tg\,\alpha$

$AOB_\triangle \rightarrow tg(180° - \alpha) = \frac{b}{a}$

$-tg\,\alpha = \frac{b}{a}$

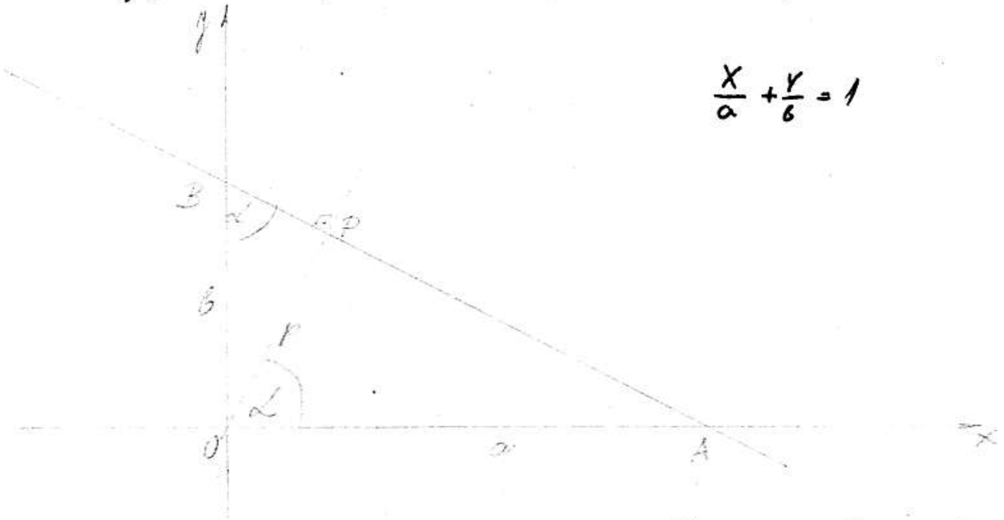
$tg\,\alpha = -\frac{b}{a} = m$

$y = -\frac{b}{a}x + b$

$y \cdot b^{-1} = -x a^{-1} + 1$

$$\frac{x}{a} + \frac{y}{b} = 1$$

5.) Az egyenes egyenletének Hesse-f. alakja



$$\frac{x}{a} + \frac{y}{b} = 1$$

$OPA_\triangle$ .... $\cos\alpha = \frac{p}{a} \rightarrow a = \frac{p}{\cos\alpha}$

$OPB_\triangle$ .... $\sin\alpha = \frac{p}{b} \rightarrow b = \frac{p}{\sin\alpha}$

$$\frac{x}{\frac{p}{\cos\alpha}} + \frac{y}{\frac{p}{\sin\alpha}} = 1$$

$$\frac{x.\cos\alpha}{p} + \frac{y.\sin\alpha}{p} = 1$$

$$x.\cos\alpha + y.\sin\alpha = p$$

$$\underline{\underline{x.\cos\alpha + y.\sin\alpha - p = 0}}$$

6.) az egyenes egyenletének általános alakja

$(ax + by + c = 0)$

$$\underline{\underline{Ax + By + C = 0}}$$

ha $B = 0 \rightarrow x = -\frac{c}{a}$ egyenes ∥ az y-nal

ha $A = 0 \rightarrow y = -\frac{c}{b}$ " ∥ x-szel

$y = mx + b$

$Ax + By + c = 0 \qquad B \neq 0$

$AB^{-1}x + y + CB^{-1} = 0$

$$y = -\frac{A}{B}x - \frac{C}{B} \qquad \left\{ \begin{array}{l} m = -\frac{A}{B} \\ b = -\frac{C}{B} \end{array} \right.$$

$$y = mx + b$$

1.) Hat. meg annak az egyenesnek az egyenletét amely az x tengely az O-tól 4, az y-t pedig -3 egységnyi távolságban metszi.

$$\frac{x}{4} - \frac{y}{3} = 1 \qquad 3x - 4y = 12$$

$$-4y = 12 - 3x$$

$$y = \frac{3x - 12}{4}$$

$$y = \frac{3}{4}x - 3$$

2.) Szám. ki az $\frac{x}{3} + \frac{y}{4} = 1$ egyenesnek a kezdőponttól mért távolságát! $p = x\cos\alpha + y \cdot \sin\alpha$

$$p = a \cdot \cos\alpha \rightarrow p = 3 \cdot \cos\alpha \qquad \sin\alpha = \frac{3}{5} \quad \cos\alpha = \frac{4}{5}$$

$$p = b \cdot \sin\alpha \rightarrow p = 4 \cdot \sin\alpha$$

$$p = 4 \cdot \frac{3}{5} = \frac{12}{5} = 2{,}4$$

3.) Írjuk fel a $3x - 5y = 10$ egyenes iránytényezős és tengelymetszetes egyenletét.

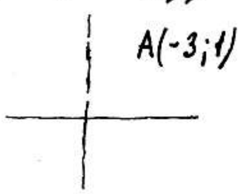
$$3x - 5y = 10 \qquad 3x - 5y = 10$$

$$-5y = 10 - 3x \qquad \frac{3}{10}x - \frac{5}{10}y = 1$$

$$5y = 3x - 10 \qquad \frac{x}{\frac{10}{3}} - \frac{y}{2} = 1$$

$$y = \frac{3}{5}x - 2$$

4.) Írjuk fel az egyenes $A(-3;1)$ ponton átmenő és $\alpha = 30°$-os irányszögű egyenes egyenletét



$$y - y_1 = m(x - x_1) \qquad tg\,30° = \frac{\sqrt{3}}{3}$$

$$y - 1 = m(x + 3)$$

$$y - 1 = \frac{\sqrt{3}}{3}(x + 3)$$

$$y = 1 + \frac{\sqrt{3}x}{3} + \sqrt{3}$$

$$y = \frac{\sqrt{3}x}{3} + (\sqrt{3} + 1)$$

5.) Hat. meg az $A(2;-1)$ és $B(3;4)$ pontokon átmenő egyenes egyenletét:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} \cdot (x - x_1) \qquad y = 5x - 11$$

$$y + 1 = \frac{5}{1}(x - 2)$$

$$y = 5x - 10 - 1$$

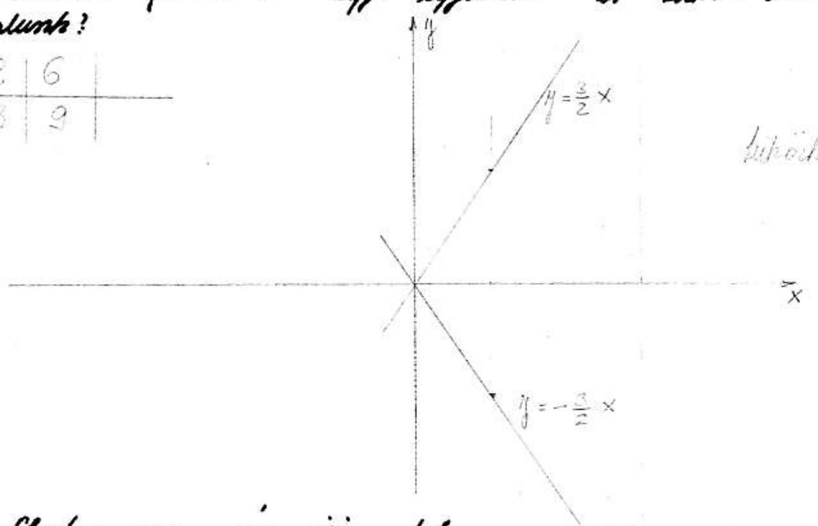
1967. IV. 30.

IV.

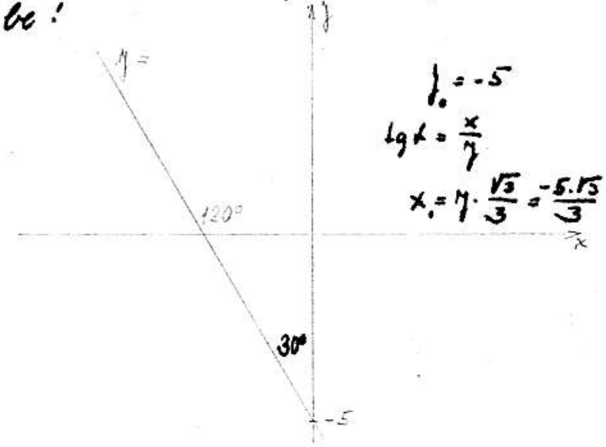
Geom. II. 31. o. 4. 5. 6. 7. 8. 9. 10. 11. 12. - 36 o.

Ábr. az $y = 3/2\ x$ egy. egyenest és tükr. az X t.-re. Mit tapasztalunk?

X	2	6
y	3	9



Szerk. meg és írj. fel annak az egy.-nek az egyenletét amely X t.-t a -5 p-ban metszi és az X t.-lyel 120°-os szöget zár be!

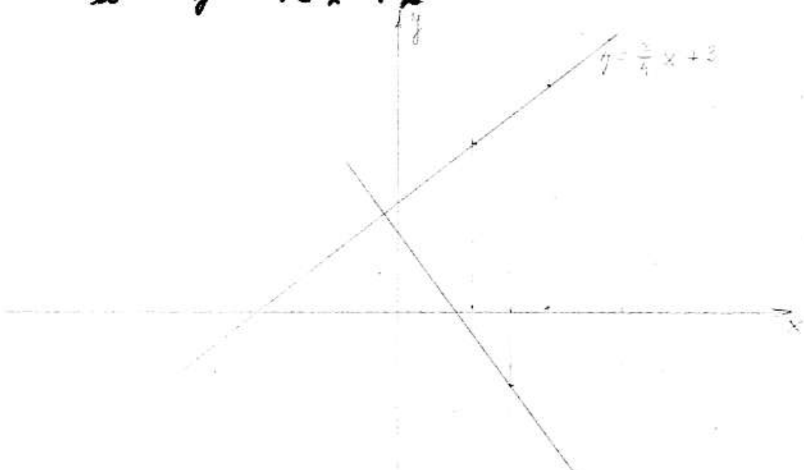


$$y_0 = -5$$
$$tg\,\alpha = \frac{x}{y}$$
$$x_0 = y \cdot \frac{\sqrt{3}}{3} = \frac{-5 \cdot \sqrt{3}}{3}$$

$$-\frac{3x}{5\sqrt{3}} - \frac{y}{5} = 1$$
$$\frac{\sqrt{3} \cdot x}{5} + \frac{y}{5} = -1$$
$$\sqrt{3} \cdot x + y = -5$$
$$y = -(x \cdot \sqrt{3} + 5)$$

Szerk. meg azt a 2 egy.-t, amelyeknek egy.-lete: $y = 3/4\ x + 3$ és $y = -4/3\ x + 2$

X	2	4
y	4,5	6
x	3	6
y	-2	-6



Igazoljuk hogy az $y = 3/4\,x + 3$ egyenesen rajta van a $P(2;4,5)$ pont.

biz.:

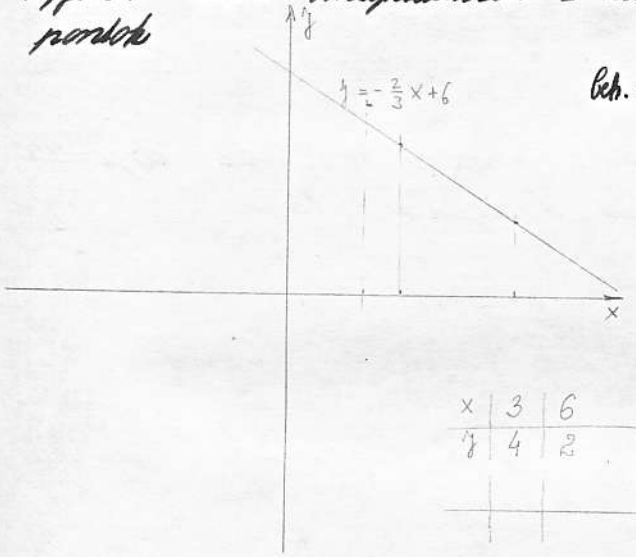
$$4,5 = \frac{3}{4}\cdot 2 + 3$$

$$4,5 = \frac{3}{2} + 3$$

$$4,5 = 1,5 + 3$$

$$4,5 = 4,5$$

Hogyan helyezkednek a koordinátatengelyek és az $y = -\frac{2}{3}x + 6$ egyenes által meghatározott Δ-höz képest a $P_1(2;5)$ és $P_2(5;1)$ pontok



x	3	6
y	4	2

biz.: $y = -\frac{2}{3}\cdot 2 + 6 = -\frac{4}{3} + 6 = -\frac{4}{3} + \frac{18}{3} = \frac{14}{3}$

$y_{P_1} = 5$

$y_{P_1} > y$

A pont a Δ-ön kívül fekszik.

Hol metszi az X és Y tengelyt a köv. egyenes: $3x + 4y = 6$

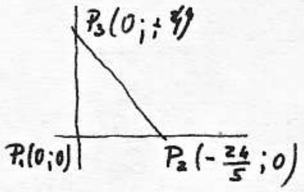
biz.: 0-t

$4y = 6$	$3x = 6$
$y = \frac{6}{4}$	$x = \frac{6}{3}$
$y = \frac{3}{2} = 1,5$	$x = 2$

Hat. meg az $y = \frac{5}{6}x + 4$ egyenes és a tengelyek által bezárt Δ területét.

$y = 4$

$x = -\frac{24}{5}$



$$T = \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right] =$$

$$T = \frac{1}{2}\left[0 + -\frac{24}{5}(4 - 0) + 0\right] = \frac{1}{2} \cdot \left[-\frac{96}{5}\right] = -9{,}6$$

$$|T| = 9{,}6$$

Hat. meg a $P(5;4)$ ponton átmenő egyenes egyenletét ha a tengellyel bezárt szögének tangense $4/3$.

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \frac{4}{3}(x - 5)$$

$$y - 4 = \frac{4}{3}x - \frac{20}{3}$$

$$y = \frac{4}{3}x - \frac{20}{3} + 4$$

$$y = \frac{4}{3}x - \frac{8}{3}$$

Hat. meg a $P_1(3;6)$ és $P_2(4;-3)$ pontokon áthaladó egyenes egyenletét.

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 6 = \frac{-3-6}{4-3}(x-3)$$

$$y - 6 = \frac{-9}{1}(x-3)$$

$$y - 6 = -9x + 27$$

$$y = -9x + 33$$

15. ó. - elm.

16. óra.

1967. X. 4.

Írjuk fel az M,N-en áth. egy. egyenletét:

$M(2;1)$

$N(5;3)$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 1 = \frac{3-1}{5-2}(x-2)$$

$$y - 1 = \frac{2}{3}(x-2)$$

$$y = 1 + \frac{2}{3}x - \frac{4}{3}$$

$$y = \frac{2}{3}x - \frac{1}{3}$$

$A(3;5)$

$B(7;4)$

$$y = 5 + \frac{-1}{4}(x-3) \quad y = -\frac{1}{4}x + \frac{\overset{23}{\cancel{17}}}{4} = -\frac{1}{4}x + \frac{23}{4}$$

$D(-7;6)$

$C(5;-3)$

$$y = 6 + \frac{-9}{12}(x+7) \quad y = -\frac{3}{4}x + 6 - \frac{21}{4} = -\frac{3}{4}x + \left(\frac{51}{12}\right) + \frac{3}{4}$$

$E(3;\frac{1}{3})$

$F(6\frac{1}{4};-5\frac{\cancel{1}}{3})$

$$y = \frac{1}{3} + \frac{-\frac{16}{5} - \frac{1}{3}}{\frac{25}{4} - 3}(x-3) = \frac{1}{3} + \frac{-\frac{48+5}{15}}{\frac{25-12}{4}}(x-3)$$

$$y = \frac{1}{3} - \frac{\frac{53}{15}}{\frac{13}{4}}(x-3) = \frac{1}{3} - \frac{212}{195}x + \frac{636}{195}$$

$$y = -\frac{212}{195}x + \frac{636+65}{195} = -\frac{212}{195}x + \frac{701}{195}$$

$O(0;0)$ $\quad y = 0 + \frac{6-0}{3-0}(x-0) = \underline{2x}$

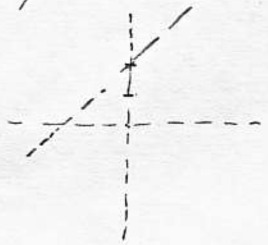
$P(3;6)$

$O(0;0)$ $\quad y = 0 + \frac{-6-0}{3-0}(x-0) = -\underline{2x} + 6 - 6 = \underline{-2x}$

$Q(3;-6)$

$\alpha = 60°$ $\quad \alpha = 60°$ $\operatorname{tg}\alpha = \sqrt{3}$

$M(0;2)$



$y = mx + b$

$\underline{y = \sqrt{3}x + 2}$

1967. X. 4. $\qquad$ 16. H.

$\alpha = 135°$ $\qquad M(0;-4{,}5)$

$M(0;3)$ $\qquad \alpha = 150°$

$\alpha = 45°$ $\qquad M(-2{,}5;0)$

$M(0;-3)$ $\qquad \alpha = 30°$

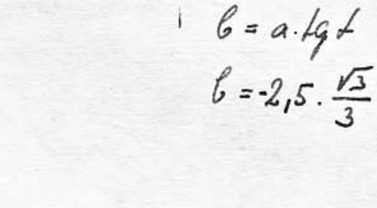
1.) $\operatorname{tg}135° = -\operatorname{tg}45° = -1$; $b = 3$ $\qquad y = mx + b$ $\qquad \underline{y = -x + 3}$

2.) $\operatorname{tg}45° = 1$ $\quad b = -3$ $\qquad \underline{y = x - 3}$

3.) $\operatorname{tg}150° = -\operatorname{tg}30° = -\frac{\sqrt{3}}{3}$ $\quad b = -4{,}5$ $\qquad \underline{y = -\frac{\sqrt{3}}{3}x - 4{,}5}$

4.) $\operatorname{tg}30° = \frac{\sqrt{3}}{3}$

$\underline{y = \frac{\sqrt{3}}{3}(x + 2{,}5)}$



$\operatorname{tg}\alpha = \frac{b}{a}$

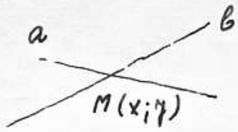
$b = a \cdot \operatorname{tg}\alpha$

$b = -2{,}5 \cdot \frac{\sqrt{3}}{3}$

17. óra. 1967. X. 5.

Két egyenes metszéspontja

$$Ax + By + C = 0$$
$$A_1x + B_1y + C_1 = 0$$



Pl.:

1.) $3x - 8y + 34 = 0$
$2x + y - 9 = 0 \longrightarrow y = 9 - 2x$

$3x - 8(9-2x) + 34 = 0$
$3x - 72 + 16x + 34 = 0$
$19x = 38$
$x = 2$

$y = 9 - 2x$
$y = 9 - 4$
$y = 5$

$M(2;5)$

2.) $2x - y + 3 = 0$
$2x - 3y + 1 = 0$
$2y + 2 = 0$
$y = -1.$

$x = \frac{y-3}{2}$
$x = \frac{-4}{2}$
$x = -2$

$M(-2;-1)$

3.) $7x + 3y - 37 = 0$
$4x - 17y - 3 = 0$

$\longrightarrow -28x \pm 12y \mp 148 = 0$
$28x - 119y - 21 = 0$
$-131y + 127 = 0$
$131y = -127$
$y = -\frac{127}{131} \doteq 0{,}9..$

$119x + 51y - 17 \cdot 37 = 0$
$12x - 51y - 9 = 0$
$131x - 638 = 0$
$x = \frac{638}{131} = 4{,}8..$

$37 \cdot 17$
259
629

4.) $3y = 2x + 3$
$y = \frac{2}{3}x + 1 \quad / \cdot 3 \rightarrow 3y = 2x + 3$

$2x + 3 = 2x + 3$
$0 = 0$

$a \equiv b.$

5.) $2x-5y-2=0$
$5x+2y-6=0$

$\pm 10x \mp 25y \mp 10 = 0$
$10x+4y-12=0$

$29y-2=0$

$y=\frac{2}{29}$

$x=\frac{12-4y}{10}$

$x=\frac{12-\frac{8}{29}}{10}$

$x=\frac{340}{290}$

$x=\frac{34}{29}$

$M\left(\frac{34}{29};\frac{2}{29}\right)$

6.) $2x-5y+6=0$ 1.4
$8x-20y+10=0$
$8x-20y+24=0$

$\left.\begin{matrix} 8x-20y+10=0 \\ 8x-20y+24=0 \end{matrix}\right\} 10 \neq 24$ párhuzamosak

1967. X. 5.   17. ff.   46. 1. 2. 3.

1.) $2x+y+7=0$
$x-3y-2=0 \rightarrow x=3y+2$

$3y+2+3y+2+y+7=0$
$7y=-11$
$y=-\frac{11}{7}$

$M\left(\frac{-19}{7};\frac{-11}{7}\right)$

$x=-\frac{33}{7}+\frac{14}{7}=-\frac{19}{7}$

2.) Két egyenes egyenlete $2x+3y=15$ és $x-y=1$. Mekkora a két egyenes és az X tengely által meghatározott háromszög területe.

I. $2x+3y-15=0$
$x-y=1 \rightarrow x=y+1$

$2y+2+3y-15=0$
$5y=13$
$y=\frac{13}{5}$

$x=\frac{18}{5}$

$M\left(\frac{18}{5};\frac{13}{5}\right)$

II. $2x=15$
$x=\frac{15}{2}$
$y=0$

$A\left(\frac{15}{2};0\right)$

III. $x=1$
$y=0$

$N(1;0)$

$T=\frac{1}{2}\left[\frac{18}{5}(0-0)+\frac{15}{2}\left(-\frac{13}{5}+0\right)+1\left(\frac{13}{5}-0\right)\right]=\frac{1}{2}\left[-\frac{195}{10}+\frac{13}{5}\right]$

$T=\frac{1}{2}\left[-\frac{195}{10}+\frac{26}{10}\right]=\frac{1}{2}\cdot-\frac{169}{10}=-\frac{169}{20}$

$|T|=\left|-\frac{169}{20}\right|=8{,}45$

3.) Hat. meg annak az egyenesnek az egyenletét, amely átmegy az $x - 3y + 9 = 0$ és $2x + 3y = 18$ egyenesek metszéspontján, és iránytangense 2,5.

$$\begin{array}{r} x - 3y + 9 = 0 \\ 2x + 3y - 18 = 0 \\ \hline 3x \quad - 9 = 0 \\ x = 3 \end{array} \qquad y = \frac{18 - 2x}{3} = \frac{18 - 6}{3} = 4$$

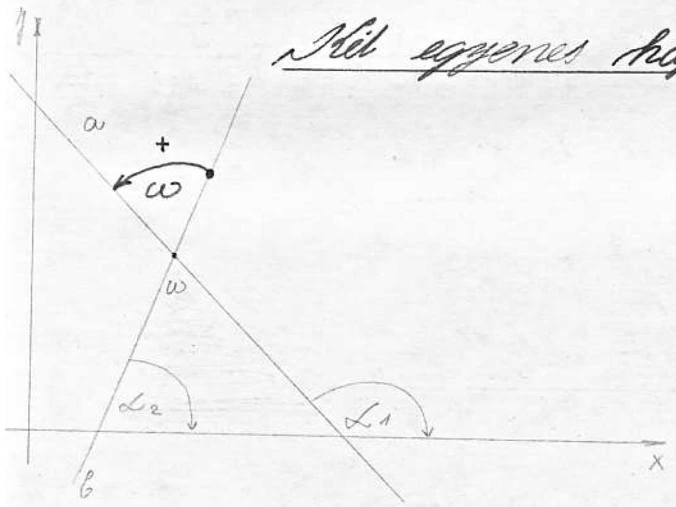
$$M(3;4)$$

$m = 2,5 \quad x_1 = 3 \quad y_1 = 4$

$$y - y_1 = m(x - x_1)$$
$$y - 4 = 2,5(x - 3)$$
$$y - 4 = 2,5x - 7,5$$
$$y = 2,5x - 3,5$$

18. óra 19.0. 1967. X. 6.

Két egyenes hajlásszöge



a.... $y = m_1x + b_1 \quad m_1 = tg\alpha_1$

b.... $y = m_2x + b_2 \quad m_2 = tg\alpha_2$

$$\alpha_1 = \alpha_2 + \omega$$
$$\omega = \alpha_1 - \alpha_2$$

$$tg\,\omega = tg(\alpha_1 - \alpha_2) = \frac{tg\alpha_1 - tg\alpha_2}{1 + tg\alpha_1 \cdot tg\alpha_2}$$

$$tg\,\omega = \frac{m_1 - m_2}{1 + m_1 \cdot m_2}$$

Ha $m_1 = m_2 \rightarrow tg\,\omega = \frac{0}{1 + m_1^2} = 0 \rightarrow \omega = 0$

Ha $a \parallel b \rightarrow tg\,0^\circ = 0. \rightarrow \omega = 0$

$m_1 = m_2$ – a párhuzamosság feltétele

1.) Keressük meg a $p \equiv 3x-2y+4=0$ és a $q \equiv -x+2y+1=0$ egyenesek hajlásszögét.

$$-2y = -3x-4 \qquad 2y = x-1$$
$$2y = 3x+4 \qquad q \equiv y = \frac{1}{2}x - \frac{1}{2}$$
$$p \equiv y = \frac{3}{2}x + 2$$

$$m_1 = \frac{3}{2} \qquad m_2 = \frac{1}{2}$$

$$\operatorname{tg}\omega = \frac{m_1 - m_2}{1 + m_1 \cdot m_2} = \frac{\frac{3}{2} - \frac{1}{2}}{1 + \frac{3}{2}\cdot\frac{1}{2}} = \frac{1}{1+\frac{3}{4}} = \frac{1}{\frac{7}{4}} = \frac{4}{7}$$

$$\operatorname{tg}\omega = \frac{4}{7} = 0{,}\overline{57142857}$$

$$\operatorname{tg}\omega = 0{,}\overline{571428} \qquad \underline{\underline{\omega \doteq 29°40'}}$$

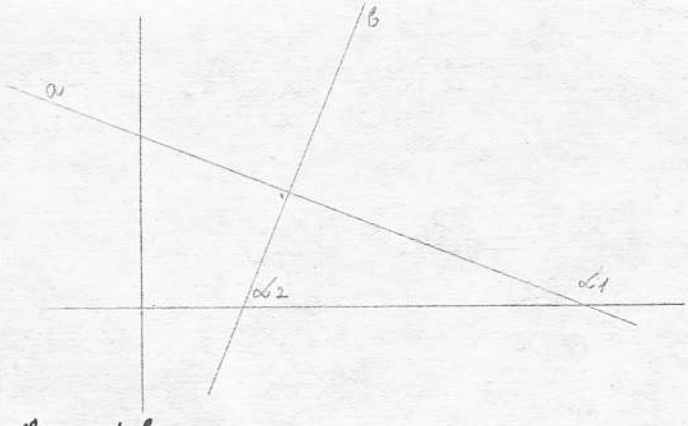
2.)
$$a \equiv 5x - y + 2 = 0 \qquad \omega = ?$$
$$b \equiv 2x - 3y + 1 = 0$$

$$y = 5x + 2$$
$$y = \frac{2}{3}x + \frac{1}{3}$$

$$\operatorname{tg}\omega = \frac{5 - \frac{2}{3}}{1 + 5\cdot\frac{2}{3}} = \frac{\frac{13}{3}}{\frac{3}{3} + \frac{10}{3}} = \frac{\frac{13}{3}}{\frac{13}{3}} = \frac{3.13}{3.13};$$

$$\operatorname{tg}\omega = \frac{1}{1} = 1. \qquad \underline{\underline{\omega = 45°}}$$

A merőlegesség feltétele.



$$\operatorname{cotg}\omega = \frac{1 + m_1 \cdot m_2}{m_1 - m_2}$$

Ha $\underline{1 + m_1 \cdot m_2} = 0$ akkor $\operatorname{cotg}\omega = 0$

$$\Downarrow$$
$$\omega = 90°$$

Ha $a \perp b$

$$\alpha_1 = 90° + \alpha_2$$

$$\operatorname{tg}\alpha_1 = \operatorname{tg}(90° + \alpha_2) = \frac{\sin(90° + \alpha_2)}{\cos(90° + \alpha_2)} =$$

$$1 + m_1 \cdot m_2 = 0$$
$$m_1 \cdot m_2 = -1$$
$$\underline{\underline{m_1 = -\frac{1}{m_2}}}$$

$$= \frac{\sin 90° \cdot \cos\alpha_2 + \cos 90° \cdot \sin\alpha_2}{\cos 90° \cdot \cos\alpha_2 - \sin 90° \cdot \sin\alpha_2} = \frac{1 \cdot \cos\alpha_2 + 0 \cdot \sin\alpha_2}{0 \cdot \cos\alpha_2 - 1 \cdot \sin\alpha_2} = \frac{\cos\alpha_2}{-\sin\alpha_2} = -\operatorname{cotg}\alpha_2$$

$$= -\frac{1}{\operatorname{tg}\alpha_2}$$

$$\operatorname{tg}\alpha_1 = -\frac{1}{\operatorname{tg}\alpha_2}$$

$$m_1 = -\frac{1}{m_2}$$ – két egyenes merőlegességének feltétele.

1.) $a \equiv 3x + 4y = 6$
$b \equiv 5x - 4y = 8$ $\quad \omega = ?$

$$\operatorname{tg}\omega = \frac{m_1 - m_2}{1 + m_1 \cdot m_2}$$

$$y_1 = \frac{-3x + 6}{4} \qquad m_1 = -\frac{3}{4}$$

$$y_2 = \frac{-5x + 8}{-4} \qquad m_2 = \frac{5}{4}$$

$$\operatorname{tg}\omega = \frac{-\frac{3}{4} - \frac{5}{4}}{1 + \frac{5}{4} \cdot -\frac{3}{4}} = \frac{-2}{1 - \frac{15}{16}} = \frac{-2}{\frac{1}{16}} =$$

$$\operatorname{tg}\omega = -32$$

$$\operatorname{tg}\omega = -32 \quad \rightarrow \quad \omega > 90°$$

$$\operatorname{tg}\omega = \frac{\frac{5}{4} + \frac{3}{4}}{1 - \frac{3}{4} \cdot \frac{5}{4}} = \frac{\frac{15}{16}}{1 - \frac{15}{16}} = \frac{\frac{15}{16}}{\frac{1}{16}} = \underline{\underline{15}}$$

$$\operatorname{tg}(180 - \alpha) = -\operatorname{tg}\alpha \qquad 180 - \omega = 88° 10'$$

$$\operatorname{tg}(180 - \omega) = 32 \qquad \underline{\underline{\omega = 91°50'}}$$

2.) Mekkora szöget zár be az $y = 3x + 4$ egyenes az $y = \frac{5}{2}x - 3$ egyenessel?

$$\operatorname{tg}\omega = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{3 - \frac{5}{2}}{1 + 3 \cdot \frac{5}{2}} = \frac{\frac{1}{2}}{\frac{17}{2}} = \frac{1}{17} = 0{,}0588$$

$$\underline{\underline{\omega = 3°20'}}$$

3.) Mekkora szöget zár be az A(− [illegible]; −4) és a $B(5;7)$ pontokon átm. egyenes az $5x - 4y = 14$ egyenessel.

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y + 4 = \frac{7 + 4}{5 + 3}(+3 + x)$$

$$y + 4 = \frac{11}{8}(x + 3)$$

$$y = \frac{11}{8}x + \frac{33}{8} - 4$$

$$\downarrow \quad m_1 = \frac{11}{8}$$

$$5x - 4y = 14$$

$$-4y = -5x + 14$$

$$y = \frac{5}{4}x + \frac{14}{-4}$$

$$\downarrow \quad m_2 = \frac{5}{4}$$

$$\operatorname{tg} \omega = \frac{\frac{11}{8} - \frac{10}{8}}{1 + \frac{11}{8} \cdot \frac{5}{4}} = \frac{\frac{1}{8}}{1 + \frac{55}{32}} = \frac{\frac{1}{8}}{-\frac{87}{32}} = \frac{32}{8 \cdot 87} = \frac{4}{87} \doteq 0{,}046$$

$$\operatorname{tg} \omega \doteq 0{,}0459 \qquad \omega = 2^\circ 30'$$

1967. X. 6. 19. ff.

$$a \equiv 7x + 2y - 37 = 0 \longrightarrow y = \frac{37 - 7x}{2} \qquad m_1 = -\frac{7}{2}$$

$$b \equiv 4x - 17y - 3 = 0 \longrightarrow y = \frac{3 - 4x}{-17} \qquad m_2 = \frac{4}{17}$$

$$\omega = ?$$

$$\operatorname{tg} \omega = \frac{m_1 + m_2}{1 + m_1 m_2} = \frac{-\frac{7}{2} - \frac{4}{17}}{1 + \frac{4}{17} \cdot -\frac{7}{2}} = \frac{\frac{-119 - 8}{34}}{\frac{34 + (-28)}{34}} = \frac{-34 \cdot 127}{34 \cdot 6} = \frac{-127}{6} = -21{,}16\dot{6}$$

$$\alpha \doteq 87^\circ 15'$$

$$\omega \doteq 92{,}7^\circ$$

$2x - 5y - 2 = 0 \equiv p \quad \rightarrow \quad y = \frac{2 - 2x}{-5} \qquad m_1 = \frac{2}{5}$

$5x + 2y - 6 = 0 \equiv q \quad \rightarrow \quad y = \frac{6 - 5x}{2} \qquad m_2 = -\frac{5}{2}$

$\omega = ?$

$$tg\,\omega = \frac{m_1 - m_2}{1 + m_1 \cdot m_2} = \frac{\frac{2}{5} + \frac{5}{2}}{1 + \frac{2}{5} \cdot \frac{5}{2}} = \frac{\frac{4 + 10}{10}}{\frac{2}{1}} = \frac{14}{20} = \frac{7}{10} = 0{,}7000$$

$\omega = 35^\circ$

$3y = 2x + 3 \equiv m \rightarrow y = \frac{2x}{3} + 1$

$y = \frac{2}{3}x + 1 \equiv n \rightarrow y = \frac{2}{3}x + 1$

$\omega = ?$

$m_1 = m_2 = \frac{2}{3} \Rightarrow$ eqbeeo''

$\omega = k\pi$

$2x - 5y + 6 = 0 \quad \rightarrow \quad y = \frac{-6 - 2x}{-5} \qquad m_1 = \frac{2}{5}$

$8x - 20y + 10 = 0 \quad \rightarrow \quad y = \frac{-10 - 8x}{-20} \qquad m_2 = \frac{2}{5}$

$\omega = ?$

$\Big\} \Rightarrow$ paihuramos

$\omega = k\pi$

Írj. f. azon. egy. egy-t. mely áth. M-en és :

I.) ∥ p-vel ,   a.) $M(5;3)$

II.) ⊥ p-re , $p \equiv 2x-3y-12=0$

b.) $M(7;-4)$

$p \equiv 9x+7y-25=0$

1.) $M(5;3) \equiv q \parallel p \equiv 2x-3y-12=0$

$$\downarrow$$

$$y = \frac{2x-12}{3} \rightarrow m = \frac{2}{3}$$

∥ Ha $m_1 = m_2$

$$q \equiv y - y_1 = m(x-x_1)$$

$$y - 3 = \frac{2}{3}(x-5)$$

$$y = 3 + \frac{2}{3}x - \frac{10}{3}$$

$$y = \frac{2}{3}x - \frac{1}{3}$$

2.) $M(5;3) \equiv q \perp p \equiv 2x-3y-12=0$

⊥ Ha $m_1 = -\frac{1}{m_2}$

$$\downarrow$$

$$m = \frac{2}{3}$$

$$q \equiv y - y_1 = m(x-x_1)$$

$$q \equiv y - 3 = -\frac{1}{m_2}(x-x_1)$$

$$y = 3 - \frac{3}{2}(x-5)$$

$$y = -\frac{3}{2}x + \frac{21}{2}$$

3.) $M(7;-4) \equiv q \parallel p \equiv 9x+7y-25=0 \longrightarrow m = -\frac{9}{7}$

$$q = y - y_1 = m(x-x_1)$$

$$y+4 = -\frac{9}{7}(x-7)$$

$$y = -4 - \frac{9}{7}x + 9$$

$$y = -\frac{9}{7}x + 5$$

4.)

$M(7;-4) \equiv q \perp p \equiv 9x+7y-25=0$ —

$\downarrow m_1 = -\frac{9}{7}$ $\qquad m = -\frac{1}{m_1}$

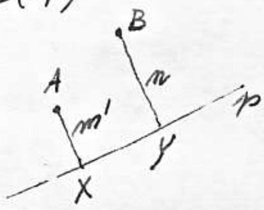
$$q \equiv y - y_1 = m(x - x_1)$$
$$y+4 = -\frac{7}{9}(x-7)$$
$$y+4 = \frac{7}{9}x - \frac{49}{9}$$
$$y = \frac{7}{9}x - \frac{85}{9}$$

20. óra

1967. X. 10

$A(3;3)$ $\quad p \equiv 4x+3y-1=0$ Hat meg a két pont távolságát az egyenestől.

$B(1;1)$



$p \equiv 4x+3y-1=0 \rightarrow m_0 = -\frac{4}{3}$ $\qquad m = \frac{3}{4}$

$$m' \equiv y - y_1 = m(x - x_1)$$
$$m' \equiv y - 3 = \frac{3}{4}(x-3)$$
$$m' \equiv y = \frac{3}{4}x + \frac{3}{4}$$

$y_1 = \frac{3}{4}x_1 + \frac{3}{4} \rightarrow 4y_1 = 3x_1 + 3 \rightarrow y = \frac{3x+3}{4}$

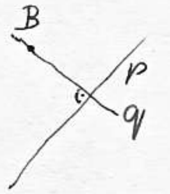
$y = -\frac{4x}{3} + \frac{1}{3} \rightarrow 3y = -4x+1$

$$3\left(\frac{3x+3}{4}\right) = -4x+1$$
$$9x+9 = -16x+4$$
$$25x = -5$$
$$x = -\frac{1}{5}$$
$$y = \frac{-\frac{3}{5} + \frac{15}{5}}{4}$$
$$y = \frac{\frac{12}{5}}{\frac{4}{1}} = \frac{12}{20} = \frac{3}{5}$$

$$d_{Ap} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$d_{Ap} = \sqrt{\left(3+\frac{1}{5}\right)^2+\left(3-\frac{3}{5}\right)^2}$$
$$d_{Ap} = \sqrt{\left(\frac{16}{5}\right)^2+\left(\frac{12}{5}\right)^2}$$
$$d_{Ap} = \sqrt{\frac{256+144}{25}} = \sqrt{\frac{400}{25}} = \frac{20}{5} = 4$$
$$d_{Ap} = 4$$

$p \equiv 4x + 3y - 1 = 0 \quad \rightarrow \quad m_1 = -\frac{4}{3} \quad \rightarrow \quad m = \frac{3}{4}$

$B(1;1)$



$$q \equiv y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{3}{4}(x - 1)$$

$$y = \frac{3}{4}x + \frac{1}{4} \quad \Rightarrow \quad 4y = 3x + 1$$

$$4y = 3x + 1 \quad \rightarrow \quad y = \frac{3x+1}{4}$$

$$\underline{3y = -4x + 1}$$

$$3\left(\frac{3x+1}{4}\right) = -4x + 1$$

$$9x + 3 = -16x + 4$$

$$25x = 1$$

$$x = \frac{1}{25}$$

$$y = \frac{\frac{3}{25} + \frac{25}{25}}{\frac{4}{1}}$$

$$y = \frac{28}{25 \cdot 4} = \frac{28}{100} = \frac{7}{25}$$

$$y = \frac{7}{25}$$

$$d_{Bp} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d_{Bp} = \sqrt{\left(1 - \frac{1}{25}\right)^2 + \left(1 - \frac{7}{25}\right)^2}$$

$$d_{Bp} = \sqrt{\left(\frac{24}{25}\right)^2 + \left(\frac{18}{25}\right)^2}$$

$$d_{Bp} = \sqrt{\frac{576}{625} + \frac{324}{625}}$$

$$d_{Bp} = \frac{\sqrt{900}}{25}$$

$$d_{Bp} = \frac{30}{25} = \frac{6}{5}$$

$$d_{Bp} = \frac{6}{5}$$

$24^2 = 576$
416
16

2.

$$p \equiv a_1x + b_1y + c_1 = 0 \quad \longrightarrow \quad m_p = -\frac{a_1}{b_1}$$

$$q \equiv a_2x + b_2y + c_2 = 0 \quad \longrightarrow \quad m_q = -\frac{a_2}{b_2}$$

Ha $a_1b_2 = b_1a_2$ akkor $p \parallel q$

Ha $p \parallel q \rightarrow m_1 = m_2 \qquad -\frac{a_1}{b_1} = -\frac{a_2}{b_2}$

$$a_1b_2 = a_2b_1 \equiv a_1b_1 = b_1a_2$$

3) Ha $p \perp q \rightarrow a_1a_2 + b_1b_2 = 0$

$p \equiv a_1x + b_1y + c_1 = 0 \rightarrow m_1 = -\frac{a_1}{b_1}$

$q \equiv a_2x + b_2y + c_2 = 0 \rightarrow m_2 = -\frac{a_2}{b_2}$

Ha $p \perp q \rightarrow m_1 = -\frac{1}{m_2}$ / beh. : $-\frac{a_1}{b_1} = -\frac{1}{-\frac{a_2}{b_2}}$

$$-\frac{a_1}{b_1} = \frac{b_2}{a_2}$$

$$-a_1a_2 = b_1b_2$$

$$\underline{\underline{0 = a_1a_2 + b_1b_2}}$$

4) $A(6;4)$

$q \perp p \equiv A$

$q \equiv 3x + 2y + 6 = 0 \rightarrow m = -\frac{3}{2}$

$m_1 = -\frac{1}{m}$

$m_1 = -\frac{1}{-\frac{3}{2}} = \frac{2}{3}$

$y - y_1 = m(x - x_1)$

$y - 4 = \frac{2}{3}(x - 6)$

$y = \frac{2}{3}x + 0$

$\underline{\underline{y = \frac{2}{3}x}}$

5) $M(5;3)$ $p \equiv 2x - 3y - 12 = 0$ $M \equiv q \parallel p$

$M \equiv q \perp p$

$\downarrow$

$m_1 = \frac{2}{3}$

Ha $\perp$. $m = -\frac{3}{2}$

$y - y_1 = m(x - x_1)$

$y - 3 = -\frac{3}{2}(x - 5)$

$y = -\frac{3}{2}x + \frac{21}{2}$

$2y = -3x + 21$

$2y + 3x - 21 = 0$

Ha $\parallel$. $y - y_1 = m(x - x_1)$

$3x + 2y - 21 = 0$

$y - 3 = \frac{2}{3}(x - 5)$

$3y = 2x - 1$

$y = \frac{2}{3}x - \frac{1}{3}$

$2x - 3y - 1 = 0$

6.) $p \equiv 3x - 4y + 5 = 0 \quad \rightarrow \quad m = -\frac{3}{4}$

$A(2;4)$

$0(0;0)$

távolságok!

$m_1 = -\frac{1}{m_2}$

$m_1 = \frac{4}{3}$

$$y - y_1 = m(x - x_1)$$
$$y - 4 = \frac{4}{3}(x - 2)$$
$$y = \frac{4}{3}x + \frac{4}{3}$$
$$3y = 4x + 4$$
$$3y - 4x - 4 = 0$$

$$3y - 4x - 4 = 0 \longrightarrow y = \frac{4x+4}{3}$$
$$\underline{-4y + 3x + 5 = 0}$$
$$-4\left(\frac{4x+4}{3}\right) + 3x + 5 = 0$$
$$\frac{-16x - 16}{3} + 3x + 5 = 0$$
$$\frac{-16x - 16 + 9x + 15}{3} = 0$$
$$\frac{-7x - 1}{3} = 0$$
$$-\frac{7}{3}x = \frac{1}{3}$$
$$-7x = 1$$
$$x = -\frac{1}{7}$$
$$y = \frac{4 \cdot -\frac{1}{7} + 4}{3} = \frac{-\frac{4}{7} + \frac{28}{7}}{3} = \frac{24}{21} = \frac{8}{7}$$

$$d_{Ap} = \sqrt{\left(2 + \frac{1}{7}\right)^2 + \left(4 - \frac{8}{7}\right)^2}$$
$$d_{Ap} = \sqrt{\left(\frac{15}{7}\right)^2 + \left(\frac{20}{7}\right)^2}$$
$$d_{Ap} = \sqrt{\frac{225}{49} + \frac{400}{49}}$$
$$d_{Ap} = \sqrt{\frac{625}{49}} = \frac{25}{7} \doteq 3{,}57\ldots$$

(1)

—

$m = -\frac{4}{3}$

$x = -\frac{3}{5}$

$y = \frac{4}{5}$

$d = 1$

1967. X. 10. 20. Hf.

1.) $A(5;6)$ magasságok hossza

$B(-2;4)$ belső szögek nagyságát

$C(6;-1)$ területét

2.) 43/5 (zöld k.)

45/6, 7, 8)

49(1)

51(2)

1,

$a \equiv y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 4 = \frac{-1-4}{6+2}(x+2)$

$y - 4 = \frac{-5}{8}(x+2)$

$a \equiv y = -\frac{5}{8}x + \frac{22}{8}$

$m_a = -\frac{5}{8}$

$m_{va} = \frac{8}{5}$ $v_a \equiv y - y_A = m_{va}(x - x_A)$

$y - 6 = \frac{8}{5}(x - 5)$

$v_a \equiv y = \frac{8}{5}x - 2$

$b \equiv y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 6 = \frac{-1-6}{6-5}(x-5)$

$y = 6 - 7x + 35$

$b \equiv y = -7x + 41$

$m_b = -7$

$m_{vb} = \frac{1}{7}$ $v_b \equiv y - 4 = \frac{1}{7}(x+2)$

$y - 4 = \frac{1}{7}x + \frac{2}{7}$

$v_b \equiv y = \frac{1}{7}x + \frac{30}{7}$

$c \equiv y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 6 = \frac{4-6}{-2-5}(x-5)$

$y = 6 + \frac{2}{7}x - \frac{10}{7}$

$c \equiv y = \frac{2}{7}x + \frac{32}{7}$

$m_c = \frac{2}{7}$

$m_{vc} = -\frac{7}{2}$ $v_c \equiv y + 1 = -\frac{7}{2}(x - 6)$

$y + 1 = -\frac{7}{2}x + \frac{42}{2}$

$y = -\frac{7}{2}x + 20$

$A'(\quad)$ $y = -\frac{5}{8}x + \frac{22}{8} = \frac{8}{5}x - 2 = \frac{8}{5}x - \frac{10}{5}$

$-25x + 110 = 64x - 80$

$190 = 89x$

$x \doteq \frac{190}{89} \doteq \cancel{1{,}18}\ 2{,}1$

$\left(y = -\frac{5}{8}x + \frac{22}{8} \right.$

$\left. y = \frac{-24+22}{8} = -\frac{1}{4} = \cancel{-0{,}25} \right)$

$y \doteq 1{,}4$

$A'(\cancel{1{,}1}\,;\ +\cancel{1{,}45})$

$B'(\quad)$ $y = -7x + 41 = \frac{1}{7}x + \frac{30}{7}$

$-49x + 287 = x + 30$

$257 = 50x$

$x \doteq 5{,}1$

$y = -7x + 41$

$y = -35{,}7 + 41$

$y = 5{,}3$

$B'(5{,}1\,;\ 5{,}3)$

$C'(\quad)$ $y = \frac{2}{7}x + \frac{32}{7} = -\frac{7}{2}x + \frac{40}{2}$

$4x + 64 = -49x + 280$

$53x = 216$

$x \doteq 4{,}1$

$y = -\frac{7}{2}x + \frac{40}{2}$

$y \doteq -\frac{28{,}7}{2} + \frac{40}{2}$

$y \doteq 5{,}7$

$C(4{,}1\,;\ 5{,}7)$

$$v_a = \sqrt{(5-2,1)^2+(6-1,4)^2} = \sqrt{2,9^2+4,6^2} = \sqrt{8,4+21,2} = \sqrt{29,6} \doteq \underline{5,45}$$

$$v_b = \sqrt{(-2-5,1)^2+(4-5,3)^2} = \sqrt{7,1^2+1,3^2} = \sqrt{50,+1,69} = \sqrt{51,7} \doteq \underline{7,2}$$

$$v_c = \sqrt{(6-4,1)^2+(-1-5,7)^2} = \sqrt{1,9^2+6,7^2} = \sqrt{3,6+45} = \sqrt{48,6} \doteq \underline{6,95}$$

$$a = \sqrt{(-2-6)^2+(4+1)^2} = \sqrt{64+25} = \sqrt{89} = 9,45$$

$$b = \sqrt{(5-6)^2+(6+1)^2} = \sqrt{1+49} = \sqrt{50} = 7,$$

$$c = \sqrt{(5+2)^2+(6-4)^2} = \sqrt{49+4} = \sqrt{53} = 7,25$$

$$\sin\alpha = \frac{v_c}{b} = \frac{6,95}{7} \doteq 0,992 \qquad \underline{\alpha \doteq 80^\circ}$$

$$\sin\beta = \frac{v_a}{c} = \frac{5,45}{7,25} \doteq 0,75 \qquad \underline{\beta \doteq 49^\circ}$$

$$\sin\gamma = \frac{v_a}{b} = \frac{5,45}{7} \doteq 0,78 \qquad \underline{\gamma \doteq 51^\circ}$$

$$T = \frac{1}{2}\left[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)\right] =$$

$$= \frac{1}{2}\left[5(4+1) + -2(-1-6) + 6(6-4)\right] =$$

$$= \frac{1}{2}(25+14+12) = \frac{1}{2}(51) = \underline{25,5}$$

Vmely △ old. egy. $x - 2y = -8$ $4x + 7y = 28$ $2x + y = 14$.
Mekkorák a △ szögei.

$2y = x + 8$ $7y = -4x + 28$ $y = -2x + 14$

$m = \frac{1}{2}$ $m = -\frac{4}{7}$ $m = -\frac{2}{1} = -2$

$$\operatorname{tg}\omega_1 = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{\frac{1}{2} + \frac{4}{7}}{1 + \frac{1}{2}\cdot\frac{4}{7}} = \frac{\frac{15}{14}}{\frac{18}{14}} = \frac{15\cdot 14}{14\cdot 18} = \frac{3}{2} = \frac{3}{2} \quad \measuredangle_1 = 56{,}3^\circ$$

$$\operatorname{tg}\omega_2 = \frac{\frac{1}{2} + \frac{4}{2}}{1 + \frac{1}{2}\cdot -\frac{4}{2}} = \frac{\frac{5}{2}}{\frac{0}{0}} = \frac{5}{2}0 = 0 \quad \measuredangle_2 = 90^\circ$$

$$\operatorname{tg}\omega_2 = \frac{-\frac{4}{7} + \frac{4}{2}}{1 + \frac{4}{7}\cdot\frac{4}{2}} = \frac{\frac{-8 + 28}{14}}{1 + \frac{16}{14}} = \frac{\frac{20}{14}}{\frac{30}{14}} = \frac{14\cdot 20}{14\cdot 30} = \frac{2}{3} \quad \measuredangle_3 = 33{,}7^\circ$$

Írj. fel. az. egy. az egy., mely álm. a $P_1(3;7)$-n és párh. $2x + 3y = 4$ egyenessel

$m = -\frac{2}{3}$

$$y - 7 = -\frac{2}{3}(x - 3)$$
$$y = +7 - \frac{2}{3}x + 2$$
$$y = -\frac{2}{3}x + 9$$

Írj fel azon. az egy. az egy, mely álm. a $P_1(6;4)$-n és ⊥ a $3x + 2y = 6$ egyenesre.

$m = -\frac{3}{2}$ $m_1 = \frac{2}{3}$

$$y - 4 = \frac{2}{3}(x - 6)$$
$$y = 4 + \frac{2}{3}x - 4$$
$$y = \frac{2}{3}x$$

Hat. meg ann. az egy. az egyenletét, mely átm. a $P_1(3;5)$ n és 45°-os szöget zár be az $x-2y=-2$ egyenessel!

$$m_1 = \frac{1}{2}$$

$$tg\,\alpha = \frac{m_2 - m_1}{1+m_1 m_2} = \frac{m_2 - \frac{1}{2}}{1+\frac{m_2}{2}}$$

$$tg\,45° = 1 \quad \rightarrow \quad 1 = \frac{m_2-\frac{1}{2}}{1+\frac{m_2}{2}}$$

$$2 + m_2 = 2m_2 - 1$$

$$3 = m_2$$

$$y-5 = 3(x-3)$$

$$y-5 = 3x-9$$

$$y = 3x-4$$

Mily. táv. van a $P_1(2;5)$ p. a $2x+3y=4$ egy.-től?

$$m_1 = -\frac{2}{3} \qquad m_2 = \frac{3}{2}$$

$$y-5 = \frac{3}{2}(x-2)$$

$$y-5 = \frac{3}{2}x-3$$

$$y = \frac{3}{2}x+2$$

$$y-\frac{3}{2}x-2=0$$

$$3y+2x-4=0$$

$$3\left(\frac{3}{2}x+2\right)+2x-4=0$$

$$\frac{9}{2}x+6+2x-4=0$$

$$6{,}5x = -2$$

$$13x = -4$$

$$x = -\frac{4}{13}$$

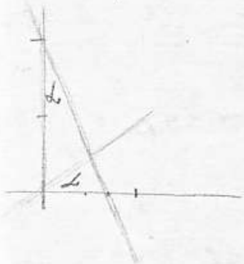
$$y = \frac{3}{2}\cdot -\frac{4}{13}+2 = -\frac{12}{26}+2$$

$$y = \frac{60}{39} = \frac{20}{13}$$

$$d = \sqrt{\left(2+\frac{4}{13}\right)^2+\left(5-\frac{20}{13}\right)^2} = \sqrt{\left(\frac{30}{13}\right)^2+\left(\frac{45}{13}\right)^2} = \frac{\sqrt{900+2025}}{13} = \frac{\sqrt{2925}}{13} = \frac{54}{13} = 4{,}1$$

Mily. távolságra van az origó a $2y + 3x - 4 = 0$ egy.-től.

$2y + 3x = 4 \qquad x = \frac{4}{3} \quad y = 2$



$$\left.\begin{array}{l} \sin\alpha = \frac{p}{2} \quad \rightarrow p = 2 \cdot \sin\alpha \\ \cos\alpha = \frac{3p}{4} \quad \rightarrow p = \frac{4}{3}\cos\alpha \end{array}\right\} 2\sin\alpha = \frac{4}{3}\cos\alpha$$

$$\operatorname{tg}\alpha = \frac{\frac{3}{4}}{2} = \frac{3}{8}$$

$$2 \cdot \sin\alpha = \frac{4}{3}\cos\alpha$$

$$\sin^2\alpha + \cos^2\alpha = 1 \quad \rightarrow \cos^2\alpha = 1 - \sin^2\alpha$$

$$2^2\sin^2\alpha = \left(\frac{4}{3}\right)^2\cos^2\alpha$$

$$4\sin^2\alpha = \frac{16}{9}\left(1 - \sin^2\alpha\right)$$

$$4\sin^2\alpha = \frac{16}{9} - \frac{16}{9}\sin^2\alpha$$

$$\frac{36\sin^2\alpha + 16\sin^2\alpha}{9} = \frac{16}{9}$$

$$52\sin^2\alpha = 16$$

$$\sin^2\alpha = \frac{16}{52}$$

$$\sin\alpha \doteq \frac{4}{7{,}2} \doteq 0{,}59$$

$$(\alpha \doteq 33{,}°40')$$

$$p = 2 \cdot \sin\alpha$$

$$p = 2 \cdot 0{,}59$$

$$p \doteq 1{,}18$$

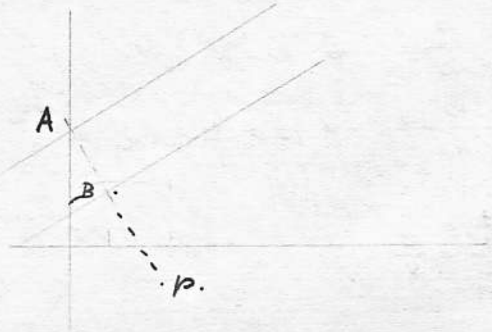
1967. X. 11.

21. óra

Két párhuzamos egyenes egyenlete:

$x - 2y + 6 = 0$

$x - 2y + 2 = 0$ Mekkora a két egyenes távolsága.



$m_1 = \frac{1}{2}$ $\quad m = -2$

$$x = 0 \rightarrow -2y + 6 = 0 \quad \left.\begin{array}{l} 6 = 2y \\ y = 3 \end{array}\right\} A(0;3)$$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -2(x - 0)$$

$$y = 3 - 2x \equiv p$$

$2x + y - 3 = 0$

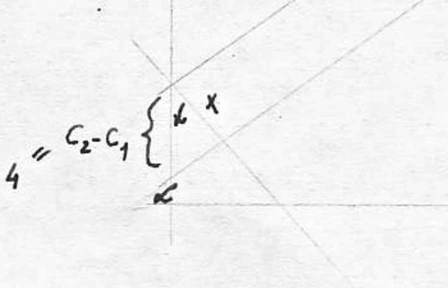
$x - 2y + 2 = 0 \rightarrow x = 2y - 2$

$$\left.\begin{array}{ll} 2(2y-2) + y - 3 = 0 & x = 2y - 2 \\ 4y - 4 + y - 3 = 0 & x = 2\cdot\frac{7}{5} - 2 \\ 5y = 7 & x = \frac{14}{5} - 2 \\ y = \frac{7}{5} & x = \frac{14-10}{5} = \frac{4}{5} \end{array}\right\} B\left(\frac{4}{5};\frac{7}{5}\right)$$

$$d = \sqrt{\left(0 - \frac{4}{5}\right)^2 + \left(3 - \frac{7}{5}\right)^2} = \sqrt{\frac{16}{25} + \frac{64}{25}} = \sqrt{\frac{80}{25}} = \frac{\sqrt{80}}{5} \doteq \frac{8{,}1}{5} \doteq 1{,}6$$

$\sqrt{80} \doteq 8{,}1$

$160 : 161.$



$tg\,\alpha = m = \frac{1}{2}$

$$\left(\begin{array}{l} ctg\,\alpha = \frac{1}{tg\,\alpha} \\ ctg\,\alpha = \end{array} \right)$$

$$\cos\alpha = \frac{1}{\sqrt{1 + tg^2\alpha}} \qquad \cos\alpha = \frac{x}{c_2 - c_1} = \frac{x}{4}$$

$tg\,\alpha = \frac{1}{2}$

$$\cos\alpha = \frac{1}{\sqrt{1 + \frac{1}{4}}} = \frac{1}{\sqrt{\frac{5}{4}}} = \frac{\frac{1}{1}}{\frac{\sqrt{5}}{2}} = \frac{2}{\sqrt{5}}$$

$\cos\alpha = \frac{x}{4}$

$x = 4 \cdot \cos\alpha$

$x = 4 \cdot \frac{2}{\sqrt{5}}$

A $P_1(-3;2)$ ponton keresztül párhuzamost húzunk a $P_2(1;-2)$ és a $P_3(4;1)$ pontokon áthaladó egyenessel. Mekkora e párh. egyenesek távolsága?

$$d = \sqrt{32}.$$

1.) $A(3;2)$ $\quad AB = ?$

$B(-1;-1)$ $\quad BC = ?$

$C(11;-6)$ $\quad AC = ?$

$d = \sqrt{16+9} = \underline{5} = \underline{AB}$

$d = \sqrt{144+25} = \sqrt{169} = \underline{13} = \underline{BC}$

$d = \sqrt{64+64} = \sqrt{128} \doteq \underline{11{,}3} = AC$

2. $A(3;-7)$ $\quad$ Hat. meg a Δ old. felezőpontjait.

$B(5;2)$

$C(-1;0)$

$C' = (4;-2{,}5)$ $\qquad x = \frac{x_1+x_2}{2}$

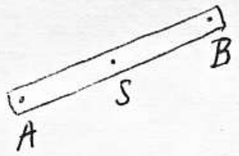
$A' = (2;1)$ $\qquad y = \frac{y_1+y_2}{2}$

$B' = (1;-3{,}5)$

Szám. ki ABC△ ter. $A(4;2)$ $B(9;4)$ $C(7;6)$

$$T=\frac{1}{2}[4(-2)+9(4)+7(-2)]=\frac{1}{2}(-8+36-14)=\frac{1}{2}(14)=\underline{7}$$

Egy homogén fémpálca súlypontja $S(5;1)$. Egyik végpontja $A(-5;-3)$. Szám ki a másik végpont koordinátái



$$x_s=\frac{x_A+x_B}{2}$$
$$2x_s=x_A+x_B$$
$$x_B=2x_s-x_A$$
$$x_B=10-(-5)$$
$$x_B=15$$

$$y_s=\frac{y_A+y_B}{2}$$
$$2y_s=y_A+y_B$$
$$y_B=2y_s-y_A$$
$$y_B=2-(-3)$$
$$y=5$$

$$\underline{B(15;5)}$$

5.) Írj. fel a $P(2;-3)$-n átmenő és a $8x-2y+3=0$ egyenessel $\parallel$ egyenes egyenletét.

$m_1=4$ $m_2=4$

$$y-y_1=m(x-x_1)$$
$$y+3=4(x-2)$$
$$y+3=4x-8$$
$$\underline{4x-y-11=0}$$

6.

⊥ Írj. fel. a $P(-1;4)$-n átm. és az $x-3y+8=0$ egyenre ⊥ egy. egyenletét.

$m=\frac{1}{3}$ $m_1'=-3$

$$y-y_1=m(x-x_1)$$
$$y=-3(x+1)+4$$
$$y=-3x-3+4$$
$$\underline{0=-3x-y+1}$$

Szám. ki a $6x-8y+3=0$ és $12x+9y-5=0$ egy. hajlásszögét.

$$6x-8y+3=0 \downarrow m=\frac{3}{4} \qquad 12x+9y-5=0 \downarrow m=-\frac{4}{3}$$

$$tg\,\omega=\frac{-\frac{4}{3}-\frac{3}{4}}{1+\frac{3}{4}\cdot-\frac{4}{3}}=\frac{-\frac{16+9}{12}}{1-\frac{12}{12}}=\frac{-\frac{25}{12}}{1-1}=\frac{-\frac{25}{12}}{0}=0$$

$$tg\,\omega=0 \rightarrow \underline{\omega=90}$$

8.

Szám. ki a $P(3;-1)$ p. távolságát a $3x-4y+2=0$ egyenestől.

$m_2=-\frac{4}{3} \leftarrow m_1=\frac{3}{4}$

$$y-y_1=m_2(x-x_1)$$
$$y+1=-\frac{4}{3}(x-3)$$
$$y+1=-\frac{4}{3}x+\frac{12}{3}$$
$$y=-\frac{4}{3}x+\frac{9}{3}$$
$$y=\frac{3x}{4}+\frac{2}{4}$$

$$-\frac{4}{3}x+\frac{9}{3}=\frac{3x}{4}+\frac{2}{4}$$
$$-16x+36=9x+6$$
$$30=25x$$

$$x=\frac{6}{5}$$
$$y=-\frac{4}{3}x+3$$
$$y=-\frac{4}{3}\cdot\frac{6}{5}+\frac{9}{3}$$
$$y=-\frac{24}{15}+\frac{9}{3}$$
$$y=-\frac{8}{5}+\frac{9}{3}$$
$$y=\frac{-24+45}{15}$$
$$y=\frac{21}{15}=\frac{7}{5}$$

$P_1\left(\frac{6}{5};\frac{7}{5}\right)$

$P(3;-1)$

$P_1\left(\frac{6}{5};\frac{7}{5}\right)$

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$d=\sqrt{\left(3-\frac{6}{5}\right)^2+\left(-1-\frac{7}{5}\right)^2}$$

$$d=\sqrt{\left(\frac{9}{5}\right)^2+\left(\frac{12}{5}\right)^2}$$

$$d=\sqrt{\frac{81}{25}+\frac{144}{25}}=\sqrt{\frac{225}{25}}=\frac{15}{5}=3$$

$$\underline{d=3}$$

9, Hat. meg a $P(2;0)$-n átmenő és a $2x-5y-6$ egyenletű egyenessel $45°$-os szöget bezáró egyenes egyenletét.

$2x-5y-6=0 \rightarrow m_1=\frac{2}{5}$ $\qquad tg\,\omega=1$

$$tg\,\omega=\frac{m_1-m_2}{1+m_1\cdot m_2}$$

$$1(1+m_1\cdot m_2)=m_1-m_2$$

$$1+m_2\cdot\frac{2}{5}=\frac{2}{5}-m_2$$

$$1+\frac{2}{5}m_2=\frac{2}{5}-m_2$$

$$\frac{2}{5}m_2+\frac{5}{5}m_2=\frac{2}{5}-\frac{5}{5}$$

$$\frac{7}{5}m_2=\frac{-3}{5}$$

$$35m_2=-15$$

$$m_2=-\frac{15}{35}=-\frac{3}{7}$$

$$y-y_1=m(x-x_1)$$

$$y-0=-\frac{3}{4}(x-2)$$

$$\underline{y=-\frac{3}{7}x+\frac{6}{7}}$$

1960 / 79; 80

21. Hf. 1967. X. 11.

I.) A $3x+4y+2=0$ egyenesre $\perp$ egyenes átmegy az $x-7y+13=0$ és a $7x+y-9=0$ egyenesek metszéspontján. Írj. fel ezen az egyenesnek az egyenletét.

$$\begin{aligned} x-7y+13&=0 \rightarrow x = 7y-13 \\ 7x+y-9&=0 \end{aligned}$$

$$3x+4y+2=0 \;\downarrow\; m_1 = -\frac{3}{4}$$

$$7(7y-13)+y-9=0$$
$$49y-91+y-9=0$$
$$50y=100$$
$$y=2$$

$$x=7y-13$$
$$x=7\cdot 2-13$$
$$x=14-13$$
$$x=1$$

$M(1;2)$

$$y-y_1 = m(x-x_1)$$
$$y-2=\frac{4}{3}(x-1)$$
$$y-2=\frac{4}{3}x-\frac{4}{3}$$
$$y=\frac{4}{3}x+\frac{2}{3}$$

$$\frac{4}{3}x-y+\frac{2}{3}=0$$
$$\underline{4x-3y+2=0}$$

II.) Szám. ki a köv. 2,2 egyenes által bezárt szög nagyságát.

a.) $2x-3y+4=0 \rightarrow \frac{2}{3}$
$x-y+1=0 \rightarrow 1$

$$\operatorname{tg}\omega = \frac{\frac{2}{3}-1}{1+1\cdot\frac{2}{3}} = \frac{-\frac{1}{3}}{\frac{5}{3}} = -\frac{3}{15} = -\frac{1}{5} = -0{,}2$$

$\alpha = 11°20'$ $\underline{\omega = 168°40'}$ ////

b.) $8x+15y+10=0 \rightarrow -\frac{8}{15}$
$y=0 \rightarrow 0$

$$\operatorname{tg}\omega = \frac{-\frac{8}{5}-0}{1+-\frac{8}{5}\cdot 0} = \frac{-\frac{8}{5}}{1} = -\frac{8}{5} = -1{,}6$$

$\alpha = 58°$ $\underline{\omega = 122°}$ /////

c.) $3x-7=0 \rightarrow -\frac{3}{0}$ $x=\frac{7}{3}$
$x+y+13=0 \rightarrow -1$ .... $\alpha = 45$ $\underline{\omega = 125°}$ ////

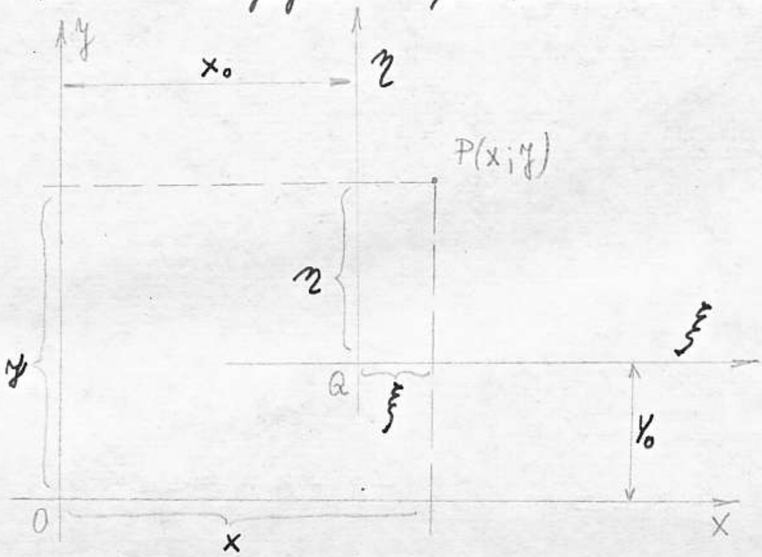
d.) $4x+5=0 \rightarrow \parallel y$
$y-7=0 \rightarrow \parallel x$ $\Big\} \rightarrow \underline{\omega = 90°}$

1967. X. 12.

22. óra.

Koordináta transzformáció

1. A koord. tengelyeket párhuzamosan eltoljuk.



P(x;y)
$Q(x_0;y_0)$

$$\boxed{\begin{aligned} \xi &= x - x_0 &\longrightarrow\quad x &= \xi + x_0 \\ \eta &= y - y_0 &\longrightarrow\quad y &= \eta + y_0 \end{aligned}}$$

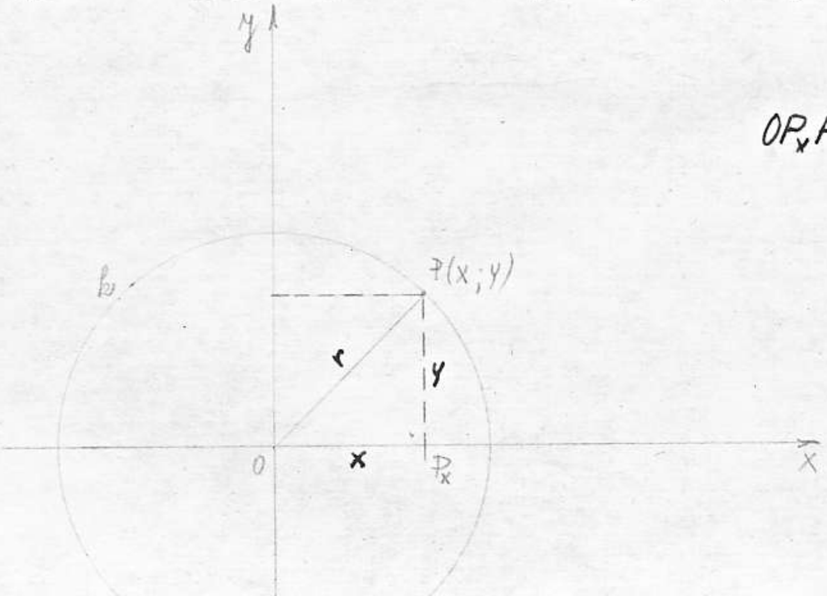
A kör egyenlete

A kör a sík. az. p. halm. melyek 1 ponttól azonos távra vannak. Középpontja, sugara.

$k \equiv (O; r)$

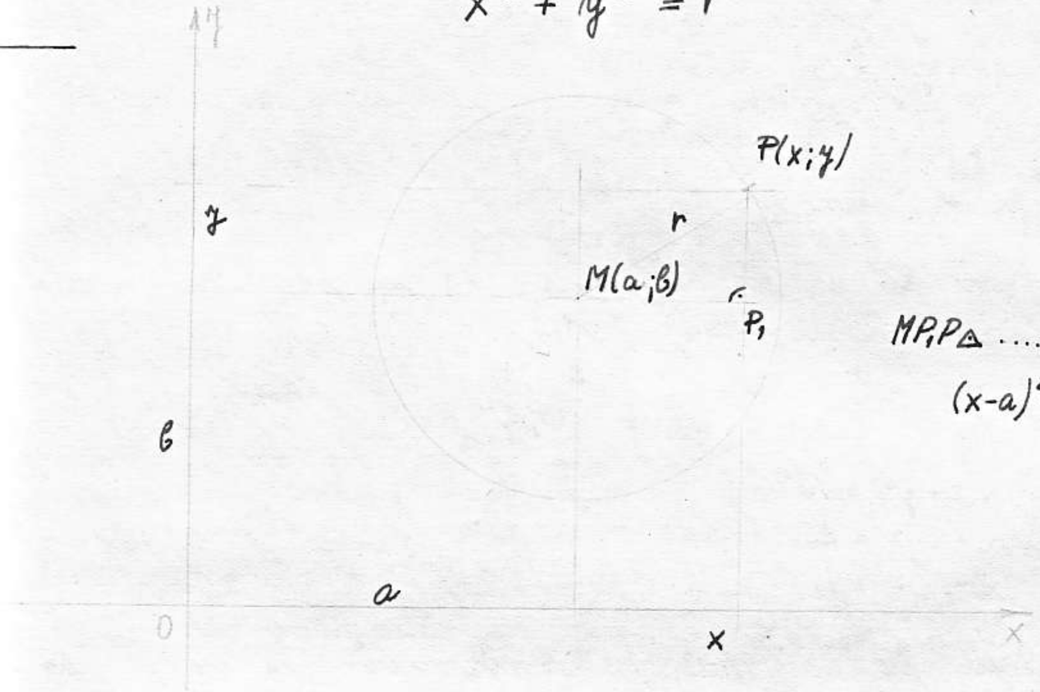
OP_xP △ ..... $OP_x^2 + P_xP^2 = OP^2$

$$x^2 + y^2 = r^2$$



A kör egyenlete ha középpontja a koordináta rendszer középpontja:

$$X^2 + y^2 = r^2$$



$k \equiv (M; r)$
$M(a; b)$

$MP_1P\triangle \ldots\ MP_1^2 + P_1P^2 = r^2$
$(x-a)^2 + (y-b)^2 = r^2$

A kör egyenlete ha középpontja nem azonos a koordináta r. kezdőpontjával:

$$(X-a)^2 + (y-b)^2 = r^2$$

23. óra

1967. X. 13.

$$X^2 + y^2 = r^2$$
$$(x-a)^2 + (y-b)^2 = r^2$$
$$x^2 + a^2 - 2ax + y^2 - 2by + b^2 = r^2$$
$$x^2 + y^2 - 2ax - 2by + (a^2 + b^2 - r^2) = 0$$

– másodfokú kiliomerellenű egyenlő együtthatójú egyenlet.

1.) Írjuk föl a kör egyenletét, ha középpontja az origóban van és sugara $r = 4$, $r = 5$, $r = 6$, $r = 7$.

$$x^2 + y^2 = 16$$
$$x^2 + y^2 = 25$$
$$x^2 + y^2 = 36$$
$$x^2 + y^2 = 49$$

2.) Írjuk fel a kör egyenletét, ha középpontja az origóban van és a kör egy pontja: $A(2;3)$
$B(3;4)$
$C(4;5)$

$$x^2 + y^2 = 13$$
$$x^2 + y^2 = 25$$
$$x^2 + y^2 = 41$$

3.) Írjuk fel az $M(-3;2)$ középpontú és $d = 8$ átmérőjű kör egyenletét.

$r = \frac{d}{2} = \frac{8}{2} = 4$

$r^2 = 16$

$$(x-a)^2 + (y-b)^2 = r^2$$
$$(x+3)^2 + (y-2)^2 = 16$$
$$x^2 + 6x + 9 + y^2 - 4y + 4 - 16 = 0$$
$$\underline{x^2 + y^2 + 6x - 4y - 3 = 0}$$

4.) Hol metszi X -t a o. $\searrow$ $y = 0$

$$x^2 + 6x - 3 = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{36 + 12}}{2} = \frac{-6 \pm \sqrt{48}}{2} =$$

$$= -3 \pm \sqrt{\frac{48}{4}} = -3 \pm \sqrt{12};$$

$\sqrt{12} \doteq 3,46$
$300 : 64 \cdot 4$
$4400 : 686 \cdot 6$
284

$x_1 \doteq -6,46$ $\quad$ $x_2 \doteq 0,46$

5.) A kör középpontja $M(4;7)$; sugara $r=6$.
Írjuk föl az egyenletet!

$$(x-a)^2+(y-b)^2=r^2$$
$$(x-4)^2+(y-7)^2=36$$
$$x^2-8x+16+y^2-14y+49=36$$
$$\underline{x^2+y^2-8x-14y+29=0}$$

6.) Írjuk föl a „O" egyenletét ha középpontja $M(6;2)$ és egy pontja $A(8;9)$.

$$(x-a)^2+(y-b)^2=r^2$$
$$(x-6)^2+(y-2)^2=r^2$$
$$x^2-12x+36+y^2-4y+4=53$$
$$\underline{\underline{x^2+y^2-12x-4y-13=0}}$$

$$d=r=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$r^2=(x_2-x_1)^2+(y_2-y_1)^2$$
$$r^2=(6-8)^2+(2-9)^2$$
$$r^2=4+49=53$$

24. óra 1967. X. 13.

7.) Hat. meg a kör középpontjának koordinátáit és a sugarát ha egyenlete:

$$x^2+y^2-6x+8y+14=0$$
$$\downarrow$$
$$(x-a)^2+(y-b)^2=r^2$$
$$(x^2-6x)+(y^2+8y)=-14$$
$$(x-3)^2+(y+4)^2=-14+9+16$$
$$\underline{(x-3)^2+(y+4)^2=11 \quad \rightarrow \quad a=3 \quad b=(-4) \quad r=\sqrt{11}}$$

8.)
$$x^2+y^2+12x-8y-12=0$$
$$(x+6)^2+(y-4)^2=12+36+16$$
$$\underline{(x+6)^2+(y-4)^2=64 \quad \rightarrow a=(-6) \quad b=4 \quad r=8}$$

9.) $x^2+y^2-20x-2y(\sqrt{69})=0$ $\quad a=?\quad b=?\quad r=?$

$$(x-10)^2+(y-\sqrt{69})^2=0+100+69=169$$

$$\underline{a=10\quad b=\sqrt{69}\quad r=13}$$

10.) Írj. fel azon kör egyenletét mely keresztülh. az A(3;7) ponton és koncentrikus az
$5x^2-5y^2+20x-45y+15=0$ körrel.

$$5x^2-5y^2+20x-45y=-15 \quad /\cdot\tfrac{1}{5}$$
$$x^2-y^2+4x-9y=-3$$
$$(x+2)^2-(y+4{,}5)^2=-3+4+20{,}25$$
$$(x+2)^2+[-(y+4{,}5)]^2=-19{,}25 \longrightarrow a=-2\quad b=-4{,}5)$$
$$(x+2)^2+(y+4{,}5)^2=21{,}25 \qquad M(-2;4{,}5)$$

A(3;7)
M(-2;-4,5)

$$(x+2)^2+(y+4{,}5)^2=r^2=(3+2)^2+(7+4{,}5)^2=25+132{,}25$$
$$\underline{(x+2)^2+(y+4{,}5)^2=157{,}25}$$

11. Írj fel a ○ egy.-t, mely áth. a köv. pontokon:

A(3;5)
B(-3;3)
C(-1;-3)

M(a;b)

$$\underline{(x-a)^2+(y-b)^2=r^2}$$
$$(3-a)^2+(5-b)^2=r^2=9-6a+a^2+25-10b+b^2 \quad (1)$$
$$(-3-a)^2+(3-b)^2=r^2=9+6a+a^2+9-6b+b^2 \quad (2)$$
$$(-1-a)^2+(-3-b)^2=r^2=1+2a+a^2+9+6b+b^2 \quad (3)$$

$(1)+(2)=2(3)$
$$9-6a+a^2+25-10b+b^2+9+6a+a^2+9-6b+b^2=$$
$$=2+4a+2a^2+18+12b+2b^2$$
$$-16b+52=4a+12b+20$$
$$4a+28b-32=0$$

$(1)+(3)=2(2)$
$$9-6a+a^2+25-10b+b^2+1+2a+a^2+9+6b+b^2=$$
$$=18+12a+2a^2+18-12b+2b^2$$
$$-4a-4b+44=12a-12b+36 \quad /\cdot\tfrac{1}{4}$$
$$-a-b+11=3a-3b+9$$
$$4a-2b-2=0$$

$$a + 7b - 8 = 0 \qquad a = 8 - 7b = 1$$
$$2a - b - 1 = 0 \qquad \underline{a = 1}$$
$$16 - 14b - b - 1 = 0$$
$$-15b = -15$$
$$\underline{b = 1}$$

$$r = \sqrt{(3-1)^2 + (5-1)^2} = \sqrt{4 + 16} = \sqrt{20}$$

$$\underline{\underline{(x-1)^2 + (y-1)^2 = \sqrt{20}^2 = 20}}$$

24. Hf

1967. X. 13.

$A(4;5)$
$B(-1;2)$
$C(-3;-4)$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$(4-a)^2 + (5-b)^2 = r^2 = 16 - 8a + a^2 + 25 - 10b + b^2$$
$$(-1-a)^2 + (2-b)^2 = r^2 = 1 + 2a + a^2 + 4 - 4b + b^2$$
$$(-3-a)^2 + (-4-b)^2 = r^2 = 9 + 6a + a^2 + 16 + 8b + b^2$$

$$a^2 + b^2 - 8a - 10b + 41 = a^2 + b^2 + 2a - 4b + 5$$
$$a^2 + b^2 - 8a - 10b + 41 = a^2 + b^2 + 6a + 8b + 25$$

$$36 = 10a + 6b$$
$$16 = 14a + 18b$$

$$18 = 5a + 3b \qquad 54 = 15a + 9b$$
$$8 = 7a + 9b \qquad \pm 8 = \pm 7a \pm 9b$$

$$46 = 8a \qquad b = \frac{5a - 18}{3} = \frac{\frac{115}{4} - \frac{72}{4}}{3}$$
$$\underline{\underline{a = \frac{23}{4}}} \qquad \underline{\underline{b = \frac{43}{12}}}$$

$$r^2 = \left(\frac{16 - 23}{4}\right)^2 + \left(\frac{60 - 43}{12}\right)^2$$
$$r^2 = \left(\frac{7}{4}\right)^2 + \left(\frac{17}{5}\right)^2 = \frac{49}{16} + \frac{289}{144}$$
$$\left(r^2 = \frac{1225 + 4624}{16 \cdot 25} = \frac{5849}{400} \doteq 14{,}6 \doteq r^2\right)$$

$$\frac{441 + 289}{144} = \frac{730}{144}$$

$$\underline{\left(x - \frac{23}{4}\right)^2 + \left(y - \frac{43}{12}\right)^2 = 14{,}6 \quad 5}$$

1967. X. 17.

25. óra

1. Állapítsuk meg az $x^2 + y^2 - 4x - 2y - 20 = 0$ egyenlettel adott kör középpontjának koordinátáit és sugarát!

$$(x-a)^2 + (y-b)^2 = r^2$$
$$(x-2)^2 + (y-1)^2 = 20 + 4 + 1$$
$$(x-2)^2 + (y-1)^2 = 25$$

$S = M(2;1)$

$r = 5$

2. Hat. meg az $A(-2;-1)$
$B(0;-5)$
$C(6;3)$ pontokon átmenő kör egyenletét.

$$(x-a)^2 + (y-b)^2 = r^2$$
$$(-2-a)^2 + (-1-b)^2 = r^2 = 4 + 4a + a^2 + 1 + 2b + b^2 \quad (1)$$
$$(0-a)^2 + (-5-b)^2 = r^2 = 0 - 0 + a^2 + 25 + 10b + b^2 \quad (2)$$
$$(6-a)^2 + (3-b)^2 = r^2 = 36 - 12a + a^2 + 9 - 6b + b^2 \quad (3)$$

(1)=(2) $\quad 4 + 4a + a^2 + 1 + 2b + b^2 = a^2 + 25 + 10b + b^2$

(1)=(3) $\quad 4 + 4a + a^2 + 1 + 2b + b^2 = 36 - 12a + a^2 + 9 - 6b + b^2$

$$4a + 2b + 5 = 10b + 25$$
$$4a + 2b + 5 = -12a - 6b + 45$$

$$4a - 8b - 20 = 0$$
$$16a + 8b - 40 = 0$$

$$20a = 60$$
$$a = 3 \qquad b = -1$$

$8b = 40 - 16a$
$8b = 40 - 48$
$8b = -8$
$b = -1$

$r^2 = 4 + 12 + 9 + 1 - 2 + 1 = 25$

$r = 5$

$$(x-3)^2 + (b+1)^2 = 25$$

3.) Milyen messze van az $A(6;7)$ p. az $x^2+y^2-6x+6y+2=0$ kör középpontjától.

$$(x-3)^2+(y+3)^2=-2+9+9$$
$$(x-3)^2+(y+3)^2=16$$

$M(3;-3)$ $\qquad d=\sqrt{(6-3)^2+(7+3)^2}$

$A(6;7)$ $\qquad d=\sqrt{9+100}=\sqrt{109}=\underline{10{,}4403}$

A kör és pont kölcsönös helyzete:

1.) A pont a körön kívül van:

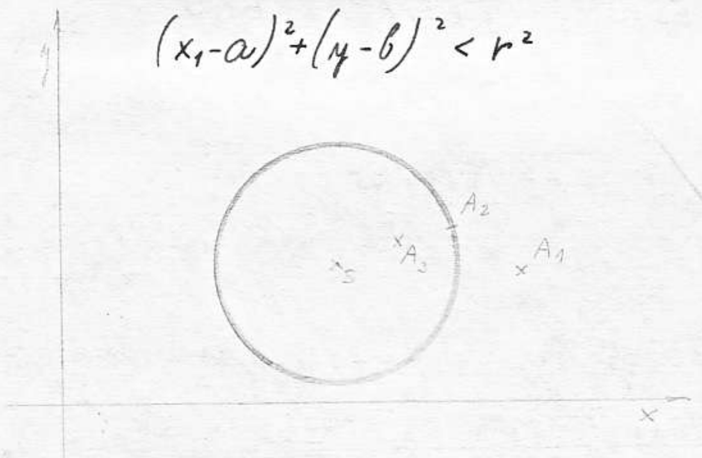
$$(x_1-a)^2+(y_1-b^2)>r^2$$

2.) A pont a kör kerületszerinti pontja:

$$(x_1-a)^2+(y_1-b)^2=r^2$$

3.) A pont a kör belsejében van:

$$(x_1-a)^2+(y-b)^2<r^2$$



A kör és az egyenes kölcsönös helyzete:

1.) nincs közös pont
2.) 1 közös pont (érintési pont) van
3.) Az egyenes a kört 2 pontban metszi.

A kör és egyenes kölcsönös helyzetét úgy határozzuk meg, hogy a kör és egyenes által meghatározott „másodfokú kétismeretlenes" egyenletrendszert megoldjuk.

Ha két különböző megoldása van, az egyenes a kör szelője.

Ha az egyenletrendszernek nincs valós megoldása, nincs közös pont.

Egy megoldás esetén egy közös pont van.

1, Hogyan helyezkedik el az $x^2+y^2-4x-8y+6$ körhöz képest az $A(-3;-4)$ pont.

$$(x-2)^2+(y-4)^2=-6+4+16$$
$$(x-2)^2+(y-4)^2=14$$
$$M(2;4) \quad r=\sqrt{14}$$
$$d=\sqrt{25+64}=\sqrt{89}=9{,}43$$

$d > r$

A pont a körön kívül van.

1967. X. 17. 25 H.

Geom.II.: 60/1 62/4 64/6 65/8
61/3 63/5 65/7 67/9

1. Hogyan helyezkedik el az $x^2+y^2-8x+12y+7=0$ körhöz képest a $P_1(6;-8)$ p.?

$$(x-4)^2+(y+6)^2=-7+16+36=45$$
$$(6-4)^2+(-8+6)^2=2^2+2^2=8$$

$8<45 \rightarrow$ A kör belsejében

3. Hat. meg az $x^2+y^2-2x-4y-15=0$ kör és a $3x-2y=7$ egyenes metszéspontjait

$$(x-1)^2+(y-2)^2=15+1+4=20 \qquad y=\frac{3x-7}{2}$$
$$(x-1)^2+\left(\frac{3x-7-4}{2}\right)^2=20$$
$$x^2-2x+1+\frac{9x^2}{4}-\frac{66x}{4}+\frac{121}{4}=20 \quad /\cdot 4$$
$$4x^2-8x+4+9x^2-66x+121-80=0$$
$$13x^2-74x+45=0$$

$74^2=5476$
4916
56 $\quad \sqrt{3136}=56$
636 : 106.6

180.13
540
2340

$$x_{1,2}=\frac{+74\pm\sqrt{5476-2340}}{26}=\frac{74\pm\sqrt{3136}}{26}=\frac{74\pm 56}{26}$$

$$x_1=\frac{130}{26}=\frac{65}{13}\doteq 5 \qquad y=\frac{8}{2}=4$$

$$x_2=\frac{18}{26}=\frac{9}{13}\doteq 0{,}7 \qquad y\doteq\frac{2{,}1-7}{2}\doteq -2{,}5$$

$$\frac{\frac{27}{13}-\frac{91}{13}}{2}=-\frac{64}{13\cdot 2}=-\frac{32}{13}$$

$P_1(5;4)$

$P_2\left(\frac{9}{13};-\frac{32}{13}\right)$

4.) Mi az egyenlete annak a körnek, amelynek középpontja a $2x + y = 8$ egyenesen fekszik, és amely átmegy a $P_1(4;5)$ és $P_2(6;7)$ pontokon?

$M \sim \overline{P_1P_2}$ felezőpontja $\rightarrow M(5;6)$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 5 = \frac{7-5}{6-4}(x-4)$$

$$y - 5 = \frac{2}{2}(x-4)$$

$$y = x + 1 \rightarrow m_1 = 1 \rightarrow m_2 = -1$$

$$y - y_1 = -1(x - x_1)$$

$$y - b = -x + a$$

$$\left.\begin{aligned} y + x - (a+b) &= 0 \\ y + 2x - 8 &= 0 \end{aligned}\right]$$

x, y - a kör. középp. koord.

$$(4-x)^2 + (5-y)^2 = (6-x)^2 + (7-y)^2$$

$$16 - 8x + x^2 + 25 - 10y + y^2 = 36 - 12x + x^2 + 49 - 14y + y^2$$

$$-8x - 10y + 41 = -12x - 14y + 85$$

$$4x + 4y - 44 = 0 \quad /:4$$

$$x + y - 11 = 0$$

$$\begin{aligned} -x \pm y \mp 11 &= 0 \\ 2x + y - 8 &= 0 \\ \hline x + 3 &= 0 \end{aligned}$$

$$\underline{x = -3}$$

$$y = 11 - x$$

$$y = 11 + 3 = \underline{14 = y}$$

$$(4+3)^2 + (5-14)^2 = r^2$$

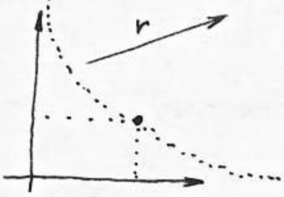
$$7^2 + 9^2 = r^2$$

$$49 + 81 = r^2$$

$$\underline{130 = r^2}$$

$$\underline{(x+3)^2 + (y-14)^2 = 130}$$

5. Írj. f. ann. a k. egyenletét, amely a $P_1(5;3)$ p.-on megy ker. és érinti a koord. tengelyeket. 1. síknegyed.



$$S \equiv M(r;r)$$

$$(x-r)^2+(y-r)^2=r^2$$

$$(5-r)^2+(3-r)^2=25$$

$$25-10r+r^2+9-6r+r^2=(25)\ r^2$$

$$r^2-16r+34=0$$

$$r_{1,2}=\frac{16\pm\sqrt{256-136}}{2} \qquad 256$$

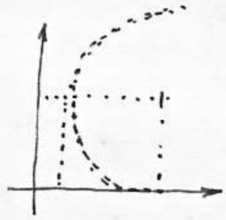
$$r_{1,2}=\frac{16\pm\sqrt{120}}{2}=\frac{16\pm\sqrt{120}}{2}=8\pm\sqrt{30} \qquad \sqrt{30}<8 \rightarrow r_1;\ r_2>0$$

$\downarrow$

$$(x-8-\sqrt{30})^2+(y-8-\sqrt{30})^2=(8+\sqrt{30})^2$$
$$(x-8+\sqrt{30})^2+(y-8+\sqrt{30})^2=(8-\sqrt{30})^2$$

6.)

Hat. meg az. kör egy. am. sug. 4 egys., átmegy $P_1(1;4)$ p-on és érinti X tengelyt.



$\rightarrow$ $M(5;4)$

$$(x-a)^2+(y-b)^2=r^2$$
$$(1-a)^2+(4-4)^2=r^2$$

$M_1(5;4)$
$M_2(-3;4)$

$$(x-5)^2+(y-4)^2=16$$

$$(x+3)^2+(y+4)^2=16$$

7.) Hat. meg a kör. kör.és a koord. t. teng. metszéspontjait:

$$x^2+y^2-8x+14y+6=0$$

1.) $x=0 \rightarrow y^2+14y+6=0$

$$y_{1,2}=\frac{-14\pm\sqrt{196-24}}{2}=\frac{-14\pm\sqrt{172}}{2}=\left(\frac{-14\pm}{\cdot}\right)$$

$$y_{1,2}=-7\pm\sqrt{43}$$

2.) $y=0$

$$x^2-8x+6=0$$

$$x_{1,2}=\frac{+8\pm\sqrt{64-24}}{2}=\frac{8\pm\sqrt{40}}{2}=4\pm\sqrt{10}$$

$P_1=(0;-7+\sqrt{43})$ $\quad P_2(0;-7-\sqrt{43})$

$P_3=(4+\sqrt{10};0)$ $\quad P_4=(4-\sqrt{10};0)$

8.) Mily. messz. van az $x^2+y^2-8x+8y+2=0$ k. középp. a $2x+3y=9$ egyenestől?

$$(x-4)^2+(y+4)^2=-2+16+16 \rightarrow S\equiv M(4;-4)$$

$3y=-2x+9$

$y=-\frac{2x}{3}+3 \rightarrow m_1=-\frac{2}{3} \rightarrow m_2=\frac{3}{2}$

$$y-y_1=m(x-x_1)$$

$$y+4=\frac{3}{2}(x-4)$$

$$y=\frac{3}{2}x-10$$

$2x+3y-9=0 \rightarrow x=\frac{-3y+9}{2}$

$3x-2y-20=0$

$-\frac{9y}{2}+\frac{27}{2}-2y-20=0 \quad /\cdot 2$

$-9y+27-4y-40=0$

$-13y-13=0$

$y=-1$

$x=6$

$A(6;-1)$

$S(4;-4)$

$$d=\sqrt{2^2+9}=\sqrt{13}$$

$$d=\sqrt{13}$$

9.) Az $x^2+y^2-10x-4y+16=0$. k.-nek mely . p-ja van legközelebb a $3y+2x=36$ egyeneshez?

$$(x-5)^2+(y-2)^2=-16+25+4=13 \rightarrow \underline{r=\sqrt{13}}$$

$$\downarrow \quad M(5;2)$$

$$y=\frac{-2x+36}{3}$$

$$\downarrow$$

$$m_1=-\frac{2}{3} \rightarrow m_2=\frac{3}{2}$$

$$y-y_1=m_2(x-x_1)$$

$$y-2=\frac{3}{2}(x-5)$$

$$y=\frac{3}{2}x-\frac{15}{2}+\frac{4}{2}=\frac{3}{2}x-\frac{11}{2}$$

$$\left.\begin{aligned} y&=\frac{3}{2}x-\frac{11}{2}\\ 3y&=-2x+36\\ y&=\frac{-2x}{3}+12 \end{aligned}\right\} \quad \frac{3}{2}x-\frac{11}{2}=\frac{-2x}{3}+12 \,/\cdot 6$$

$$9x-33=-4x+72$$

$$13x=105$$

$$x\doteq 8=\frac{105}{13}$$

$$y=\frac{3}{2}\cdot\frac{105}{13}-\frac{11}{2}$$

$$y=\frac{315}{26}-\frac{143}{26}=\frac{172}{26}=\frac{86}{13}$$

$$y=\frac{86}{13}\doteq 6{,}5$$

$$P_2\left(\frac{105}{13};\frac{86}{13}\right)$$

$$M(5;2)$$

$$d=\sqrt{\left(\frac{105}{13}-5\right)^2+\left(\frac{86}{13}-2\right)^2}-\sqrt{13}$$

$$d=\sqrt{\left(\frac{105-65}{13}\right)^2+\left(\frac{86-26}{13}\right)^2}-\sqrt{13}=\sqrt{\left(\frac{40}{13}\right)^2+\left(\frac{60}{13}\right)^2}-\sqrt{13}=$$

$$=\sqrt{\frac{1600}{169}+\frac{3600}{169}}-\sqrt{13}=\sqrt{\frac{5200}{169}}-\sqrt{13}\doteq\frac{\sqrt{5200}}{13}-\sqrt{13}\doteq\underline{2}$$

26 óra 1967. X. 18.

Két kör kölcsönös helyzete

1, sugarak összege ~~nagyobb~~ kisebb mint a középpontok távolsága

$r+R < \overline{S_1S_2}$

2., $r+R = \overline{S_1S_2}$

3., $r+R > \overline{S_1S_2}$

4., $R-r = \overline{S_1S_2}$ belülről érinti

5., a kisebb a nagyobbon belül van

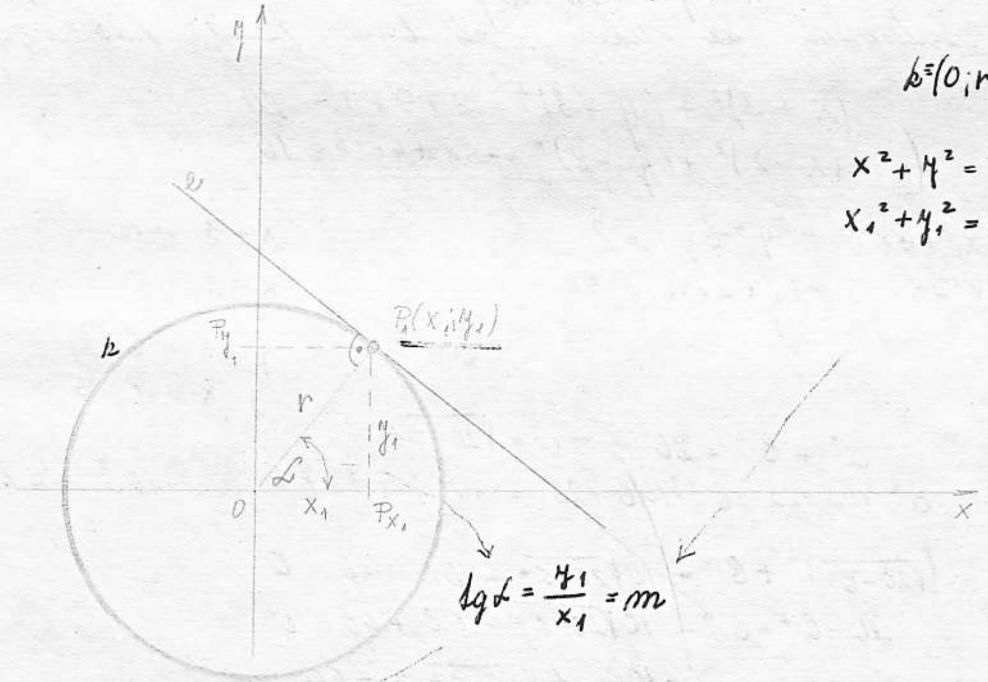
6., koncentrikus körök.

A kör érintője

Érintő $\perp$ az ér. ponton áthaladó sugár.

$k \equiv (0;r)$

$$\left.\begin{aligned} x^2+y^2&=r^2 \\ x_1^2+y_1^2&=r^2 \end{aligned}\right\} \text{mert } P_1 \in k$$



$$\operatorname{tg}\alpha = \frac{y_1}{x_1} = m$$

$P_1(x_1;y_1)$

$$m_1 = -\frac{1}{m_2} = -\frac{x_1}{y_1}$$

$$y-y_1 = m(x-x_1)$$

$$y-y_1 = -\frac{x_1}{y_1}(x-x_1)$$

$$y_1(y-y_1) = -x_1(x-x_1)$$

$$y y_1 - y_1^2 = -x x_1 + x_1^2$$

$$x x_1 + y y_1 = \boxed{x_1^2+y_1^2}$$

$$x \cdot x_1 + y \cdot y_1 = r^2$$

$$\underline{\underline{x \cdot x_1 + y \cdot y_1 = r^2}}$$

A kör érintőjének egyenlete, ha középpontja a kezdőpont.

$x_1 ; y_1$ ~ érintési pont koordin.

1, Írjuk fel az $x^2+y^2=5$ kör $P_1(1;-2)$ pontjához húzott érintőjének egyenletét!

$$x \cdot x_1 + y \cdot y_1 = r^2$$
$$x \cdot 1 + y(-2) = 5$$
$$x - 2y = 5$$
$$\underline{x - 2y - 5 = 0}$$

2, Geom. 2.

Hat. meg az $x^2+y^2+6x-6y-2$ egyenlő 0 és az $x^2+y^2-4x-16y+58 = 0$ körök metszéspontjait, valamint az ezeken áthaladó közös húr egyenletét.

$$(x+3)^2+(y-3)^2 = 2+9+9 = 20$$
$$(x-2)^2+(y-8)^2 = -58+4+64 = 10$$

$$(x^2+6x)+(y^2-6y) = 2$$
$$(x^2-4x)+(y^2-16y) = -58$$

$$x+3 = a$$
$$x-2 = a-5$$
$$y-3 = b$$
$$y-8 = b-5$$

$$a^2+b^2 = 20 \longrightarrow a = \sqrt{20-b^2}$$
$$a^2-10a+25+b^2-10b+25 = 10 \;;\; a^2+b^2-10a-10b = -40$$

$$(\sqrt{20-b^2})^2+b^2 - 10\sqrt{20-b^2}-10b+40 = 0$$
$$20-b^2+b^2 - 10\sqrt{20-b^2}-10b+40 = 0$$
$$-10b-10\sqrt{20-b^2} = -60$$
$$10b+10\sqrt{20-b^2} = 60$$
$$10\sqrt{10-b^2} = 60-10b \quad /^2$$
$$100(10-b^2) = 3600-1200b+100b^2$$
$$1000-100b^2 = 3600-1200b+100b^2$$
$$0 = 200b^2-1200b+2600$$
$$0 = 2b^2-12b+26$$
$$0 = b^2-6b+13$$

$$b_{1,2} = \frac{6 \pm \sqrt{36-52}}{2} = 3 \pm \frac{\sqrt{-16}}{2}$$

$b_1 = 3 + \frac{\sqrt{-16}}{2}$ $\quad b_2 = 3 - \frac{\sqrt{-16}}{2}$ $\quad b = y - 3 \Rightarrow y = b+3$

$y_1 = 3+2i+3$ $\quad y_2 = 3-2i+3$

$$\underline{y_1 = 6+2i \quad y_2 = 6-2i}$$

$a = \sqrt{20-b^2}$

$(3+2i)^2 = 9+6i+6i-4 = 5+12i$

$(3+2i)(3+2i)$

$a = \sqrt{20-(3\pm 2i)^2}$

$(3-2i)(3-2i) = 9-6i-6i-4 = 5-12i$

$a_1 = \sqrt{20-(3+2i)^2}$

$a_1 = \sqrt{20-5+12i}$ $\quad a_2 = \sqrt{20-5+12i}$ $\quad x = a-3$

$x^2+y^2+6x-6y-2=0$

$x^2+y^2-4x-16y+58=0$

(1)-(2) $\quad 10x+10y-60=0$

$5x+5y-30=0$ – a szelő egyenlete

$y = \frac{6-x}{1} = 6-x$

$(x+3)^2+(6-x-3)^2=20$

$x^2+6x+9+9-6x+x^2=20$

$2x^2=2$

$x = \pm 1$

27. ó.

Gyak.

1967. X. 20.

28. 29 óra.

$x^2+y^2=r^2$ $\quad P(x_1,y_1)$ $\quad x_1\cdot x+y_1\cdot y=r^2$

$(x-a)^2+(y-b)^2=r^2$ $\quad P(x_1;y_1)$, $S(a;b)$ $\quad \underline{\underline{(x-a)(x_1-a)+(y-b)(y_1-b)=r^2}}$

Levezetés: $\quad \xi^2+\eta^2=r^2$

$\downarrow$

érintő: $\xi_1\cdot\xi+\eta_1\cdot\eta=r^2$

behely.: $(x-a)(x_1-a)+(y-b)(y_1-b)=r^2$

1./ Írjuk fel az $(x-1)^2+(y-2)^2=25$ kör $P_1(5;5)$ pontjához szerkesztett érintő egyenletét!

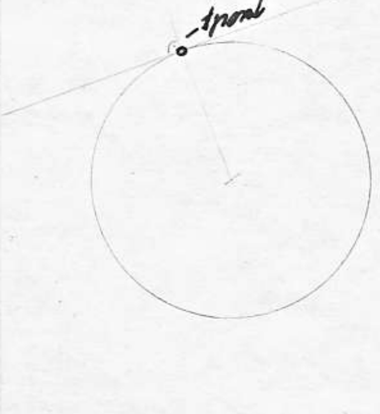
$P_1(5;5)$
$S(1;2)$
$r=5$

$$(x-a)(x_1-a)+(y-b)(y_1-b_1)=r^2$$
$$(x-1)(5-1)+(y-2)(5-2)=25$$
$$(x-1)\cdot 4+(y-2)\cdot 3=25$$
$$4x-4+3y-6=25$$
$$\underline{4x+3y=35}$$

2./ Írjuk fel annak a körnek az egyenletét, mely középpontja $S(6;4)$ és $y=3x-4$ egyenest érinti.

$y-3x=-4 \;/\cdot(-1)$

$3x-y=4$



$$(x-a)^2+(y-b)^2=r^2$$
$$3x-y=4$$
$$(x-6)^2+(y-4)^2=r^2$$
$$y=3x-4$$
$$(x-6)^2+(3x-4-4)^2=r^2$$
$$x^2-12x+36+9x^2-48x+64=r^2$$
$$10x^2-60x+100=r^2$$
$$x^2-6x+10=\frac{r^2}{10}$$
$$x^2-6x+\left(10-\frac{r^2}{10}\right)=0$$

$$x_{1,2}=\frac{6\pm\sqrt{36-4\cdot\left(10-\frac{r^2}{100}\right)}}{2}$$

$$\underline{(x-6)^2+(y-4)^2=10\cancel{0}}$$

$D=0 \rightarrow 36-4\left(10-\frac{r^2}{100}\right)=0$

$36-40\cancel{0}+\frac{r^2}{25}=0$

$r_1^2=100$

$\underline{\underline{r_1=10}}$

$\underline{r^2=10}$

3., Írj. föl annak a körnek az egyenletét, melynek középpontja $S(5;7)$ és az $y=2x-6$ egyenest érinti.

$$(x-5)^2+(y-7)^2=r^2$$
$$y=2x-6$$
$$(x-5)^2+(2x-13)^2=r^2$$
$$169-52x+x^2-10x+25+4x^2=r^2$$
$$5x^2-62x+(194-r^2)=0$$
$$x_{1,2}=\frac{62\pm\sqrt{62^2-4\cdot5(194-r^2)}}{10}$$
$$D=0=\sqrt{62^2-20\cdot(194-r^2)}$$

$$0=D=\sqrt{3844-3880+20r^2}=\sqrt{-36+20r^2}$$
$$0=-36+20r^2\rightarrow r^2=\frac{36}{20}=\frac{18}{10}=\frac{9}{5}$$
$$(x-5)^2+(y-7)^2=\frac{9}{5}$$

74/2 4., Írj. f. az $x^2+y^2=41$ körnek azokat az érintőit, amelyek párh. az $5x+4y=12$ egyenessel.

$\nwarrow q$

$5x+4y=12\rightarrow m_1=-\frac{5}{4}\quad m_2=\frac{4}{5}$

$a\perp q\equiv y-y_1=m_2(x-x_1)$

$y=\frac{4}{5}x$

$$x^2+y^2=41$$
$$x^2+\frac{16}{25}x^2=41$$
$$41x^2=41\cdot25$$
$$x^2=25$$
$$x=\pm5$$
$$y^2=41-x^2$$
$$y^2=41-25$$
$$y^2=16$$
$$y=\pm4$$

$P_1(5;4)$

$P_2(-5;-4)$

$$y-y_1=m(x-x_1)$$
$$y-4=-\frac{5}{4}(x-5)$$
$$y-4=-\frac{5}{4}x+\frac{5\cdot5}{4}$$
$$\underline{y=-\frac{5}{4}x+\frac{41}{4}}$$

$$y-y_1=m(x-x_1)$$
$$y+4=-\frac{5}{4}(x+5)$$
$$y+4=-\frac{5x}{4}-\frac{25}{4}$$
$$\underline{y=-\frac{5x}{4}-\frac{41}{4}}$$

Hat. meg az $x^2+y^2=25$ körnek az $x=4$ abszcisszájú pontjaihoz tartozó érintői egyenletét.

$$y^2=-x^2+25$$
$$y^2=-16+25$$
$$y^2=9$$
$$y=\pm\sqrt{9}$$
$$y=\pm3$$

$P_1(4;3)$ $m_1=-\frac{4}{3}$

$P_2(4;-3)$

$r_1\equiv y=\frac{3}{4}x$

$r_2\equiv y=-\frac{3}{4}x$ $m_2=\frac{4}{3}$

$$y-y_1=m_1(x-x_1)$$
$$y-3=-\frac{4}{3}(x-4)$$
$$y-3=-\frac{4}{3}x-\frac{16}{3}$$
$$\underline{y=-\frac{4}{3}x-\frac{7}{3}}$$

$$y-y_1=m_2(x-x_1)$$
$$y+3=\frac{4}{3}(x-4)$$
$$\underline{y=\frac{4}{3}x-\frac{25}{3}}$$

77.4 Hat. meg az $x^2+y^2-12x-6y+25=0$ körnek az $x_1=10$ absz. tartozó pontjait valamint az ezekhez húzható érintők egyenleteit.

$$(x-6)^2+(y-3)^2=-25+36+9=20$$
$$(10-6)^2+(y-3)^2=20$$
$$y^2-6y=-9+20-16$$
$$y^2-6y+5=0$$

$$y_{1,2}=\frac{6\pm\sqrt{36-20}}{2}=\frac{6\pm4}{2}$$
$$y_1=5;\quad y_2=1$$
$$P_1(10;5)\quad P_2(10;1)\quad S(6;3)$$

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$
$$y-3=\frac{5-3}{10-6}(x-6)$$
$$y=3+\frac{1}{2}x-3$$
$$y=\frac{1}{2}x\quad\ldots P_1(10;5)$$

$$y-y_1=m(x-x_1)$$
$$y-5=-2(x-10)$$
$$y=-2x+25$$

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$
$$y-3=\frac{1-3}{10-6}(x-6)$$
$$y=3-\frac{1}{2}x+3$$
$$y=-\frac{1}{2}x+6\quad\ldots P_2(10;1)$$

$$y-y_1=m(x-x_1)$$
$$y-1=2(x-10)$$
$$y=2x-19$$

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30. óra.

Ismétlés, felelés.

1.) Szám. ki a Δ területét ha $A(-2;-1)$ $B(1;6)$ $C(6;1)$.

$$T=\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]=$$
$$=\frac{1}{2}\left[-2(6-1)+1(1+1)+6(-1-6)\right]=$$
$$=\frac{1}{2}\left[-10+2-42\right]=\left|\frac{1}{2}\cdot(-50)\right|=\underline{25}$$

2.) Mi az egyenlete annak az egyenesnek amely a $(4;5)$ ponton megy keresztül és a $3x+2y=12$ egyenessel párh.

$$m_1=m_2=-\frac{3}{2}$$

$$y-y_1=m(x-x_1)$$
$$y-5=-\frac{3}{2}(x-4)$$
$$y=-\frac{3}{2}x+6+5$$
$$\underline{y=-\frac{3}{2}x+11}$$

3.) Mi az egy. ann. az egy., mely álm. az origón és az $5x - 7y = 35$ egyenesre merőleges?

$$m_1 = \frac{5}{7} \rightarrow m_2 = -\frac{7}{5}$$

$$y - y_1 = m(x - x_1)$$
$$y = mx$$
$$\underline{y = -\frac{7}{5}x}$$

4.) Mily. távra van a $(2;5)$ pont a $3x + 4y = 6$ egyenestől?

$$m_1 = -\frac{3}{4} \rightarrow m_2 = \frac{4}{3}$$

$$y - 5 = \frac{4}{5}(x - 2)$$
$$y - 5 = \frac{4}{5}x - \frac{8}{5} \quad /\cdot 5$$
$$5y - 25 = 4x - 8$$
$$y = \frac{4}{5}x + \frac{17}{5}$$

$$y - 5 = \frac{4}{3}(x - 2)$$
$$y - 5 = \frac{4}{3}x - \frac{8}{3}$$
$$y = \frac{4}{3}x + \frac{7}{3} \quad /\cdot 3$$

$$3x + 4y = 6$$
$$3y - 4x = 7$$

$$3x + 4y = 6 \quad /\cdot 4$$
$$-4x + 3y = 7 \quad /\cdot 3$$

$$12x + 16y = 24$$
$$-12x + 9y = 21$$
$$25y = 45$$
$$y = \frac{45}{25} = \frac{9}{5}$$

$$3x = 6 - 4y$$
$$3x = 6 - \frac{36}{5} \quad /\cdot 5$$
$$15x = 30 - 36$$
$$15x = -6$$
$$x = -\frac{2}{5}$$

II. $A\left(-\frac{2}{5}; \frac{9}{5}\right)$

I. $B(2;5)$

$$d = \sqrt{\left(-\frac{2}{5} - 2\right)^2 + \left(\frac{9}{5} - 5\right)^2} = \sqrt{\left(-\frac{12}{5}\right)^2 + \left(-\frac{16}{5}\right)^2} = \sqrt{\frac{144 + 256}{25}}$$

$$d = \frac{\sqrt{400}}{5} = \frac{20}{5} = \underline{4 = d}$$

Mi az egy ann. a körnek mely álm. a $(8;5)$-n és középp. a $2x + y + 1 = 0$ és $3x - y + 9 = 0$ egyenesek metszéspontjában van.

$$2x + y = -1$$
$$3x - y = -9$$
$$5x = -10$$
$$x = -2$$

1. $y = -1 - 2x$

$$y = -1 + 4$$
$$y = 3$$

$M(-2;3)$ $\quad d^2 = r^2 = (8+2)^2 + (5-3)^2 = 100 + 4 = 104$

$A(8;5)$

$$\underline{(x+2)^2 + (y-3)^2 = 104}$$

6.1 Mi az egy. ann. a körnek mely a (-1;1) (4;2) (4;-4) pontokon megy keresztül.

$$(x-a)^2+(y-b)^2=r^2$$

$$(-1-a)^2+(1-b)^2=r^2$$

$$\left.\begin{cases}(4-a)^2+(2-b)^2=r^2\\(4-a)^2+(-4-b)^2=r^2\end{cases}\right\} \quad 4-4b+b^2=16+8b+b^2$$

$$-12b=12$$

$$b=-1$$

$$1+2a+a^2+4=16-8a+a^2+9$$

$$10a=20$$

$$a=2$$

$S(2;-1)$

$$(x-2)^2+(y+1)^2=13$$

$$9+4=13$$

Szám. ki az $(x-1)^2+(y-2)^2=20$ kör és a $3x-2y=7$ egyenes metszéspontjainak koordinátáit.

$$(x-1)^2+\left(\frac{3x-7}{2}-2\right)^2=20$$

$$x^2-2x+1+\frac{9x^2-66x+121}{4}=20 \quad /\cdot 4$$

$$4x^2-8x+4+9x^2-66x+121=80$$

$$13x^2-74x+45=0$$

$$x_{1,2}=\frac{74\pm\sqrt{2476-2340}}{26}=\frac{74\pm\sqrt{136}}{26}$$

$$x_1\doteq\frac{74+11{,}6}{26}\doteq\frac{85{,}6}{26}\doteq 3{,}2 \quad x_2\doteq\frac{62{,}6}{26}\doteq 2{,}4$$

$$\left(\frac{2y+7}{3}-1\right)^2+(y-2)^2=20$$

$$\frac{4y^2+16y+16}{9}+y^2-4y+4=20 \quad /\cdot 9$$

$$4y^2+16y+16+9y^2-36y+36=180$$

$$13y^2-20y-128=0$$

$$y_{1,2}=\frac{20\pm\sqrt{400+6656}}{26}=\frac{20\pm 84}{26}$$

$$y_1=4 \quad y_2=-2{,}4$$

$P_1(2{,}4;4)$ $P_2(3{,}2;-2{,}4)$

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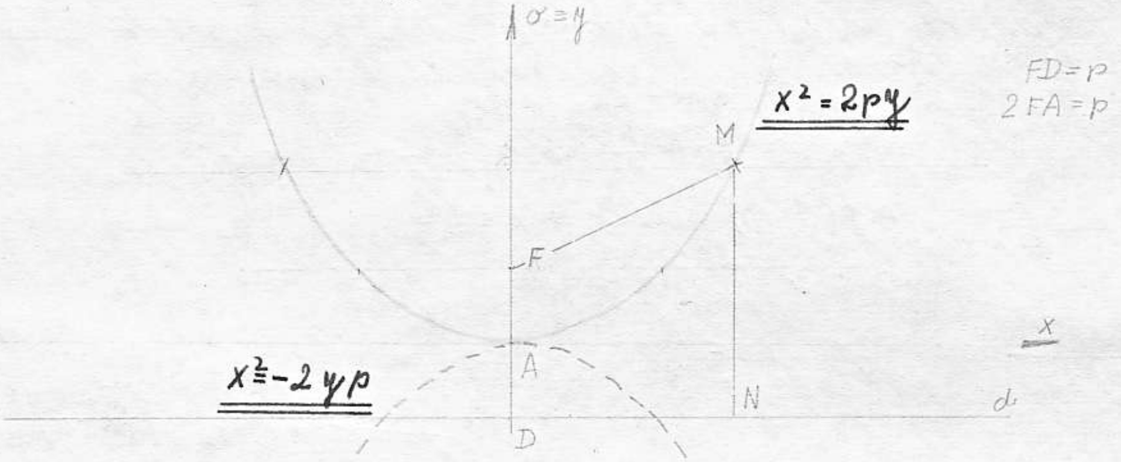
31. óra

A parabola egyenletei

A par. a sík azon pontjainak mértani helye, melyek egy adott egyenestől és egy adott ponttól egyenlő távolságra vannak. Gyújtópont, vezérvonal, direktrix, paraméter



$MF = MN$; $M(x;y)$

$F(0;\frac{p}{2})$ $\quad MF = d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\quad d = NF = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$N(x;-\frac{p}{2})$ $\quad \searrow = \swarrow$

$$\sqrt{(x-0)^2+(y-\frac{p}{2})^2} = \sqrt{(x-x)^2+(y+\frac{p}{2})^2} \quad /^2$$

$$(x-0)^2+(y-\frac{p}{2})^2 = (x-x)^2+(y+\frac{p}{2})^2$$

$$x^2+y^2-yp+\frac{p^2}{4} = y^2+py+\frac{p^2}{4}$$

$$\underline{x^2 = 2py}$$

A parabola egyenlete ha csúcsa a koordinátarendszer origója, tengelye az y tengely

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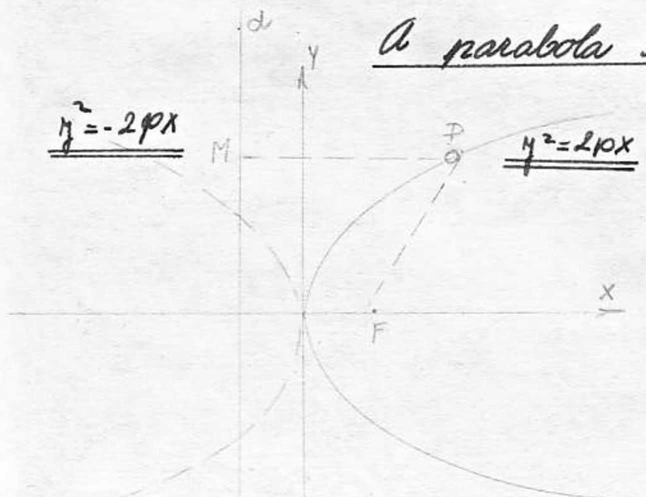
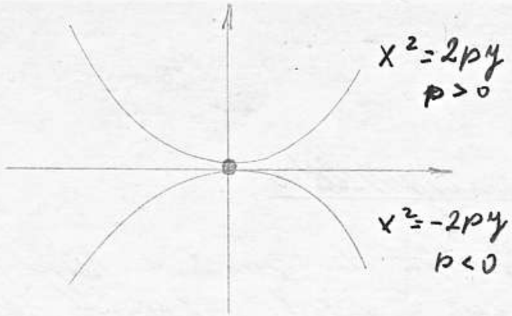
A parabola egyenletei

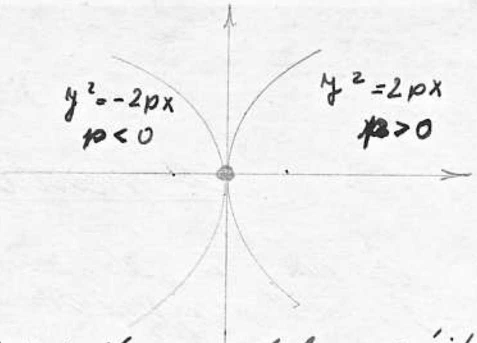


$F(\frac{p}{2};0)$ $\quad d = d$

$P(x;y)$ $\quad FP = MP.$

$M(-\frac{p}{2};y)$ $\quad \underline{y^2 = 2px}$

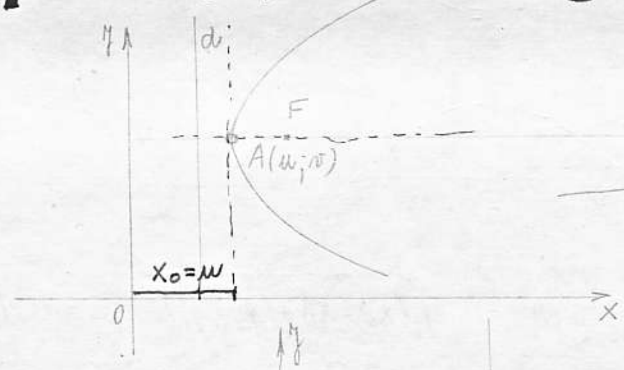




1, Hat. meg az $M(3;2)$ ponton áthaladó parabola gyújtópontjának koordinátáit és a direktrix egyenletét. A parabola csúcsa egybeesik az origóval, a tengelye pedig az x tengely.

$$y^2=2px \rightarrow 2p=\frac{y^2}{x} \rightarrow p=\frac{y^2}{2x}=\frac{4}{6}=\frac{2}{3};$$

$$p=\frac{2}{3} \rightarrow F\left(\frac{1}{3};0\right) \rightarrow d \equiv x=-\frac{2}{3}$$

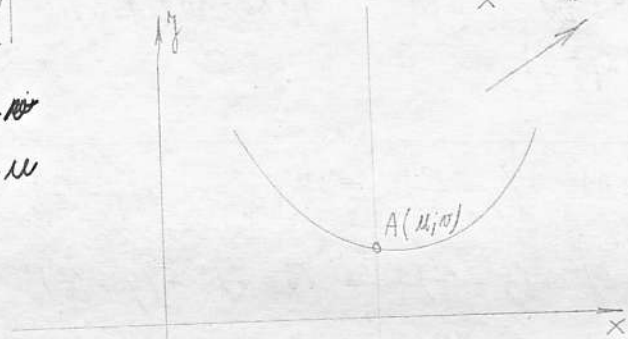


$$(y-v)^2=2p(x-u)$$

$$(x-u)^2=2p(y-v)$$

$\eta = y-v$

$\xi = x-u$



$$y^2-2vy+v^2=2px-2pu$$
$$\underline{y^2-2vy-2px+v^2+2pu=0}$$

$$x^2-2ux+u^2=2py-2pv$$
$$\underline{x^2-2ux-2py+u^2+2pv=0}$$

Hat. meg a parabola egyenletét, ha csúcsa az origóban van és $p=4$. Tengelye az x tengely pozitív fele.

$$y^2=2px \rightarrow \underline{y^2=8x}$$

Hat. meg a parabola gyújtópontjának koordinátáit és a vezérvonal egyenletét, ha $y=-\frac{1}{8}x^2 \rightarrow x^2=-8y$.

$$x^2=-8y \rightarrow p=\frac{-8}{2}; \quad F\left(0;-\frac{4}{2}\right) \qquad y=4 \equiv d$$