1. óra 1966. IX. 1

Tankönyvek szétosztása

2. óra 1966. IX. 2

Ismétlés

1966. IX. 6.

3. óra

Ismétlés

$x_1 = 1$
$x_2 = 2$

$x_1 \cdot x_2 = q$

$x_1 + x_2 = -p$

$x^2 - 3x + 2 = 0$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = 1,2$

$y = x^2 \quad y = 3x - 2$ grafikon.

$\frac{5-3x}{3-5x} + \frac{3-5x}{5-3x} = \frac{5}{2}$

$7x^2 - 50x + 7 = 0$

$x_1 = 7 \quad x_2 = \frac{1}{7}$

185/16 b c d Házi feladat 1966. IX. 6.

$\frac{5}{x-2} + \frac{3}{x-3} - \frac{7}{x-1} = 0$

$x^2 + 6x - 21 = 0$

$x_1 \doteq \frac{5}{2} \quad x_2 \doteq -8{,}5$

$\frac{4}{x+4} - \frac{4}{x-4} + \frac{x^2-20}{x^2-16} = 0$

$9x^2 + 48 = 0$

$x_{1,2} = \pm\frac{7}{3}$

$\frac{2}{x-2} - \frac{7}{x+1} = \frac{3}{x}$

$12x^2 - 5x - 3 = 0 \quad x = -\frac{1}{2}$

4. óra 1966. IX. 7.

Ismétlés

$2 - \frac{3x-58}{6} > 4x$

$x < 3\frac{2}{27}$

$\left(3x + 5 - \frac{1}{4}\right)\left(10x + 34\right) < 0$

$x < 1\frac{3}{4}$

1966. IX. 7. **Házi feladat**

79 56 cde
58 abcd

$\frac{2x+1}{x+2} > 1$ $\quad x > 1$

$\frac{4x-1}{x+3} > 2$ $\quad x > 3\frac{1}{2}$

$x > 3\frac{1}{2}$

$\frac{3x-2}{x+1} < 1$ $\quad x < \frac{3}{2}$

$\frac{5x-1}{x+6} < 1$ $\quad x < 1\frac{3}{4}$

$x < \frac{3}{2}$

$|x-2| < 5$

$|x+4| > 9$

$x > x \geqq 2$

$\frac{x+a}{5} - \frac{x-5}{a} = 2$ $\quad x = a-5$

$\frac{a}{3+x} = \frac{2}{x}$ $\quad x = \frac{6}{a+2}$

$a-4 + \frac{2}{x-1} = 0$ $\quad x = \frac{a+6}{a-4}$

$\frac{a-1}{a} \cdot x = -a+1$ $\quad x = -\frac{a+1}{1} = 1-a$

1966. IX. **5. óra**

$a = 5$ $\quad a = 7 \; b = 3$ $\quad x_1 = 2-\sqrt{2}$ $\quad x_1 = \frac{3}{2} - \sqrt{5}$ $\quad \sqrt[5]{a^{-3}\sqrt{3}}$

$m_a = 4$ $\quad c = 6 \; d = 2{,}5$ $\quad x_2 = \frac{3}{4}$

$b = 6$ $\quad x_2 = -1$ $\quad \sqrt{x^8 y^2 \sqrt{xy}}$

1966. IX. 9. **6. óra**

$y = a^x \; ; \; a > 0$ Exponenciális függvény

y = függő változó
x = független "
a = alap

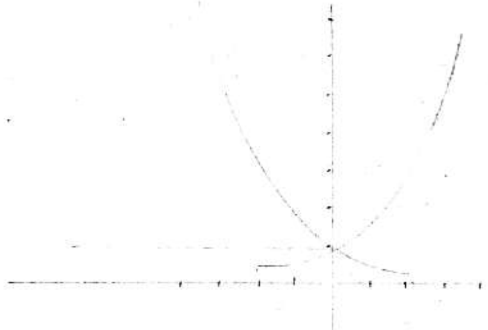
Az olyan függvényt, amelyben a független változó a kitevőben szerepel exponenciális függvénynek nevezzük.

$y = 2^{x}$ $a > 1$

$y = 2^{-x} = \left(\frac{1}{2}\right)^{x}$ $0 < a > 1$

$a = 1$

x	-4	-3	-2	-1	0	$\frac{1}{2}$	1	2	3	4
y	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	$\sqrt{2}$	1	4	8	1
y'	16	8	4	2	1	$\sqrt{2}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$
y''	1	1	1	1	0	1	1	1	1	1



Az exponenciális f. képe az exponenciális görbe.

1. Az x tengely fölött fekszik
2. Keresztül halad a 0;1 ponton
3. Ha $a > 1$ az $y = a^x$ f. növekvő
4. Ha $a = 1$ a f. képe egy egyenes, mely keresztül halad az y tengely 1-es pontján és párhuzamos az x tengellyel.
5. Ha $a < 1$ a függvény csökkenő.

7 óra 1966. IX. 10.

$|x - 7| < 2$ $9 > x > 7$

$x > 5$ $9 > x > 5$

$x_1 = \frac{3}{4}$ $x^2 + \frac{11}{20}x - \frac{3}{20}$

$x_2 = -\frac{1}{5}$

$20x^2 + 11x - 3 = 0$

A logaritmus fogalma

$3 \cdot 3 = 3^2 = 9$ | $3^2 = x$

$x^2 = 9$ $x = \sqrt{9}$ | $x^2 = 9$ $x = \log_3 9$

$3^x = 9$ | $3^x = 9$ $\log_3 9 = 2$

A logaritmus az a szám amelyre az alapot fel kell emelni, hogy az adott számot megkapjuk.
A logaritmus kitevő. Negatív szám logaritmusa nincs értelmezve,
Egy pedig nem lehet. $a > 0$; $a \neq 1$

$a^x = m$ $x = \log_a m$

1966. IX. 15.

8. óra

$\log_5 25 = 2$ mert $5^2 = 25$

$\log_2 32 = 5$ " $2^5 = 32$

$\log_4 \frac{1}{16} = -2$ " $4^{-2} = 1/16$

$\log_5 1/125 = -3$ " $5^{-3} = 1/125$

$\log_7 49 = 2$ " $7^2 = 49$

$\log_3 27 = 3$ " $3^3 = 27$

$\log_x 1 = 0$ "

$5^x = 25$

$x = \log_5 25$

$\log_a 1 = 0$ $\quad a \neq 1$, $a > 1$

Egynek a logaritmusa bármilyen alap mellett egyenlő nullával.

$\log_a a = 1$

Az alap logaritmusa mindig egyenlő eggyel.

Negatív számnak és 0-nak nincs értelmezve a logaritmusa.

$\log_2 16 = 4$ mert $2^4 = 16$

$\log_6 \frac{1}{36} = -2$ mert $6^{-2} = 1/36$

$\log_2 \sqrt[3]{2} = \frac{1}{3}$ mert $2^{\frac{1}{3}} = \sqrt[3]{2}$

$\log_2 \sqrt[3]{8} = \frac{3}{3} = 1$ mert $2^1 = 2$

$3^2 = x \ (9)$

$(3)\ x^2 = 9 \quad x = \sqrt[2]{9}$

$3^x = 9 \quad \log_3 9 = x$

1966. IX. 15.

8. Házi feladat

118/21 22 23

$\log_2 \sqrt[3]{\frac{1}{8}} = \frac{-3}{3} = -1$ mert $2^{-1} = \frac{1}{2} = \sqrt[3]{\frac{1^3}{2^3}} = \sqrt[3]{\frac{1}{8}}$

$\log_5 \frac{1}{\sqrt{125}} = -\frac{3}{2}$ " $5^{-\frac{3}{2}} = \frac{1}{5^{\frac{3}{2}}} = 1/\sqrt{5^3}$

$\log_8 8 = 1$ " $8^1 = 8$

$\log_a a^2 = 2$ " $a^2 = a^2$

$\log_3 3^m = m$ " $3^m = 3^m$

$\log_{12} 12^x = x$ " $12^x = 12^x$

21. $\log_7 49 = 2$ mert $7^2 = 49$

$\log_{10} 0{,}01 = -2$ " $10^{-2} = 1/100$

$\log_2 1/4 = -2$ " $2^{-2} = 1/4$

$\log_{1/2} 4 = -2$ " $\left(\frac{1}{2}\right)^{-2} = 1/{1/4} = 4$

$\log_{10} 0{,}1 = -1$ " $10^{-1} = 1/10$

22. $\log_2 x = 4 \quad x = 16$

$\log_{10} x = -1 \quad x = 0{,}1$

$\log_{\frac{1}{3}} x = -2 \quad x = 9$

$\log_{\sqrt{2}} x = 4 \quad x = 4$

$\log_5 x = 0 \quad x = 1$

$\log_6 x = 1 \quad x = 6$

$\log_{\frac{1}{2}} x = -1 \quad x = 2$

$\log_{0{,}1} x = -1 \quad x = 10$

23. $\log_x 216 = 3 \quad x^3 = 216 \quad x = \sqrt[3]{216} \quad x = 6$

$\log_x 1/27 = 3 \quad x = 1/3$

$\log_x 1/64 = -3 \quad x = 4$

$\log_x \sqrt{8} = 3/4 \quad x = 4$

$\log_x 10 = 6 \quad x = \sqrt[6]{10}$

$\log_x 3 = 3 \quad x = \sqrt[3]{3}$ mert $(\sqrt[3]{3})^3 = 3$

$\log_x \frac{1}{2} = \frac{1}{2} \quad x = \sqrt[1/2]{1/2}$

$\log_x m = m \quad x = \sqrt[m]{m}$

9. óra 1966. IX. 16.

Műveletek logaritmussal

a) Szorzat logaritmusa:

$a > 0$, b } ismert

$$a^x = b \qquad x = \log_a b$$
$$a^y = c \qquad y = \log_a c$$
$$a^{x+y} = b \cdot c \qquad x + y = \log_a bc$$
$$\log_a bc = x + y$$
$$\log_a bc = \log_a b + \log_a c$$

A szorzat logaritmusa egyenlő a tényezők logaritmusainak összegével.

Pl.: $x = k l m n \qquad \log x = \log(k \cdot l \cdot m \cdot n) = \log k + \log l + \log m + \log n$

$x = 2 \cdot 3 \qquad \log x = \log 2 + \log 3$

$$a^x = b \qquad x = \log_a b$$
$$a^y = c \qquad y = \log_a c$$
$$\frac{a^x}{a^y} = \frac{b}{c} \qquad x - y = \log_a b - \log_a c$$
$$a^{x-y} = \frac{b}{c} \qquad \log_a \frac{b}{c} = x - y$$
$$\log_a \frac{b}{c} = \log_a b - \log_a c$$

Hányados, tört logaritmusa egyenlő a számláló és a nevező logaritmusainak a különbségével.

Pl: $x = \frac{a \cdot b}{c}$

$$\log x = \log \frac{a \cdot b}{c} = \log(a \cdot b) - \log c = \log a + \log b - \log c$$

$$x = \frac{k}{m \cdot n}$$

$$\log x = \log\left(\frac{k}{m \cdot n}\right) = \log k - \log m - \log n$$

9. Házi feladat

122/1 abcde

1966. IX. 16.

9. Házi feladat

$$\log_2 a2 = \log_2 a + \log_2 2 = \log_2 a + 1$$
$$\log_2 a 2^2 = \log_2 a + \log_2 2^2 = \log_2 a + 2$$
$$\log_2 a 2^3 = \log_2 a + \log_2 2^3 = \log_2 a + 3$$
$$\log_2 a/2 = \log_2 a - \log_2 2 = \log_2 a - 1$$
$$\log_2 a/2^2 = \log_2 a - \log_2 2^2 = \log_2 a - 2$$

1966. IX. 17.

10. óra Gyakorlat

$$\begin{array}{c} a^x = b \\ a^y = c \\ \hline a^{x+y} = bc \end{array} \qquad \begin{array}{l} x = \log_a b \\ y = \log_a c \end{array} \qquad x + y = \log_a bc$$

$$\log_a(bc) = \log_a b + \log_a c$$

$$\begin{array}{c} a^x = b \\ a^y = c \\ \hline a^{x-y} = b/c \end{array} \qquad \begin{array}{l} x = \log_a b \\ y = \log_a c \end{array} \qquad x - y = \log_a \frac{b}{c}$$

$$\log_a \frac{b}{c} = \log_a b - \log_a c$$

A hatvány logaritmusa

$$a^x = b \quad /^n \qquad x = \log_a b$$
$$a^{xn} = b^n \qquad x\, n = \log_a b^n$$

$$\underline{\underline{\log_a b^n = n . \log_a b}}$$

Hatvány logaritmusa egyenlő az alap logaritmusának és a kitevőnek a szorzatával.

Gyök logaritmusa

$$a^x = b \quad /\sqrt{\ } \qquad x = \log_a b$$
$$\sqrt[n]{a^x} = \sqrt[n]{b}$$
$$a^{\frac{x}{n}} = \sqrt[n]{b} \qquad \frac{x}{n} = \log_a \sqrt[n]{b}$$

$$\frac{\log_a(b)}{n} = \log_a \sqrt[n]{b} \qquad \underline{\underline{\log_a \sqrt[n]{b} = \frac{1}{n} \cdot \log_a b}}$$

A gyök logaritmusa egyenlő a gyökjel alatti mennyiség logaritmusának és a gyökkitevőnek a hányadosával.

$$\log_{2} 2\cdot\sqrt{a} = \log_2 2 + \log_2\sqrt{a} = 1 + \frac{1}{2}\cdot\log_2 a$$
$$\log_2 \sqrt{a\cdot 2} = \frac{1}{2}\log_2 a\cdot 2 = \frac{1}{2}(\log_2 a + \log_2 2) = \frac{1}{2}(\log_2 a + 1).$$

$$T_\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$
$$\log T = \frac{1}{2}(\log[s(s-a)(s-b)(s-c)])$$
$$\log T = \frac{1}{2}[\log s + \log(s-a) + \log(s-b) + \log(s-c)].$$

$$x = \frac{1}{4}\sqrt{a\cdot\sqrt{b}}$$
$$\log x = \log\frac{1}{4}\cdot\sqrt{a\cdot\sqrt{b}} = \log\frac{1}{4} + \log\sqrt{a\cdot\sqrt{b}} = \log 1 - \log 4 + \frac{1}{2}\log a\sqrt{b} =$$
$$= \log 1 - \log 4 + \frac{1}{2}(\log a + \log\sqrt{b}) = \log 1 - \log 4 + \frac{1}{2}\log a + \frac{1}{4}\log b.$$

122. o. 2, 3 10. Házi feladat 1966. IX. 17.

$$\log_2 ab = \log_2 a + \log_2 b$$
$$\log_2 a^2 b = \log_2 a^2 + \log_2 b = 2\log_2 a + \log_2 b$$
$$\log_2 a^2 b^2 = 2\log_2 a + 2\log_2 b$$
$$\log_2 a/b = \log_2 a - \log_2 b$$
$$\log_2 a^2/b = 2\log_2 a - \log_2 b$$
$$\log_2 a/b^2 = \log_2 a - 2\log_2 b$$
$$\log_2 a\sqrt{b} = \log_2 a + \frac{1}{2}\log_2 b$$
$$\log_2 \sqrt{a^2} = \log_2 a$$
$$\log_2 \sqrt{a^3}\cdot\sqrt[5]{b^4} = \frac{3}{2}\log_2 a + \frac{4}{5}\log_2 b$$
$$\log_2 \sqrt[3]{a}/b^2 = \frac{1}{3}\log_2 a - 2\log_2 b$$
$$\log_2 \frac{a 2^2}{b} = \log_2 a + 2 - \log_2 b$$
$$\log_2 2^2/ab^2 = 2 - \log_2 a - 2\log_2 b$$
$$\log_2 2^2/a\sqrt{b} = 2 - \log_2 a - \frac{1}{2}\log_2 b$$
$$\log_2 \sqrt{abc} = \frac{1}{2}(\log_2 a + \log_2 b + \log_2 c)$$
$$\log_2 \sqrt[3]{\frac{b^2}{2}} = \frac{1}{3}(2\log_2 b - \log_2 2) = \frac{1}{3}(2\log_2 b - 1) = \frac{2}{3}\log_2 b - \frac{1}{3}$$

$$\log_2(abc) = \log_2 a + \log_2 b + \log_2 c$$
$$\log_2 5cd = \log_2 5 + \log_2 c + \log_2 d$$
$$\log_2 3(x+y) = \log_2 3 + \log_2(x+y)$$
$$\log_2(a^2-b^2) = \log_2(a+b) + \log_2(a-b)$$
$$\log_2 2(x^2-y^2) = \log_2 2 + \log_2(x+y) + \log_2(x-y)$$
$$\log_2 2ab/c = \log_2 2 + \log_2 a + \log_2 b - \log_2 c$$
$$\log_2\frac{2(a+b)}{3(a-b)} = \log_2 2 + \log_2(a+b) - \log_2 3 - \log_2(a-b)$$
$$\log_2\frac{s(s-a)(s-b)(s-c)}{4abc} = \log_2 s + \log_2(s-a) + \log_2(s-b) + \log_2(s-c) -$$
$$- \log_2 4 - \log_2 a - \log_2 b - \log_2 c$$
$$\log_2 3a^2 = \log_2 3 + 2\log_2 a$$
$$\log_2 4a^2 b = \log_2 4 + 2\log_2 a + \log_2 b$$
$$\log_2 12a^2b^3c^5 = \log_2 12 + 2\log_2 a + 3\log_2 b + 5\log_2 c$$
$$\log_2\frac{2m^2}{3(m^2-1)} = \log_2 2 + 2\log_2 m - \log_2 3 - \log_2(m+1) - \log_2(m-1)$$
$$\log_2 2^3 \lg\alpha/abc = \cancel{3\log_2 2} = 3 + \log_2\lg\alpha - \log_2 a - \log_2 b - \log_2 c$$

$$\log_2 \frac{10(a^2-b^2)}{3c^2d^4} = \log_2 10 + \log_2(a+b) + \log_2(a-b) - \log_2 3 - 2\log_2 c - 4\log_2 d$$

$$\log_2 a\sqrt{b} = \log_2 a + \tfrac{1}{2}\log_2 b$$

$$\log_2 a\sqrt[3]{b} = \log_2 a + \tfrac{1}{3}\log_2 b$$

$$\log_2 3a\sqrt[3]{3b^2} = \log_2 3 + \log_2 a + \tfrac{1}{3}\log_2 3 + \tfrac{2}{3}\log_2 b$$

$$\log_2 \sqrt[3]{\frac{a}{b}} = \tfrac{1}{3}\log_2 a - \tfrac{1}{3}\log_2 b$$

$$\log_2 \sqrt[4]{\frac{a^3}{b}} = \tfrac{3}{4}\log_2 a - \tfrac{1}{4}\log_2 b$$

$$\log_2 3a\sqrt[5]{a^3(a+b)^2} = \log_2 3 + \log_2 a + \tfrac{3}{5}\log_2 a + \tfrac{2}{5}\log_2(a+b)$$

$$\log_2 15r^2\cdot\sqrt[3]{2r^2(p-q)^3} = \log_2 15 + 2\log_2 r + \tfrac{1}{3}\log_2 2 + \tfrac{2}{3}\log_2 r + \log_2(p-q)$$

$$\log_2 \frac{3m^2n}{4\cdot\sqrt{5mn}} = \log_2 3 + 2\log_2 m + \log_2 n - \log_2 4 - \tfrac{1}{2}\log_2 5 - \tfrac{1}{2}\log_2 m - \tfrac{1}{2}\log_2 n$$

$$\log_2 \frac{6a\sqrt{2(a-b)c}}{5(a-b)^2} = \log_2 6 + \log_2 a + \tfrac{1}{2}\log_2 2 + \tfrac{1}{2}\log_2(a-b) + \tfrac{1}{2}\log_2 c - \log_2 5 - \log_2(a-b) - \log_2(a-b).$$

$$\log_2\left(\sqrt[5]{\frac{a}{2b}}\right)^3 = \tfrac{3}{5}\left[\log_2 a - \log_2 2 - \log_2 b\right]$$

$$\log_2\left(\sqrt[4]{\frac{ab}{3b}}\right)^3 = \tfrac{3}{4}\left[\log_2 a - \log_2 3\right]$$

1966. IX. 26.

11. óra

A logaritmikus függvény ábrázolása

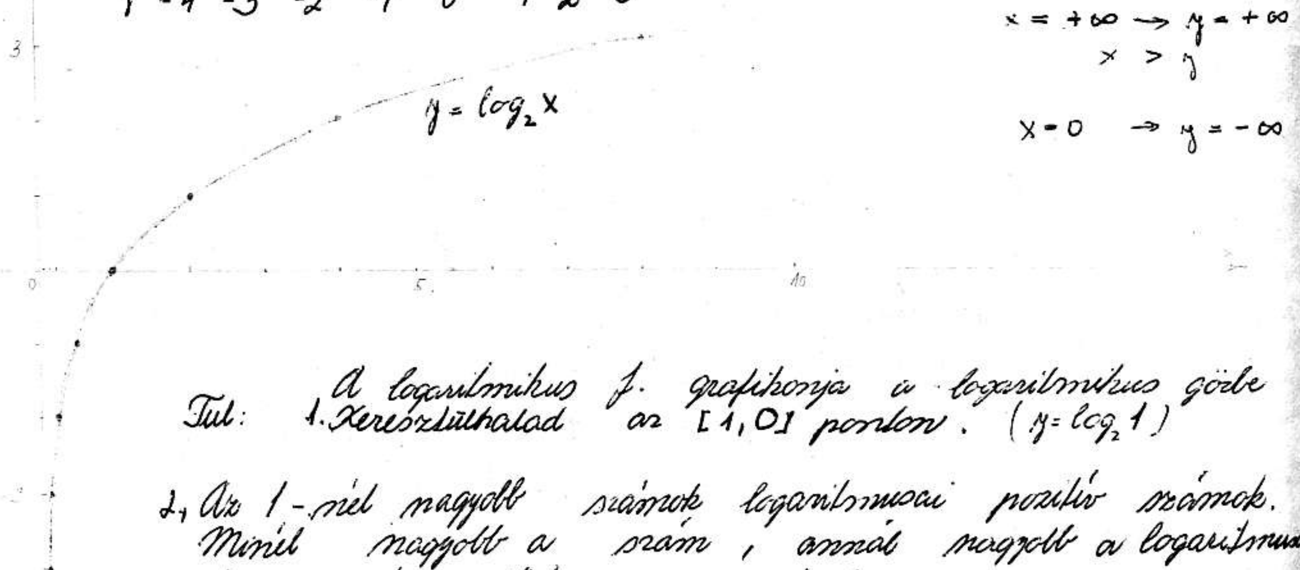
$y = \log_a x \qquad a > 0 \qquad a \neq 1 \qquad (a^y = x)$

x	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
y	-4	-3	-2	-1	0	1	2	3

$x = +\infty \rightarrow y = +\infty$

$x > y$

$x = 0 \rightarrow y = -\infty$



Tul: A logaritmikus f. grafikonja a logaritmikus görbe

1. Keresztülhalad az $[1,0]$ ponton. $(y = \log_2 1)$

2., Az 1-nél nagyobb számok logaritmusai pozitív számok. Minél nagyobb a szám, annál nagyobb a logaritmusa, de nem (egyenlő) egyenes arányban.

3., Az 1-nél kisebb pozitív számok logaritmusai negatív számok.

0-nak, negatív szám logaritmusai nincsenek értelmezve (komplex számok)

12. óra Gyakorlat 1966. IX. 28.

1.) $x = \dfrac{\sqrt{a\sqrt{a\sqrt{a}}}}{\sqrt[3]{a^3\sqrt{a}}}$ $\quad \log x = \log \dfrac{\sqrt{a}\cdot\sqrt[4]{a}\cdot\sqrt[8]{a}}{\sqrt[3]{a}\cdot\sqrt[9]{a}} = \frac{1}{2}\log a + \frac{1}{4}\log a + \frac{1}{8}\log a - \frac{1}{3}\log a - \frac{1}{9}\log a$

$\log x = \log \dfrac{\sqrt[8]{a^4}\cdot\sqrt[8]{a^2}\cdot\sqrt[8]{a}}{\sqrt[9]{a^3}\cdot\sqrt[9]{a}} = \log \dfrac{\sqrt[8]{a^7}}{\sqrt[9]{a^4}} = \frac{1}{8}\log a^7 - \frac{1}{9}\log a^4$

2.) $x = \dfrac{a+b}{x^2-y^2} \Rightarrow \log x = \log \dfrac{a+b}{x^2-y^2} = \log(a+b) - \log(x+y) - \log(x-y)$

3.) $\log v = \frac{3}{4}\log 5 + 7\log a \qquad v = 5^{\frac{3}{4}}\cdot a^7$

Tízes alapú logaritmus

Ugyanazon alapra vonatkozó logaritmusok logaritmusrendszert alkotnak.

Briggs ... 10 (dekadikus)
Napier e (természetes)

$\log 1 = 0$ $\qquad \log 0{,}1 = -1$
$\log 10 = 1$ $\qquad \log 0{,}01 = -2$
$\log 100 = 2$ $\qquad \log 0{,}001 = -3$
.... $\qquad$ ....

$$10^2 < 108 < 10^3$$
$$\log 10^2 < \log 108 < \log 10^3$$
$$2 < \log 108 < 3 \qquad \log 108 = 2, \ldots\ldots$$

Minden szám logaritmusa egy egész számú részből - karakterisztika és egy tizedes részből - mantissza áll.
A mantisszát keressük a táblázatban.

$\log 1966 = 3{,}\overset{2\,9\,3}{\ldots}$

1966. IX. 30. 13. óra

$$\log\frac{\sqrt[4]{425^6\cdot a^7}}{\sqrt{425^9\cdot a^{14}}}=\log\frac{\sqrt[4]{425^6}\cdot\sqrt[4]{a^7}}{\sqrt{425^9}\cdot\sqrt{a^{14}}}=\frac14\cdot6\log425+\frac14\cdot7\log a-9\cdot\frac12\log425-\frac12\cdot14\log a=$$
$$=\frac32\log425+\frac74\log a-\frac92\log425-7\log a\ .$$

$$\log\frac{\sqrt[4]{425^6\cdot a^7}}{\sqrt{425^9\cdot a^{14}}}=\log\sqrt[4]{\frac{425^6\cdot a^7}{425^{18}\cdot a^{28}}}=\log\sqrt[4]{\frac{1}{425^{12}\cdot a^{21}}}=\frac14\left(\log1-12\log425-21\log a\right)=$$
$$=-3\log425-\frac{21}{4}\log a\ .$$

$$\log x=\log\frac ab+\log\frac bc-\log\frac{ay}{cz}$$
$$x=\frac{\frac ab\cdot\frac bc}{\frac{ay}{cz}}=\frac{\frac ac}{\frac{ay}{cz}}=\frac{acz}{acy}=\frac zy\ .$$

$$\log\sqrt[4]{\frac{3x}{5y}\cdot\sqrt{\frac{2a^4b^3}{3c}}\cdot\sqrt[3]{\frac{a\cdot\sqrt c}{b^2\cdot\sqrt x}}}=\frac14\log\frac{3x}{5y}\cdot\sqrt{\frac{2a^4b^3}{3c}}\cdot\sqrt[3]{\frac{a\cdot\sqrt c}{b^2\cdot\sqrt x}}=$$
$$=\frac14\left(\log\frac{3x}{5y}+\log\sqrt{\frac{2a^4b^3}{3c}}+\log\sqrt[3]{\frac{a\cdot\sqrt c}{b^2\cdot\sqrt x}}\right)=$$
$$=\frac14\left[\log3x-\log5y+\frac12\log\frac{2a^4b^3}{3c}+\frac13\log\frac{a\cdot\sqrt c}{b^2\cdot\sqrt x}\right]=$$
$$=\frac14\left[\log3+\log x-\log5-\log y+\frac12\left(\log2a^4b^3-\log3c\right)+\frac13\left(\log a\cdot\sqrt c-\log b^2\sqrt x\right)\right]=$$
$$=\frac14\log3+\frac14\log x-\frac14\log5-\frac14\log y+\frac18\left(\log2+\log a^4+\log b^3-\log3-\log c\right)+$$
$$+\frac1{12}\left(\log a+\log\sqrt c-\log b^2+\log\sqrt x\right)=$$
$$=\frac14\log3+\frac14\log x-\frac14\log5-\frac14\log y+\frac18\log2+\frac12\log a+\frac38\log b-\frac18\log3-\frac18\log c+$$
$$+\frac1{12}\log a+\frac1{24}\log c-\frac16\log b+\frac1{24}\log x.$$

1966. IX. 30. 13. H. f.

$$x_1={}^{\frac23}\!\sqrt{a^6b^{2x-1}c^5}\cdot\sqrt{a^{2x-6}bc^3}\qquad x_2=\left\{\left[\left(a^{\frac12}\right)^{\frac12}\right]^{\frac12}\right\}^{\frac12}$$

1) $x=\sqrt[3]{(a^6b^{2x-1}c^5)^2}\cdot\sqrt{a^{2x-6}bc^3}=\sqrt[6]{(a^6b^{2x-1}c^5)^4}\cdot\sqrt[6]{(a^{2x-3}bc^3)^3}=\sqrt[6]{(a^{24}b^{8x-4}c^{20})(a^{6x-9}b^3c^9)}=$

$=\sqrt[6]{a^{6x+15}\cdot b^{8x-1}\cdot c^{29}}\qquad \log x=\frac16\cdot(6x+15)\log a+\frac16(8x-1)\cdot\log b+\frac16\cdot29\cdot\log c.$

2) $x=a^{\frac1{16}}=\sqrt[16]{a}\qquad \log x=\frac1{16}\log a.$

14. óra 1966. X. 1.

Gyakorlat MO példák

$10^x = 123$ $x = \log 123$

$10^2 = 100$ $10^3 = 1000$ $\Rightarrow 2 < x < 3$

$\log 123 = 2,\alpha$

$\log 20 = 1,3010$ $\longrightarrow 10^{1,301} = 20$
$\log 205 = 2,3118$

15. óra 1966. X. 5.

Számolás logarléccel

16. óra Gyakorlat 1966. X. 6.

A logaritmusos egyenletek

Az olyan egyenletet amelyben az ismeretlen logaritmusa fordul elő, logaritmusos egyenletnek nevezzük.

$\log x = 3,\not{0}2175$
$x = 1650$

$\log(2+3x) = \log 20$ elhagyjuk a logaritmust: $\log 10 = \log 10 \rightarrow 10 = 10$
$2+3x = 20$
$x = 6$

$\log(2x+12) + \log(3x-8) = \log 18$
$\log[(2x+12)(3x-8)] = \log 18$
$(2x+12)(3x-8) = 18$
$6x^2+36x-16x-96-18 = 0$
$6x^2+20x-114 = 0$
$3x^2+10x-57 = 0$

$x_{12} = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-10 \pm \sqrt{100+684}}{6} =$
$= \frac{-10 \pm \sqrt{784}}{6} = \frac{-10 \pm 28}{6} =$

$x_1 = \frac{18}{6} = 3$ $x_2 = \frac{-38}{6} = -6,\dot{3}$

1966. X. 7.

17. óra

$\log(x-13) - \log(x-3) = 1 - \log 2$

$\log \frac{x-13}{x-3} = \log \frac{10}{2}$

$x - 13 = 5x - 15$

$2 = 4x$

$x = \frac{1}{2}$

$\log 12{,}5 - \log 2{,}5 = \log 10 - \log 2$

$\vdots$

$\log 5 = \log 5$

$\log(7x+6) = 1 + \log 3(x - 4$

$\log(7x+6) = \log 10(3x-4)$

$7x + 6 = 30x - 40$

$46 = 23x$

$x = 2$

$\log 20 = 1 + \log 2$

$\log 20 = \log 10 \cdot 2$

$\log 20 = \log 20$

$\log(x+2) - \log(x-1) = 2 + \log(x+2)$

$\log \frac{(x+2)}{(x-1)} = \log 100(x+2)$

$\frac{(x+2)}{(x-1)} = 100x + 200$

$x + 2 = (x-1)(100x+200)$

$x + 2 = 100x^2 - 100x + 200x - 200$

$0 = 100x^2 + 99x - 202$

$0 = x^2 + 0{,}99x - 2{,}02$

$x_{12} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q} = -\frac{0{,}99}{2} \pm \sqrt{\frac{0{,}99^2}{4} + 2{,}02} = -0{,}495 \pm \sqrt{\frac{0{,}9801}{4} + 2{,}02} =$

$= -0{,}495 \pm \sqrt{0{,}245 + 2{,}02} = -0{,}495 \pm \sqrt{2{,}265} = -0{,}495 \pm 1{,}505$

$x_1 = -2{,}000$; $x_2 = 1{,}01$

$18.$ óra

1966. X. 8.

$$\log x^2 + \log x^3 + \log x^4 + \log x^5 = 6$$
$$\log x^{14} = 6$$
$$14 \cdot \log x = 6$$
$$\log x = \frac{3}{7}$$
$$\log x = 0{,}42857$$
$$x \doteq 2{,}68$$

$$5 \cdot \log \sqrt[3]{x} - 4 \cdot \log \sqrt[6]{x} + \frac{1}{2} \log x^8 = 9 - \log x^5$$
$$\log \sqrt[3]{x^5} - \log \sqrt[6]{x^4} + \log x^4 = 9 - \log x^5$$
$$5/3 \log x - \frac{2}{3} \log x + 4 \cdot \log x + 5 \log x = 9$$
$$10 \log x = 9$$
$$\log x = 0{,}9$$
$$x \doteq 7{,}95$$

$$\frac{3 + \log x}{2 - \log x} = 4$$

$$3 + \log x = 8 - 4 \log x$$
$$\log x + 4 \log x = 8 - 3$$
$$5 \log x = 5$$
$$\log x = 5/5$$
$$\log x = 1$$
$$x = 10$$

$$\log(1+x) - \log(1-x) + \log(2-x) - \log(2+x) = \log(2x+1) - \log(2x-1)$$
$$\log \frac{(1+x)(2-x)}{(1-x)(2+x)} = \log \frac{(2x+1)}{(2x-1)}$$
$$\frac{(1+x)(2-x)}{(1-x)(2+x)} = \frac{2x+1}{2x-1}$$
$$\frac{2+2x-x-x^2}{2-2x+x-x^2} = \frac{2x+1}{2x-1}$$
$$(2+x-x^2)(2x-1) = (2-x-x^2)(2x+1)$$
$$4x + 2x^2 - 2x^3 - 2 - x + x^2 = 4x - 2x^2 - 2x^3 + 2 - x - x^2$$
$$-2 + 3x + 3x^2 = -3x^2 + 3x + 2$$
$$6x^2 = 4$$
$$x^2 = \frac{2}{3}$$
$$x = \sqrt{0{,}666}$$
$$x = \pm 0{,}816$$

1966. X. 8.

18. Házi feladat

$$2\log x = \log 36$$
$$\log x^2 = \log 36$$
$$x^2 = 36$$
$$x = 6$$

$$x^{\log x} = 10\,000$$

$$x = 100$$

$$\frac{1}{3}\log x^6 = \frac{1}{2}\log 8$$
$$\log x^2 = \log \sqrt{8}$$
$$x^2 = \sqrt{8}$$
$$x = \sqrt[4]{8}$$
$$x \doteq 1{,}68$$

$$\log(2x+12) + \log(3x-8) = \log 18$$
$$\log(2x+12)(3x-8) = \log 18$$
$$6x^2 + 36x - 16x - 96 = 18$$
$$6x^2 + 20x - 114 = 0$$
$$x_{12} = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-20 \pm \sqrt{400+2736}}{12} =$$
$$= \frac{-20 \pm \sqrt{3136}}{12} = \frac{-20 \pm 56}{12}$$
$$x_1 = 3 \qquad x_2 = -6{,}3\ldots$$

$$\log(x+2) + \log(x+3) = \log(x-2) + \log x$$
$$\log(x+2)(x+3) = \log(x-2)\cdot x$$
$$x^2 + 2x + 3x + 6 = x^2 - 2x$$
$$7x = -6$$
$$x = -\frac{6}{7}$$

18. óra Számolás logarléccel

1966. X. 10.

Házi feladat

10 osztás, 10. szorzás log. léccel - de leírni.

$316 \cdot 163 \doteq 51500,$
$125 \cdot 324 \doteq 40500,$
$3{,}14 \cdot 159 \doteq 499,$
$525 \cdot 12 \doteq 6300,$
$125 \cdot 125 \doteq 15620,$
$32 \cdot 122 \doteq 3902,$
$43 \cdot 28 \doteq 120{,}3$
$154 \cdot 32 \doteq 4940,$
$17 \cdot 52{,}4 \doteq 890,$
$187 \cdot 237 \doteq 44300,$

$527 : 32 = (17)\ (16{,}43)\ 16{,}445$
$927 : 41 = 22{,}61$
$32 : 175 = 0{,}1828$
$157 : 3 = 52{,}35$
$12 : 13 = 0{,}9228$
$3 : 7 = 0{,}428$
$125 : 32 = 3{,}93$
$1457 : 18 = 886,$
$192 : 3 = 64{,}08$
$67 : 15 = 4{,}47$

19. óra. 1966. X. 12.

Házi feladat
10, 10. p (sok nullát!)

$0{,}028 \cdot 0{,}305 \doteq 0{,}008543$	$0{,}602 : 6{,}01 = 0{,}100$
$0{,}0032 \cdot 6{,}2 \doteq 0{,}01985$	$6{,}28 : 12 = 0{,}523$
$0{,}505 \cdot 0{,}2 \doteq 0{,}101$	$0{,}28 : 3{,}1 = 0{,}0935$
$15{,}6 \cdot 1{,}8 \doteq 283$	$102 : 0{,}3 = 3397$
$1{,}05 \cdot 0{,}203 \doteq 0{,}213$	$32 : 528 = 0{,}0607$
$0{,}02 \cdot 3{,}12 \doteq 0{,}06243$	$8 : \sqrt{2} = 5{,}66$
$0{,}0032 \cdot 0{,}025 \doteq 0{,}00008$	$99 : 12 = 8{,}25$
$3{,}35 \cdot 0{,}73 \doteq 2{,}445$	$0{,}3 : 28 = 0{,}017$
$9{,}17 \cdot 6{,}2 \doteq 56{,}8$	$0{,}305 : 12 = 0{,}0251$
$9{,}27 \cdot 9{,}2 \doteq 85{,}25$	$0{,}05 : 0{,}2 = 0{,}25.$

20. óra 21. óra 1966. X. 13.

Gyakorlat.

$$\frac{\log x^3 - 2}{\log^3 x} + \frac{3}{\log x} = 2 \quad / \cdot \log^3 x$$

$$\log x^3 - 2 + 3\log^2 x = 2 \cdot \log^3 x$$
$$\log x^3 - 2 + 3\log^2 x - 2\log^3 x = 0 \qquad \log x = y$$
$$3y - 2 + 3y^2 - 2y^3 = 0$$
$$2y^3 - 3y^2 - 3y + 2 = 0$$
$$2y^3 + 2 - (3y^2 + 3y) = 0 \qquad y_1 = -1 \quad y_2 = 2 \quad y_3 = 0{,}5$$
$$(y+1)(2y^2 - 5y + 2) = 0$$

Exponenciális egyenletek.

Az ismeretlen a kitevőben fordul elő.

$a^x = b$ Kétféle képpen oldjuk meg:
1. logaritmus segítségével
2. logaritmus nélkül. (egyforma alappal)

1,
$$\log a^x = \log b$$
$$x \log a = \log b$$
$$x = \frac{\log b}{\log a}$$

2,
$$5^x = 25$$
$$5^x = 5^2$$
$$x = 2$$

$$\log 5^x = \log 25$$
$$x \log 5 = \log 25$$
$$x = \frac{\log 25}{\log 5} = 2$$

$$5^{3x-1} = 25$$
$$5^{3x-1} = 5^2$$
$$3x-1 = 2$$
$$3x = 3$$
$$x = 1$$

$$\left(\frac{1}{2}\right)^{2-3x} = 3^x$$
$$\log\left(\frac{1}{2}\right)^{2-3x} = \log 3^x$$
$$(2-3x)\log\frac{1}{2} = x\log 3$$
$$(2-3x)(\log 1 - \log 2) = x\log 3$$
$$2\log 1 - 3x\log 1 - 2\log 2 + 3x\log 2 = x\log 3$$
$$-3x\log 1 + 3x\log 2 - x\log 3 = 2\log 2 - 2\log 1$$
$$x(3\log 2 - 3\log 1 - \log 3) = 2\log 2 - 2\log 1$$
$$x = \frac{2\log 2 - 2\log 1}{3\log 2 - 3\log 1 - \log 3}$$

$$x = \frac{2.0{,}30403 - 2.0}{3.0{,}30403 - 3.0 - 0{,}4771} = \frac{0{,}60206}{0{,}90309 - 0{,}47710} = \frac{0{,}60206}{0{,}42599} = 1{,}414$$

1966-X. 15. 22. óra

$$4^{x+1} - 8.4^{x-1} = 32$$
$$2^{(x+1).2} - 2^3.2^{(x-1).2} = 2^5$$
$$2^{2x+2} - 2^3.2^{2x-2} = 2^5$$
$$2^{2x+2} - 2^{2x+1} = 2^5$$
$$\frac{2^{2x+1}}{2} = 2^5$$
$$2^{2x+2} = 2^6$$
$$2x+2 = 6$$
$$2x = 4$$
$$x = 2$$

$$2^{3x+1}.2^{2x+3} = 2^{5x+1}.2^{x+2}$$
$$2^{3x+1+2x+3} = 2^{5x+1+x+2}$$
$$2^{5x+4} = 2^{6x+3}$$
$$1 = x$$

$$3^x + 3^{x+1} + 3^{x+2} = 5^x + 5^{x+1} + 5^{x+2}$$
$$3^x + 3^x.3 + 3^x.3^2 = 5^x + 5^x.5 + 5^x.5^2$$
$$3^x(1+3+9) = 5^x(1+5+25)$$
$$3^x.13 = 5^x.31$$
$$\frac{3x}{5x} = \frac{31}{13}$$
$$\left(\frac{3}{5}\right)^x = \frac{31}{13}$$
$$\log\left(\frac{3}{5}\right)^x = \log\frac{31}{13}$$
$$x.\log\frac{3}{5} = \log\frac{31}{13}$$
$$x = \frac{\log\frac{31}{13}}{\log\frac{3}{5}} = \frac{\log 31 - \log 13}{\log 3 - \log 5} = \frac{1{,}4914 - 1{,}1139}{0{,}4771 - 0{,}6990} = \frac{0{,}3775}{-0{,}2219}$$
$$x = -1{,}7$$

22. Házi feladat

1966. X. 15

138. o. 5 b c d

139. o 9 a b c e f

$$8^{5x-3} : 8^{2x-1} = 8^{3x+2} : 8^{4x-4}$$

$$8^{5x-3-2x+1} = 8^{3x+2-4x+4}$$

$$3x-2 = -x+6$$

$$4x = 8$$

$$x = 2$$

$$2^{3x} \cdot 4^{3x-3} = 8^{2x+1}$$

$$2^{3x} \cdot 2^{6x-6} = 2^{6x+3}$$

$$2^{3x+6x-6} = 2^{6x+3}$$

$$9x-6 = 6x+3$$

$$3x = 9$$

$$x = 3$$

$$27^{5x-6} \cdot 81^{2x+3} = 9^{4x-2} \cdot 3^{7x-2}$$

$$3^{15x-18} \cdot 3^{8x+12} = 3^{8x-4} \cdot 3^{7x-2}$$

$$3^{15x-18+8x+12} = 3^{8x-4+7x-2}$$

$$23x-6 = 15x-6$$

$$23x = 15x$$

$$8x = 0$$

$$x = 0$$

$$\left(\frac{2}{3}\right)^{x+1} \cdot \left(\frac{3}{4}\right)^{x+1} \cdot \left(\frac{1}{8}\right)^{x} = \frac{1}{96}$$

$$(x+1)\log\left(\frac{2}{3}\right) + (x+1)\log\left(\frac{3}{4}\right) + x\log\left(\frac{1}{8}\right) = \log\frac{1}{96}$$

$$\left(2.3^{-1}\right)^{x+1} \cdot \left(3.2^{-2}\right)^{x+1} \cdot \left(2^{-3}\right)^{x} = 96^{-1}$$

$$2^{x+1} \cdot 3^{-x-1} \cdot 3^{x+1} \cdot 2^{-2x-2} \cdot 2^{-3x} = 96^{-1}$$

$$2^{x+1-2x-2-3x} \cdot 3^{-x-1+x+1} = 96^{-1}$$

$$2^{-4x-1} = 96^{-1}$$

$$-4x-1 \cdot \log 2 = -1 \cdot \log 96$$

$$-4x-1 = \frac{-1 \cdot \log 96}{\log 2}$$

$$-4x = \frac{-1 \cdot \log 96}{\log 2} + 1$$

$$-x = \frac{\frac{-1 \cdot \log 96 + \log 2}{\log 2}}{4}$$

$$-x = \frac{-1 \cdot \log 96 + \log 2}{4 \cdot \log 2}$$

$$x = -\frac{-1 \cdot \log 96 + \log 2}{4 \log 2} = -\frac{-1 \cdot 1{,}9823 + 0{,}3010}{4 \cdot 0{,}3010} =$$

$$= -\frac{-1{,}9823 + 0{,}3010}{1{,}2040} = -\frac{-1{,}6813}{1{,}2040} = 1{,}322$$

$$5^x + 5^{x-1} = 750$$
$$5^x + 5^x \cdot 5^{-1} = 750$$
$$5^x(1+5^{-1}) = 750$$
$$5^x = \frac{750}{1+\frac{1}{5}}$$
$$5^x = \frac{750}{6/5}$$
$$5^x = \frac{5 \cdot 750}{6} = \frac{3750}{6} = 625$$
$$5^x = 5^4$$
$$x = 4$$

$$3^x + 3^{x+1} + 3^{x+2} = 5^x + 5^{x+1} + 5^{x+2}$$
$$x = -1,7$$

$$3^{x+2} - 3^{x-1} = 702$$
$$3^x \cdot 3^2 - 3^x \cdot 3^{-1} = 702$$
$$3^x(3^2 - 3^{-1}) = 702$$
$$3^x = \frac{702}{9-\frac{1}{3}} = \frac{702}{\frac{26}{3}} = \frac{2106}{26} = 81$$
$$3^x = 3^4$$
$$x = 4$$

$$4^x + 3^{x+4} = 4^{x+3} - 3^{x+2}$$

$$4^x + 3^x \cdot 3^4 = 4^x \cdot 4^3 - 3^x \cdot 3^2$$
$$4^x - 4^x \cdot 4^3 = -3^x \cdot 3^4 - 3^x \cdot 3^2$$
$$4^x(1-4^3) = -3^x(3^4+3^2)$$
$$\frac{4^x}{3^x} = \frac{-(3^4+3^2)}{1-4^3}$$
$$\left(\frac{4}{3}\right)^x = -\frac{90}{-63} = \frac{30}{21} = \frac{10}{7}$$
$$x \log\frac{4}{3} = \log\frac{10}{7}$$
$$x = \frac{\log\frac{10}{7}}{\log\frac{4}{3}} = \frac{\log 10 - \log 7}{\log 4 - \log 3} = \frac{1 - 0,8451}{0,6021 - 0,4771} = \frac{0,1549}{0,1250} = 1,219 \doteq 1,22$$

$$3^{x+1} + 3^x = 4^{x-1} + 4^x$$
$$3^x(3+1) = 4^x(4^{-1}+1)$$
$$\frac{3^x}{4^x} = \frac{(1+4^{-1})}{4} = \frac{5/4}{4} = \frac{5}{16}$$
$$\left(\frac{3}{4}\right)^x = \frac{5}{16}$$
$$x \log\frac{3}{4} = \log\frac{5}{16}$$
$$x = \frac{\log 5 - \log 16}{\log 3 - \log 4} = \frac{0.6990 - 1,2041}{0,4771 - 0,6021} = \frac{-0,5051}{-0,1250} \doteq 4,043$$

23. ó 1966. X. 17.

$112^2 = 12540,$ $268^2 = 71900,$ $425^2 = 180400,$

$2.2 = 4$ $2.2 = 4$ $2.2 = 4+1=5$

$\sqrt{64{,}61} = 80{,}4$

$\sqrt{2{,}14{,}65} = 146{,}3$

Ha az első csoportban 2 jegy van, a 10-en felül keressük.
Annyi egész szám, ahány csoport.

23. Házi f. 1966. X. 17.

5 oszt, szor, hatv, gyökv.

$0{,}00305 . 0{,}305 = 0{,}00093$ $324 : 12 = 26{,}95$

$15 . 324 = 4850,$ $305 : 0{,}2 = 1513,$

$325 . 625 = 20700,$ $1292 : 3{,}2 = 404{,}0$

$0{,}306 . 25{,}8 = 7{,}8$ $105 : 325 = 0{,}323$

$1{,}25 . 3{,}2 = 4,$ $12 : 1304 = 0{,}009125$

$\sqrt{2{,}38{,}53} = 154{,}5$ $32^2 = 1012$

$\sqrt{2{,}38} = 154{,}2$ $34^2 = 1156$

$\sqrt{\pi} = 1{,}772$ $\pi^2 = 989$

$\sqrt{\pi^3} = 5555$ $(4\pi)^2 = 157{,}7$

$\sqrt{3} = 1{,}732$ $(\pi^3)^2 = 962{,}0$

24. óra 1966. X. 20.

Gyakorlat

$$\log x^3 - \log x^4 + \log x^5 = 8$$
$$3 . \log x - 4 . \log x + 5 \log x = 8$$
$$4 \log x = 8$$
$$\log x = 2$$
$$x = 100$$

$$12^x = 144 \qquad 12^x = 12^2 \qquad x = 2$$

$$\left(\frac{3}{4}\right)^x = \frac{81}{256} \qquad \left(\frac{3}{4}\right)^x = \left(\frac{3}{4}\right)^4 \qquad x = 4$$

$$\sqrt[5]{\sqrt[x-3]{2}} = \sqrt{\sqrt[-x]{\tfrac{1}{2}}}$$
$$\sqrt[5(x-3)]{2} = \sqrt[-2x]{0{,}5}$$
$$2^{\frac{1}{5x-15}} = 0{,}5^{\frac{1}{-2x}}$$
$$2^{5x-15} = 2^{+2x}$$
$$5x - 15 = 2x$$
$$3x = 15$$
$$x = 5$$

$$\sqrt[2x-3]{\frac{4}{11}} = \sqrt[x+5]{2{,}\frac{3}{4}}$$
$$\sqrt[2x-3]{4/11} = \sqrt[x+5]{11/4}$$
$$\frac{4}{11}^{\frac{1}{2x-3}} = \frac{11}{4}^{\frac{1}{x+5}}$$
$$\frac{4}{11}^{\frac{1}{2x-3}} = \frac{4}{11}^{-\frac{1}{x+5}}$$
$$\frac{1}{2x-3} = \frac{1}{-x-5}$$
$$3x = -2 \qquad x = -\frac{2}{3}$$

$$\frac{\log x}{1-\log 2} = 2$$

$$\frac{\log x}{\log 10 - \log 2} = 2$$

$$\log x = 2(\log 10 - \log 2)$$

$$\log x = \log 100 - \log 4$$

$$\log x = \log \frac{50}{2}$$

$$x = 25$$

1966. X. 20

24. Házi feladat

140,σ 14, 15

139,σ 11 bcdef

138,σ 4 abc ·

6

27.-re

$$\log(4x+6) - \log(2x-1) = 1$$

$$\log \frac{4x+6}{2x-1} = \log 10$$

$$4x+6 = 20x-10$$

$$16 = 16x$$

$$x = 1$$

$$\log(x+3) - \log 5 = \log(x-3) - \log 2$$

$$\frac{x+3}{5} = \frac{x-3}{2}$$

$$2x+6 = 5x-15$$

$$21 = 3x$$

$$x = 7$$

$$\log(x+1) + \log(x-1) - \log x = \log(x+2)$$

$$\frac{(x+1)(x-1)}{x} = x+2$$

$$x^2 - 1 = x^2 + 2x$$

$$x = -\frac{1}{2}$$

$$\log(x+3) + \log 12 - \log(x-5) = 1,7782$$

$$\frac{x+3}{x-5} = 0,699$$

$$x+3 = 0,699x - 3,495$$

$$0,311x = -6,495$$

$$311x = -6495$$

$$x = -\frac{6495}{311}$$

$$x = -21$$

$$2 + \log 5x = \log(6x+7) + \log 25$$

$$500x = 150x + 175$$

$$350x = 175$$

$$x = \frac{175}{350}$$

$$x = 0,5$$

$$\log(2x+9) - 2\log x + \log(x-4) = 2 - \log 50$$
$$\frac{(2x+9)(x-4)}{x^2} = \frac{100}{50}$$
$$2x^2 + 9x - 8x - 36 = 2x^2$$
$$x = 36$$

$$\log(x^2-1) - \log(x+1) = 2$$
$$\frac{x^2-1}{x+1} = 100$$
$$x - 1 = 100$$
$$x = 101$$

$$\frac{1}{3}\log(x-1) = 2 - \log 50$$
$$(x-1)^{\frac{1}{3}} = 2 \quad /^3$$
$$x - 1 = 8$$
$$x = 9$$

$$\log(3-x) - \log 2(1+x^2) + \log 2 = -\log(1-x)$$
$$\frac{2.(3-x)(1-x)}{2+2x^2} = 0$$
$$2.(3-x)(1-x) = 0$$
$$2(3 - x - 3x + x^2) = 0$$
$$2x^2 - 8x + 6 = 0$$
$$x^2 - 4x + 3 = 0$$
$$x_{12} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q} = +2 \pm \sqrt{4-3}$$
$$x_{12} = 2 \pm \sqrt{1}$$
$$x_1 = 3 \qquad x_2 = 1$$

$$\log \frac{2(3x-1)}{x-2} = 1$$
$$6x - 2 = 10x - 20$$
$$4(2\cancel{6})x = 18$$
$$x = \frac{18}{4} = \frac{9}{2} = 4{,}5$$

$$\log(x+7) - \log(x-7) = 0{,}9031$$
$$\frac{x+7}{x-7} = 8$$
$$x + 7 = 8x - 56$$
$$63 = 7x$$
$$x = 9$$

$$\log(x-3) + \log(x+3) = 2\log(3-x)$$
$$(x-3)(x+3) = (3-x)^2$$
$$x^2 - 9 = 9 - 6x + x^2$$
$$6x = 18$$
$$x = 3$$

$$\log x^3 + \frac{1}{2}\log x^2 + 7.\log x^4 + 64 = 0$$
$$\log x^3 . x . x^{28} = -64$$
$$\log x^{32} = -64$$
$$\log x = -2$$
$$x = 0{,}01.$$

$$3\log x + \log x^4 - \log \sqrt[3]{x} = 5$$
$$3.\log x + 4.\log x - \frac{1}{3}\log x = 5$$
$$6\tfrac{2}{3}\log x = 5$$
$$\log x = \frac{5}{6{,}666}$$
$$\log x = 0{,}7515$$
$$x \doteq 6$$

$\frac{1}{2}\log x = 2\log 2$
$\sqrt{x} = 4$
$x = 16$

$4\log x = 3\log 16$
$x^4 = 16^3$
$x^4 = 4096$
$x = 8$

$2\log x = -3\log 25$
$x^2 = 25^{-3}$
$x = 5^{-3}$
$x = \frac{1}{125}$

$\sqrt[x+2]{27} = \sqrt[x+1]{9}$
$(3^3)^{\frac{1}{x+2}} = (3^2)^{\frac{1}{x+1}}$
$3^{\frac{3}{x+2}} = 3^{\frac{2}{x+1}}$
$3x+3 = 2x+4$
$x = 1$

$\sqrt[x+\frac{1}{2}]{729} = \sqrt[x-\frac{1}{2}]{9}$
$729^{\frac{2}{2x+1}} = 9^{\frac{2}{2x-1}}$
$3^{\frac{10}{2x+1}} = 3^{\frac{4}{2x-1}}$
$20x-10 = 8x+4$
$12x = 14$
$x = \frac{7}{6}$

$\sqrt[3]{2^{2x-3}} = \sqrt[7]{0{,}5^{3-x}}$
$2^{\frac{2x-3}{3}} = (2^{-1})^{\frac{3-x}{7}}$
$\frac{2x-3}{3} = \frac{x-3}{7}$
$14x-21 = 3x-9$
$11x = 12$
$x = \frac{12}{11} \doteq 1$

1380 6p.

$\sqrt[x]{3^{x+3m}}\cdot\sqrt[x]{3^{x-3m}} = 27$
$\sqrt[x]{3^{x+3m}\cdot 3^{x-3m}} = 27$
$\sqrt[x]{3^{2x}} = 27$
$3^{\frac{2x}{x}} = 27 = 3^3$
$3^2 \neq 3^3$

$(0{,}25)^{2-x} = \frac{256}{2^{x+3}}$
$(2^{-2})^{2-x}\cdot 2^{x+3} = 2^8$
$2^{2x-4}\cdot 2^{x+3} = 2^8$
$3x-1 = 8$
$x = 3$

$2^{x+2} - 2^x = 96$
$2^x = \frac{96}{2^2-1} = \frac{96}{3} = 32$
$2^x = 2^5$
$x = 5$

$3^{x+2} + 3^{x-2} = 82$
$3^x\cdot 3^2 + 3^x\cdot\frac{1}{3^2} = 82$
$3^x(3^2+\frac{1}{9}) = 82$
$3^x = \frac{82}{\frac{82}{9}} = \frac{82\cdot 9}{82} = 3^2$
$x = 2$

$3^{2x-1} + 3^{2x-2} - 3^{2x-4} = 315$
$3^{2x}\cdot 3^{-1} + 3^{2x}\cdot 3^{-2} - 3^{2x}\cdot 3^{-4} = 315$
$3^{2x}(3^{-1}+3^{-2}-3^{-4}) = 315$
$3^{2x}(\frac{1}{3}+\frac{1}{9}-\frac{1}{81}) = 315$
$3^{2x} = \frac{315}{35/81} = 729$
$9^x = 9^3$
$x = 3$

$2^{x-1} + 2^{x-2} + 2^{x-3} = 448$
$2^x\cdot 2^{-1} + 2^x\cdot 2^{-2} + 2^x\cdot 2^{-3} = 448$
$2^x(2^{-1}+2^{-2}+2^{-3}) = 448$
$2^x = \frac{448}{7/8} = \frac{3584}{7}$
$2^x = 512 = 2^9$
$x = 9$

$4\cdot 3^{x+1} - 72 = 3^{x+2} + 3^{x-1}$
$4\cdot 3^{x+1} - 3^{x+2} - 3^{x-1} = 72$
$3^x(4\cdot 3 - 3^2 - 3^{-1}) = 2^6$
$3^x = \frac{2^6}{8/3} = 27$
$x = 3$

$0{,}6^x + \left(\frac{3}{5}\right)^{x-1} = \frac{40}{9}$
$\left(\frac{3}{5}\right)^x\cdot\left[1+\left(\frac{3}{5}\right)^{-1}\right] = \frac{40}{9}$
$\left(\frac{3}{5}\right)^x \cdot \frac{8}{3} = \frac{40}{9}$
$\left(\frac{3}{5}\right)^x = \frac{\frac{40}{9}}{\frac{8}{3}} = \frac{120}{72}$

$x\cdot\log\frac{3}{5} = \log\frac{120}{72}$
$x = \frac{\log 120 - \log 72}{\log 3 - \log 5} = \frac{2{,}0792 - 1{,}8573}{0{,}4771 - 0{,}6990}$
$x = \frac{0{,}2219}{-0{,}2219} = -1$

Gyakorlat

$$\sqrt{2^{5-7x}} = \sqrt[3]{4^{3-5x}} \;/\; 2\cdot\sqrt{2}$$

$$2\cdot\sqrt{2}\cdot\sqrt{2^{5-7x}} = \sqrt[3]{4^{3-5x}}$$

$$\sqrt{8}\cdot 2^{\frac{5-7x}{2}} = 4^{\frac{3-5x}{3}}$$

$$8\cdot 2^{\frac{5-7x}{2}}$$

$$\sqrt{8\cdot 2^{5-7x}} = \sqrt[3]{4^{3-5x}}$$

$$\sqrt{2^3\cdot 2^{5-7x}} = \sqrt[3]{(2^2)^{3-5x}}$$

$$2^{\frac{8-7x}{2}} = 2^{\frac{6-10x}{3}}$$

$$24-21x = 12-20x.$$

$$3+9 = x$$

$$x = 12$$

$$\sqrt[x]{2^{x+3}}\cdot\sqrt[2x]{2^{x-3}}\cdot\sqrt[4]{2} = 4$$

$$2^{\frac{x+3}{x}}\cdot 2^{\frac{x-3}{2x}}\cdot 2^{\frac{1}{4}} = 2^2$$

$$2^{\frac{x+3}{x}+\frac{x-3}{2x}+\frac{1}{4}} = 2^2$$

$$\frac{4x+12+2x-6+x}{4x} = 2$$

$$7x+6 = 8x$$

$$6 = x$$

$$x = 6$$

$$\sqrt[x-1]{5} = \sqrt[x+1]{6}$$

$$5^{\frac{1}{x-1}} = 6^{\frac{1}{x+1}}$$

$$\frac{1}{x-1}\cdot\log 5 = \frac{1}{x+1}\cdot\log 6$$

$$\frac{\log 5}{x-1} = \frac{\log 6}{x+1}$$

$$(x+1)\cdot\log 5 = (x-1)\log 6$$

$$x\cdot\log 5+\log 5 = x\cdot\log 6-\log 6$$

$$x\cdot\log 5-x\log 6 = -\log 6-\log 5$$

$$x = \frac{-\log 6-\log 5}{\log 5-\log 6}$$

$$x = \frac{\log 6+\log 5}{\log 6-\log 5} = \frac{\log 30}{\log 1{,}2}$$

$$\sqrt[7x]{7^{5x+7}} = \sqrt[5x]{5^{7x+5}}$$

$$7^{\frac{5x+7}{7x}} = 5^{\frac{7x+5}{5x}}$$

$$\frac{5x+7}{7x}\cdot\log 7 = \frac{7x+5}{5x}\cdot\log 5$$

$$\frac{5x\cdot\log 7+7\cdot\log 7}{7x} = \frac{7x\cdot\log 5+5\cdot\log 5}{5x}$$

$$25x^2\log 7+35x\cdot\log 7 = 49x^2\log 5+35x\cdot\log 5$$

$$25x\cdot\log 7+35\cdot\log 7 = 49x\cdot\log 5+35\cdot\log 5$$

$$25x\cdot\log 7-49x\cdot\log 5 = 35\log 5-35\log 7$$

$$x = \frac{35(\log 5-\log 7)}{25\cdot\log 7-49\cdot\log 5}$$

15. Házi feladat

$$3^{x+1}+4^{y+2} = 54$$

$$3^{x+2}+4^{y+1} = 30$$

$$3^x = a \qquad 4^y = b$$

$$3^x\cdot 3+4^y\cdot 4^2 = 54$$

$$3^x\cdot 3^2+4^y\cdot 4 = 30$$

$$a\cdot 3+b\cdot 16 = 54$$

$$a\cdot 9+b\cdot 4 = 30$$

$$3a+16b = 54 \;/\cdot 3$$

$$9a+4b = 30$$

$$9a+48b = 162$$

$$9a+4b = 30 \;/\cdot -1$$

$$44b = 132$$

$$b = 3$$

$$a = \frac{54-48}{3} = \frac{6}{3}$$

$$a = 2$$

$$3^x = 2 \qquad 4^y = 3$$

$$x = \frac{\log 2}{\log 3} \qquad y = \frac{\log 3}{\log 4}$$

$$x = \frac{0{,}3010}{0{,}4771} \qquad y = \frac{0{,}4771}{0{,}6021}$$

$$x \doteq 0{,}63 \qquad y \doteq 0{,}79$$

1966. X. 24.

26. óra

Számolás logarlécen

$31^2 = 961$ $\quad 1.2 = 2$

$618^2 = 382000,$ $\quad 2.2 + 1 = 5$

$\sqrt{706} = 26,56$ $\quad \sqrt{48055} = 219,2$

$\sqrt{679\,840} = 824,6$

$2^3 = 8$ $\quad 0.3 + 0 = 0$

$3^3 = 27$ $\quad 0.3 + 1 = 1$

$6^3 = 216$ $\quad 0.3 + 2 = 2$

$36^3 = 46680,$ $\quad 1.3 + 1 = 4$

$\sqrt[3]{625} = 8,55$

$\sqrt[3]{58725} = 38,8$

1966. X. 27.

27. óra

A komplex szám fogalma

(1.) Természetes számok : + • (1 ~ n ...)
(2.) 0 - jel : −
(3.) Negatív egész sz. : − } egész sz. halmaza
(4.) Törtek : : ($\frac{1}{2}; \frac{1}{3}; \frac{1}{5}; \frac{1}{7}$) racionális sz. h.
Hatványozás, gyökvonás : a^x ; $\sqrt[x]{a}$ (5.) - irracionális számok

⇓ valós sz. halmaza

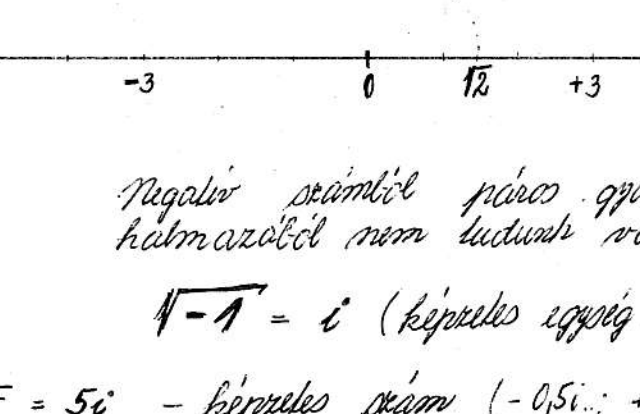
A valós számokat a számegyenesen ábrázoljuk:

$x^2 + 1 = 0$
$x^2 = -1$
$x = \sqrt{-1}$

Negatív számból páros gyököt a valós számok halmazából nem tudunk vonni.

$\sqrt{-1} = i$ (képzetes egység) immaginárius szám.

$\sqrt{-25} = \sqrt{-1} \cdot \sqrt{25} = 5i$ - képzetes szám ($-0,5i$.; $+3i$.)



Gauss f. számsík

3+2i - komplex szám

$z = a + bi$ – komplex szám = valós rész + képzetes rész

$a;\ b$ – valós szám

i – képzetes egység

$$z_1 = a_1 + b_1$$
$$z_2 = a_2 + b_2$$
$$z_1 = z_2 \rightarrow \begin{matrix} a_1 = a_2 \\ b_1 = b_2 \end{matrix}$$

Két komplex szám akkor egyenlő, ha külön a valós és képzetes részek is egyenlők.

Az olyan komplex számokat, melyek csak a képzetes rész előjelében különböznek konjugált komplex számoknak nevezzük.

$z = a + bi$	$2 + 3i$	$0{,}5 - \frac{2}{3}i$	$z = a + bi$	$a = 0$
$\bar{z} = a - bi$	$2 - 3i$	$0{,}5 + \frac{2}{3}i$	$z = 0 + bi$	
			$z = bi$	

Az olyan komplex szám, mely valós része 0, képzetes szám.
A képzetes szám olyan komplex szám, mely valós része 0.

$z = a + bi \quad b = 0$

$z = a + 0i$

$z = a$

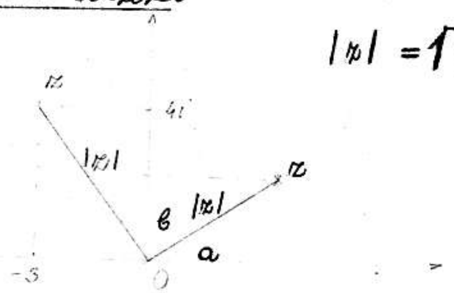
Az olyan komplex szám, mely képzetes része 0, valós szám.
A valós szám olyan komplex szám, mely képzetes része 0.

Komplex szám abszolut értéke

$$|z| = \sqrt{a^2 + b^2}$$

$z = -3 + 4i$

$|z| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = 5$



A valós és képzetes rész négyzetösszegéből vont négyzetgyök a komplex szám abszolut értéke.

$i = \sqrt{-1}$	i
$i^2 = -1$	-1
$i^3 = i^2 \cdot i = -i$	$-i$
$i^4 = -1 \cdot -1 = 1$	1

$i^5 = i^3 \cdot i^2 = -i \cdot -1 = i$

$i^6 = i^3 \cdot i^3 = -i \cdot -i = i^2 = -1$

$i^7 = i^4 \cdot i^3 = 1 \cdot -i = -i$

1966. X. 31.

28. óra

Műveletek komplex számokkal

Összeadás. $z_1 = a_1 + b_1 i$ $\quad z_1 + z_2 = (a_1 + b_1 i) + (a_2 + b_2 i) = a_1 + b_1 i + a_2 + b_2 i =$
$z_2 = a_2 + b_2 i$ $\quad = (a_1 + a_2) + (b_1 + b_2) i$

A valós és a képzetes részt külön összeadjuk.

$z_1 = 3 + 5i$ $\quad z_1 + z_2 = 3 + 5i + 4 + i = 7 + 6i$
$z_2 = 4 + i$

$z_1 = 2 + 4i$ $\quad z_1 + z_2 = 6 + 2 + 4i + 2i = 8 + 6i$
$z_2 = 6 - 2i$

$z_1 = 5 - 4i$ $\quad z_1 + z_2 = 5 + 3 - 4i + i = 8 - 3i$
$z_2 = 3 + i$

$z_1 = 6 + 2i$ $\quad z_1 + z_2 = 12$
$z_2 = 6 - 2i$

Komplex számok összege szintén komplex szám.
Konjugált komplex számok összege valós szám

$z_1 = a_1 + b_1 i$ $\quad (a_1 + b_1 i) + (\bar{a}_1 - b_1 i) = a_1 + \bar{a}_1 = 2a_1$
$\bar{z}_1 = a_1 - b_1 i$

Kivonás $\quad z_1 = a_1 + b_1 i$ $\quad z_1 - z_2 = (a_1 + b_1 i) - (a_2 + b_2 i) =$
$z_2 = a_2 + b_2 i$ $\quad = a_1 + b_1 i - a_2 - b_2 i =$
$= (a_1 - a_2) + (b_1 - b_2) i$

$z_1 = 6 + 7i$ $\quad z_1 - z_2 = (6 + 7i) - (3 + 4i) = 6 + 7i - 3 - 4i =$
$z_2 = 3 + 4i$ $\quad = 6 - 3 + 7i - 4i = 3 + 3i$

Komplex számok különbsége komplex szám, mely valós része a valós részek, képzetes része a képzetes részek különbsége.

$z_1 = 2 + 4i$ $\quad z_1 - z_2 = -5 + 7i$
$z_2 = 7 - 3i$

$z_1 = 5 - 4i$ $\quad z_1 - z_2 = 5 - 4i - 3 - i = 2 - 5i$
$z_2 = 3 + i$

$z_1 = 4 + 2i$ $\quad z_1 - z_2 = 4 - 4 + 2i + 2i = 4i$
$z_2 = 4 - 2i$

Konjugált komplex számok különbsége képzetes szám.

$z_1 = a_1 + b_1 i$ $\quad z_1 - \bar{z}_1 = (a_1 - a_1) + (b_1 i + b_1 i) = 2b_1 i$
$\bar{z}_1 = a_1 - b_1 i$

Szorzás $\quad z_1 = a_1 + b_1 i \quad z_1 \cdot z_2 = (a_1 + b_1 i)\cdot(a_2 + b_2 i) =$
$z_2 = a_2 + b_2 i$

$$= a_1 a_2 + a_2 b_1 i + a_1 b_2 i + b_1 b_2 i^2 =$$
$$= a_1 a_2 + a_2 b_1 i + a_1 b_2 i + (-b_1 b_2) =$$
$$= (a_1 a_2 - b_1 b_2) + (a_1 b_2 + a_2 b_1) i$$

$z_1 = 2 + 4i \quad z_1 \cdot z_2 = (2+4i)\cdot(6-2i) = 12 + 24i - 4i - 8i^2 =$
$z_2 = 6 - 2i \quad = 12 + 20i - (-8) = 20 + 20i$

Komplex szám szorzata általában komplex szám.
Komplex számokat úgy szorzunk, mint valós kéttagú algebrai kifejezéseket, de figyelembe kell venni: $i^2 = -1$

$z_1 = 2 + 3i \quad z_1 \cdot z_2 = (2+3i)(2-3i) = 4 - 9i^2 = 4 + 9 = 13$
$z_2 = 2 - 3i$

Ha komplex számot szorzunk a konjugáltjával, akkor a szorzat a komplex szám abszolut értékének négyzete, ami valós szám.

$z_1 = a_1 + b_1 i \quad z_1 \cdot \bar{z}_1 = (a_1 + b_1 i)(a_1 - b_1 i) = a_1^2 - b_1^2 i^2 =$
$\bar{z}_1 = a_1 - b_1 i \quad = a_1^2 + b_1^2$

Komplex szám négyzetét úgy kap. meg, mintha kéttagú kifejezést emelnénk négyzetre.

$z = 3 + 2i \quad ; \quad z^2 = (3+2i)^2 = 9 + 12i + (-4) =$
$= 5 + 12i$

136 10 12 28. H.f. 1966. X. 2.

$(23 - 15i) + (-128 + 364i) = -105 + 349i$
$(52664 - 322215i) + (-24230 + 420000i) = 28434 + 97785i$
$(-3{,}28 - 4{,}15i) + (-7{,}12 + 4{,}15i) = -10{,}4$
$15i \underline{-24} - 12i \underline{+38} - 72i \underline{+35} = 49 - 69i$

$-1{,}8 \cdot (2{,}4 - 5i) = -4{,}32 + 9i$

$25{,}8 \cdot (-3{,}08 - 16{,}4i) = 79{,}95 - 423i$

1966. XI. 3 29. óra

Osztás:

$$z_1 = a_1 + b_1 i \quad z_2 = a_2 + b_2 i \quad \frac{z_1}{z_2} = \frac{a_1 + b_1 i}{a_2 + b_2 i} = \frac{(a_1+b_1 i)(a_2 - b_2 i)}{(a_2 + b_2 i)(a_2 - b_2 i)} =$$

$$= \frac{a_1 a_2 + a_2 b_1 i - a_1 b_2 i + b_1 b_2}{a_2^2 + a_2 b_2 i - a_2 b_2 i + b_2 b_2} = \frac{a_1 a_2 + b_1 b_2 + i(a_2 b_1 - a_1 b_2)}{a_2^2 + b_2^2} =$$

$$= \frac{a_1 a_2 + b_1 b_2}{a_2^2 + b_2^2} + \frac{(a_2 b_1 - a_1 b_2) i}{a_2^2 + b_2^2}$$

Hogy két komplex szám osztását el tudjuk végezni, mind az osztandót (számlálót), mind az osztót (nevezőt) megszorozzuk az osztó konjugált értékével.

$$\frac{4+2i}{3-4i} = \frac{(4+2i)(3+4i)}{(3-4i)(3+4i)} = \frac{12 + 6i + 16i + 8i^2}{9 - 12i + 12i + 16i^2} = \frac{12 + 22i - 8}{9 + 16} =$$

$$= \frac{4 + 22i}{25} = \frac{4}{25} + \frac{22}{25} i$$

Két komplex szám hányadosa általában ismét komplex szám.

$$\frac{2+3i}{2-3i} = \frac{(2+3i)(2+3i)}{(2-3i)(2+3i)} = \frac{4 + 12i + 9i^2}{4+9} = \frac{4 - 9 + 12i}{13} = \frac{-5}{13} + \frac{12i}{13}$$

Konjugált komplex szám hányadosa ismét komplex szám.

$$\frac{5i}{4i} = \frac{5}{4} \qquad \frac{6i}{i} = 6 \qquad \frac{-2i}{-3i} = \frac{2}{3}$$

Tiszta képzetes számok hányadosa valós szám.

A képzetes egység fordított értékének a meghatározása:

$\frac{2}{3}$ ford. ért. $\frac{3}{2}$ $\qquad -\frac{1}{4}$ f. é. -4 $\qquad 3$ f. é. $\frac{1}{3}$

i f. é. $\frac{1}{i} = \frac{1 \cdot i}{i \cdot i} = \frac{i}{i^2} = \frac{i}{-1} = -i \qquad \underline{\underline{\frac{1}{i} = -i}}$

$$\frac{3-5i}{5+2i} = \frac{(3-5i)(5-2i)}{29} = \frac{15 - 25i - 6i - 10}{29} = \frac{5 - 31i}{29} = \frac{5}{29} - \frac{31i}{29};$$

29. H. f.

153 o. 39 a b c $\qquad \frac{6-3i}{2+5i} \quad \frac{2+4i}{5-2i} \quad \frac{6+4i}{9-3i} \quad \frac{3-2i}{4+5i}$

155 o. elolvasni

$(a+bi)^2 = a^2 + 2abi + b^2 i^2 = (a^2 - b^2) + 2abi$

$(a+bi)^3 = a^3 + 3.a^2.bi + 3b^2i^2a + b^3i^3 = a^3 - 3ab^2 + 3a^2bi - b^3i =$

$= a^3 + 3ab^2 + i(a^2b.3 - b^3)$

Komplex számok hatványa általában ismét komplex számok.

$(3+2i)^2 = 9 + 12i + 4i^2 = 9 - 4 + 12i = 5 + 12i$

$(1+2i)^3 = 1 + 6i - 12 - 8i = -11 - 2i$

$(5-4i)^2 = 25 - 40i - 16 = 9 - 40i$ H.f. 1966.XI.3.

$(2-3i)^3 = 8 - 36i + 54 + 27i = 62 - 9i$

$(1+i)^2 = 1 + 2i - 1 = 2i$

$(7-5i)^2 = 49 - 70i + 25 = 74 - 70i$

$(2+3i)^2 = 4 + 12i - 9 = 12i - 5$

$(5x-6yi)^2 = 25x^2 + 60xyi + 36y^2$

$(a+bi)^2 + (a-bi)^2 = a^2 + 2abi - b^2 + a^2 - 2abi + b^2 = 2a^2$

$(1+i)^3 = 1 + 3i + 3 - 1i = 2i - 2$

$(1-i)^3 = 1 + 3i + 3 + i = 4 - 2i$

$(9-4i)^3 = 729 - 922i + 432 + 64i = 1161 - 858i$

$(-2x+5yi)^3 = -8x^3 + 60x^2yi + 150xy^2 - 125y^3i$

$(1-i)^4 = (1 - 3i + 3 + i)(1-i) = (4-2i)(1-i) = 4 - 2i - 4i - 2 = 2 - 6i$

$(a+bi)^5 = (a^3 + 3a^2bi - 3ab^2 - b^3i)(a^2 + 2abi - b^2) = a^5 + 3a^4bi - 3a^3b^2 - a^2b^3i +$

$+ 2a^4bi - 6a^3b^2 - 6a^2b^3i + 2ab^4 - a^3b^2 - 3a^2b^3i + 3ab^4 + b^5i =$

$= a^5 + 5a^4bi - 10a^3b^2 - 10a^2b^3i + 5ab^4 + b^5i$

Végezzük el a kij. osztást.:

$\frac{1}{1+i} = \frac{1-i}{(1+i)(1-i)} = \frac{1-i}{1+1} = \frac{1}{2} - \frac{i}{2}$

$\frac{i-1}{i+1} = \frac{(i-1)(i-1)}{-1+1} = \frac{-1-2i+1}{0} = -\frac{2i}{0}$

$\frac{i+1}{3-i} = \frac{(i+1)(3+i)}{9+1} = \frac{3i+3-1+i}{10} = \frac{2+4i}{10} = \frac{1}{5} + \frac{2}{5}i$

$$\frac{4-5i}{3-4i} = \frac{(4-5i)(3+4i)}{9+16} = \frac{12-15i+16i+20}{25} = \frac{32+i}{25} = \frac{32}{25} + \frac{i}{25}$$

$$\frac{9+10i}{9-10i} = \frac{(9+10i)(9+10i)}{81+100} = \frac{81+180i-100}{181} = \frac{180i-19}{181} = \frac{180}{181}i - \frac{19}{181}$$

$$\frac{(3+i)\cdot\sqrt{2}}{3-i\sqrt{2}} = \frac{(3+i\sqrt{2})(3+i\sqrt{2})}{9+2} = \frac{9+8i-2}{11} = \frac{7+8i}{11} = \frac{7}{11} + \frac{8i}{11}$$

$$\frac{7+8i}{5-7i} = \frac{(7+8i)(5+7i)}{25+49} = \frac{35+40i+49i-56}{74} = \frac{89i-21}{74} = \frac{89i}{74} - \frac{21}{74}$$

Végezzük el:

$$(1+i)\cdot(1-i) = 1+i-i+1 = 2$$
$$(2+3i)\cdot(3+4i) = 6+9i+8i+12 = 17i-6$$
$$(5+8i)\cdot(5-8i) = 25+40i-40i+64 = 89$$
$$(-9+3i)\cdot(-9-3i) = 81+9$$
$$(\sqrt{2}+i\sqrt{3})\cdot(\sqrt{2}-i\sqrt{3}) = 2+3$$
$$\left(\frac{4}{5}-\frac{3}{8}i\right)\cdot\left(\frac{2}{3}+\frac{1}{2}i\right)\cdot\left(\frac{3}{4}-\frac{5}{6}i\right) = \left(\frac{8}{15}-\frac{6}{24}i+\frac{4}{10}i+\frac{3}{16}\right)\left(\frac{3}{4}-\frac{5}{6}i\right) =$$
$$= \left(\frac{173}{250}+\frac{3i}{20}\right)\left(\frac{3}{4}-\frac{5}{6}i\right) = \frac{519}{1000} + \frac{9i}{80} - \frac{865i}{1500} + \frac{15}{120} =$$
$$= \frac{519}{1000} + \frac{9i}{80} + \frac{3}{24} - \frac{173i}{3000};$$

$$\frac{6+4i}{9-3i} = \frac{(6+4i)(9+3i)}{81+9} = \frac{54+36i+18i-12}{90} = \frac{42+54i}{90} = \frac{42}{90} + \frac{54i}{90}$$

$$\frac{3-2i}{4+5i} = \frac{(3-2i)(4-5i)}{16+25} = \frac{12-8i-15i-10}{41} = \frac{2-23i}{41} = \frac{2}{41} - \frac{23i}{41}$$

$$\frac{6-3i}{2+5i} = \frac{(6-3i)(2-5i)}{4+25} = \frac{12-6i-30i-15}{29} = \frac{-3-36i}{29} = \frac{-3}{29} - \frac{36i}{29}$$

$$\frac{2+4i}{5-2i} = \frac{(2+4i)(5+2i)}{25+4} = \frac{10+20i+4i-8}{29} = \frac{2+24i}{29} = \frac{2}{29} + \frac{24i}{29}$$

$$\frac{3+5i}{4+5i} = \frac{(3+5i)(4-5i)}{16+25} = \frac{12+20i-15i+25}{41} = \frac{37+5i}{41} = \frac{37}{41} + \frac{5i}{41}$$

$$\frac{3,5-0,8i}{-0,1-2,6i} = \frac{(3,5-0,8i)(-0,1+2,6i)}{0,01+6,76} = \frac{-0,35+0,08i+9,1i+1,84}{6,77} = \frac{1,49+9,18i}{6,77}$$

$$\frac{i}{3-5i} = \frac{i(3+5i)}{9+25} = \frac{3i-5}{34} = \frac{3i}{34} - \frac{5}{34}$$

$$\frac{2}{1+2\sqrt{3i}} = \frac{2(1-2\sqrt{3i})}{1+12i} = \frac{2(1-2\sqrt{3i})(1-12i)}{145} = \frac{2-4\sqrt{3i}-24i+48i\sqrt{3i}}{145} =$$
$$= \frac{2-24i-4\sqrt{3i}+48\sqrt{-3i}}{145}$$

$$\frac{1}{i} = \frac{i}{-1} = -i$$

$$\frac{(0,7-2,4i)}{(6,4-0,6i)} = \frac{(0,7-2,4i)(6,4+0,6i)}{40,96+0,36} = \frac{4,48-15,36i+0,42i+1,44}{41,32} =$$

$$= \frac{5,92-14,94i}{41,32}$$

$$\frac{6-27i}{3i} = \frac{(6-27i)\cdot 3i}{-9} = \frac{18i+81}{-9} = -2i-9$$

30. óra 1966. XI. 5.

H. f.

Vonjuk össze a köv. képz. számokat:

$$3i+5i+9i-7i+2i-8i = 4i$$
$$2i\sqrt{3}+5i\sqrt{2}-6i\sqrt{3}-15i\sqrt{2}+9i\sqrt{3} = 5i\sqrt{3}-10i\sqrt{2} = i(5\sqrt{3}-10\sqrt{2})$$
$$\sqrt{-4}+\sqrt{-9}+\sqrt{-25}+\sqrt{-81}-\sqrt{-64}-\sqrt{-100} = 2i+3i+5i+9i-8i-10i = i$$

Adjuk össze a köv. k. számokat:

$$(8+5i)+(-7+4i)+(6-5i) = 7+4i$$
$$(3-6i)+(9-5i)+(-5+11i) = 7$$
$$(25+13i)+(25-13i) = 50$$
$$(x+yi)+(x-yi) = 2x$$

Vonjuk ki egymásból a köv. k. számokat:

$$(3+5i)-(2+8i) = 3+5i-2-8i = 1-3i$$
$$(9+12i)-(9-4i) = 9+12i-9+4i = +16i$$
$$(-10+8i)-(-10-8i) = -10+8i+10+8i = 16i$$

Végezzük el a kij. műveleteket:

$$(2+3i)\cdot(2+i)+(1+i)(1+2i) = \underline{4}+6i+2i\underline{-3}+\underline{1}+i+2i\underline{-2} = 11i$$
$$(2+3i)\cdot(1-4i)-(2-3i)(1+4i) = \underline{2}+3i-8i\underline{+12}\underline{-2}+3i-8i\underline{-12} = -10i$$
$$(3-i)\cdot(2+3i)+(1-4i)(2-i) = \underline{6}-2i+9i\underline{+3}\underline{+2}-8i-i\underline{-4} = 7-2i$$
$$[(1+2i)-(4+3i)]\cdot[-i) - (4+3i) = (1+2i-4-3i)\cdot(-i) - 4-3i =$$
$$= (-3-i)\cdot(-i) - 4 - 3i = 3i-1-4-3i = \underline{-5}$$

$$(-2-4i)\left[-\frac{1}{2}+\frac{i}{2}\right] \pm (1-2i) = 1+2i-i+2 \pm (1-2i) = (3+i) \pm (1-2i)$$

$$3+i+1-2i = 4-i$$
$$3+i-1+2i = 2+3i$$

$$\left[(-1-3i)-\left(-2+\frac{i}{2}\right)\right]\left[(2-i)-(1-2i)\right] = \left(-1-3i+2-\frac{i}{2}\right)\left(2-i-1+2i\right) =$$
$$= \left(1-\frac{7i}{2}\right)(1+i) = 1-\frac{7i}{2}+i+\frac{7}{2} = \frac{9}{2}-\frac{5}{2}i = \frac{9-5i}{2}$$

$$\frac{2+i}{3-i}+(i-2)\cdot(i-4) = \frac{(2+i)(3+i)}{(3-i)(3+i)}+(-1-2i-4i+8) = \frac{5+5i}{10}+7-6i = 7\tfrac{1}{2}-5\tfrac{1}{2}i$$

$$\left|\frac{-2-3i}{3-2i}\right| = \left|\frac{(-2-3i)(3+2i)}{13}\right| = \left|\frac{-6-9i-4i+6}{13}\right| = \left|-\frac{13i}{13}\right| = |-i| =$$

$$\left|\frac{1-3i}{2i+5}\right| = \left|\frac{(1-3i)(2i-5)}{(2i+5)(2i-5)}\right| = \left|\frac{2i+6-5+15i}{4+25}\right| = \left|\frac{1+17i}{29}\right| = \left|\frac{1}{29}+\frac{17i}{29}\right| =$$
$$= \sqrt{\left(\frac{1}{29}\right)^2+\left(\frac{17i}{29}\right)^2} = \sqrt{\frac{1}{841}+\frac{+289}{841}} = \sqrt{\frac{290}{841}} = \frac{17,..}{29}$$

$$\left|\frac{1-3i}{5+2i}\right| = \left|\frac{(1-3i)(5-2i)}{(5+2i)(5-2i)}\right| = \left|\frac{5-15i-2i-6}{25+4}\right| = \left|\frac{-1-17i}{29}\right| = \sqrt{\left(-\frac{1}{29}\right)^2+\left(-\frac{17}{29}\right)^2}$$

$$\frac{1+i}{1-2i} \pm \frac{i}{1+i} = \frac{(1+i)(1+2i)}{5} \pm \frac{i(1-i)}{2} = \frac{1+i+2i-2}{5} \pm \frac{i+1}{2} = \frac{3i-1}{5} \pm \frac{i+1}{2} =$$
$$= \frac{6i-2}{10} \pm \frac{5i+5}{10} = \qquad \frac{11i+3}{10} = \frac{11i}{10}+\frac{3}{10} \qquad \frac{i-7}{10} = \frac{i}{10}-\frac{7}{10}$$

$$\frac{2+3i}{3+4i} \pm \frac{3-4i}{2-3i} = \frac{(2+3i)(3-4i)}{25} \pm \frac{(3-4i)(2+3i)}{13} = \frac{18+i}{25} \pm \frac{18+i}{13}$$

$$\frac{18}{25}+\frac{18}{13}+\frac{i}{25}+\frac{i}{13}; \qquad \frac{18}{25}-\frac{18}{13}+\frac{i}{25}-\frac{i}{13};$$

$$\left|1+2i-\frac{2-5i}{3-i}\right| = \left|1+2i-\frac{(2-5i)(3+i)}{10}\right| = \left|1+2i-\frac{11-13i}{10}\right| = \left|-\frac{1}{10}+\frac{33}{10}i\right| =$$
$$= \sqrt{\left(-\frac{1}{10}\right)^2+\left(\frac{33}{10}\right)^2} = \sqrt{\frac{1}{100}+\frac{1089}{100}} = \sqrt{\frac{1090}{100}} = 0,33$$

$$\left|\frac{1-3i}{2+5i}\right| = \left|\frac{(1-3i)(2-5i)}{29}\right| = \left|\frac{-13-11i}{29}\right| = \sqrt{\left(-\frac{13}{29}\right)^2+\left(\frac{11}{29}\right)^2} = \sqrt{\frac{169}{29^2}+\frac{121}{29^2}} =$$
$$= \sqrt{\frac{290}{29^2}} = \sqrt{\frac{10}{29}}$$

31. óra 1966. XI. 7.

Műveletek komplex számokkal.

Az olyan komplex számot, mely abszolút értéke egy, komplex egységnek nevezzük. Grafikonjuk 0 középpontos kör.

$(-8\sqrt{-1}) : 4\sqrt{-1} = \frac{-8i}{4i} = -2$

$28 : (-7i) = -4 : i = -\frac{4}{\sqrt{-1}} = -\frac{4\cdot\sqrt{-1}}{-1} = \frac{4i}{1} = 4i$

i^{2860} $\quad 2860 : 4 = 715$ - maradék 0 $\Rightarrow i^{2860} = 1$

$\frac{abi}{c^2} \cdot \left(-\frac{bci}{a^2}\right) \cdot \left(-\frac{aci}{b^2}\right) = \frac{abi}{c^2} \cdot -\frac{bci}{a^2} \cdot -\frac{aci}{b^2} = -\frac{a^2b^2c^2 \quad i}{a^2b^2c^2} = -i = i^3$

$-6-4i$ - komplex egység-e?

$|-6-4i| = \sqrt{(-6)^2+(-4)^2} = \sqrt{36+16} = \sqrt{52} \neq 1$

$\left|(a+b)+(a-b)i\right| = \sqrt{(a+b)^2+(a-b)^2} = \sqrt{a^2+2ab+b^2+a^2-2ab+b^2} = \sqrt{2a^2+2b^2} = \sqrt{2(a^2+b^2)}$

$\frac{x+i}{x-i} = \frac{(x+i)(x+i)}{x^2+(-1)} = \frac{x^2+2xi-1}{x^2+1} = \frac{x^2-1}{x^2+1} + \frac{2xi}{x^2+1}$

$\left|\sqrt{6}+i\sqrt{10}\right| = \sqrt{\sqrt{6}^2+\sqrt{10}^2} = \sqrt{16} = 4$

$i^{27} =$ $\quad 27:4 = 6 \ (+3) \Rightarrow i^{27} = -i$

$\left|\frac{3}{5}+\frac{2}{3}i\right| = \sqrt{\left(\frac{3}{5}\right)^2+\left(\frac{2}{3}\right)^2} = \sqrt{\frac{9}{25}+\frac{4}{9}} = \sqrt{\frac{81+100}{9\cdot25}} = \frac{\sqrt{181}}{15}$

$i^{33} = 8 \ (+1) \Rightarrow i^{33} = i$

$\sqrt{-39} : \sqrt{-13} = 39i : 13i = 3$

$\frac{1}{\sqrt{-16}} = -\frac{1}{4}i$

$i^{48} = 1$
$i^{122} = -1$
$i^{3611} = -i$
$(-i)^{93} = -i$
$(-i)^{110} = -1$

$i^{56307} = -i$
$i^{6666} = -1$
$(-i)^{83} = +i$
$(-i)^{100} = 1$

$\frac{a}{a+bi} = \frac{a(a-bi)}{a^2+b^2} = \frac{a^2-abi}{a^2-b^2}$

$\frac{m+ni}{m-ni} = \frac{(m+ni)(m+ni)}{m^2+n^2} = \frac{m^2+2mni-n^2}{m^2+n^2}$

$$\frac{5c^3-7d^2i}{3d^2+5c^3i} = \frac{(5c^3-7d^2i)(3d^2-5c^3i)}{9d^4+25c^6} = \frac{15c^3d^2-21d^4i-25c^6i-35c^3d^2}{9d^4+25c^6} = -\frac{20c^3d^2+21d^4i+25c^6}{9d^4+25c}$$

$$\left(-\frac{2i}{3}\right):\left(-\frac{5}{6}i\right) = -\frac{\frac{2i}{3}}{\frac{5i}{6}} = -\frac{12i}{15i} = -\frac{4}{5}$$

$$xi:(-yi) = \frac{xi}{-yi} = -\frac{x}{y}$$

$$-36a^2b^2i:(-9a^2b) = 4bi$$

$$150a^2b^mx^{r-2}:25a^2b^mx^{3r+5}i = 6x^{r-2-3r-5}i^{-1} = -6x^{-7-4r}i$$

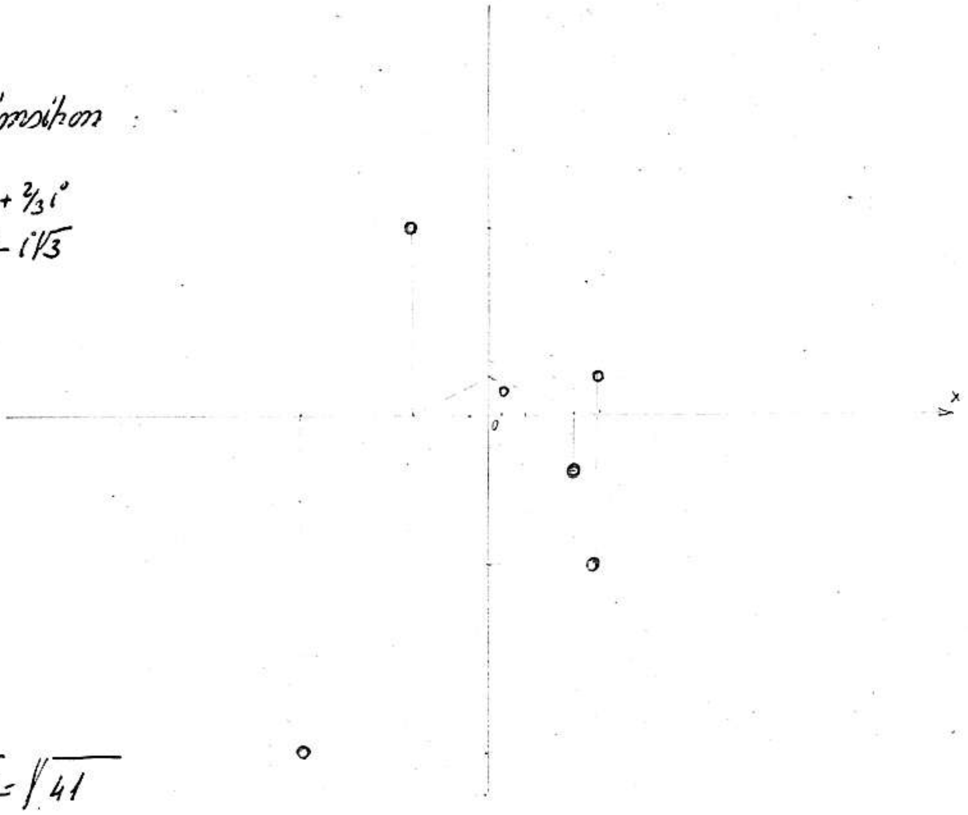
Ábrázoljuk a számsíkon :

$3+i$ $\quad \frac{1}{4}+\frac{2}{3}i$

$-2+5i$ $\quad \sqrt{5}-i\sqrt{3}$

$3-4i$

$-5-9i$



$$|4+5i| = \sqrt{16+25} = \sqrt{41}$$

$$|4-5i| = \sqrt{16+25} = \sqrt{41}$$

$$|-4+5i| = \sqrt{16+25} = \sqrt{41}$$

$$|-4-5i| = \sqrt{16+25} = \sqrt{41}$$

$$\left(\frac{1-i}{1+i}\right)^2-\left(\frac{3-4i}{2-3i}\right)^2=\left(-\frac{2i}{2}\right)^2-\left(\frac{18+i}{13}\right)^2=-1-\frac{323}{169}-\frac{36i}{169}=-\frac{492}{169}-\frac{36}{169}i$$

$$\left(\frac{1-i}{1+i}\right)^2-\left(\frac{1+i}{1-i}\right)^2=\left(\frac{1-2i-1}{2}\right)^2-\left(\frac{1+2i-1}{2}\right)^2=-1+1=0$$

$$(-1+i\sqrt{3})^3=-1+3i\sqrt{3}+9+i^3\sqrt{3^3}=8+3i\sqrt{3}-3i\sqrt{3}=8$$

$$(-1+i\sqrt{3})^3-\frac{-4+6i}{2-3i}=8-\frac{-8+12i-12i-18}{13}=8-\frac{-26}{13}=10$$

$$\left|\sqrt{\frac{2+\sqrt{2}}{2}}+\sqrt{\frac{2-\sqrt{2i}}{2}}\right|=\sqrt{\frac{2+\sqrt{2}}{4}+\frac{2-\sqrt{2i}}{4}}=\sqrt{\frac{4+\sqrt{2}-\sqrt{2i}}{2}}=\sqrt{\frac{4}{2}}=1$$

H.

$$\frac{2-3i}{3-2i}=\frac{(2-3i)(3+2i)}{13}=\frac{6-9i+4i+6}{13}=\frac{12-5i}{13}$$

$$\frac{3-i}{1+3i}=\frac{(3-i)(1-3i)}{10}=\frac{3-i-9i-3}{10}=-i$$

$$\frac{4+i}{1-3i}=\frac{(4+i)(1+3i)}{10}=\frac{4+i+12i-3}{10}=\frac{1+13i}{10}$$

32. óra — 1966. XI. 10.

Műveletek komplex számokkal

$$\frac{1}{(1+i)^2}+\frac{1}{(1-i)^2}=\frac{1}{1+2i-1}+\frac{1}{1-2i-1}=\frac{1}{2i}+\frac{-1}{2i}=\frac{0}{2i}=0$$

$$(5-i)(i+3)(8-i)=(5i+1+15-3i)(8-i)=(2i+16)(8-i)=16i+128+2-16i=130$$

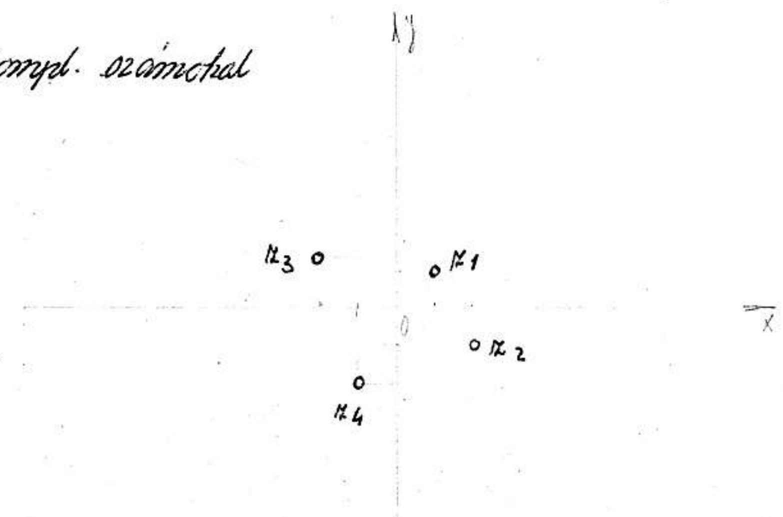
$$\left(\frac{1+i\sqrt{3}}{2}\right)^3=\frac{1+3i\sqrt{3}-9-i\sqrt{27}}{8}=\frac{-8}{8}=-1$$

$$\left(\frac{1+i}{1-i}\right)^6=\left(\frac{(1+i)(1+i)}{(1-i)(1+i)}\right)^6=\left(\frac{1+2i-1}{2}\right)^6=\left(\frac{2i}{2}\right)^6=i^6=-1$$

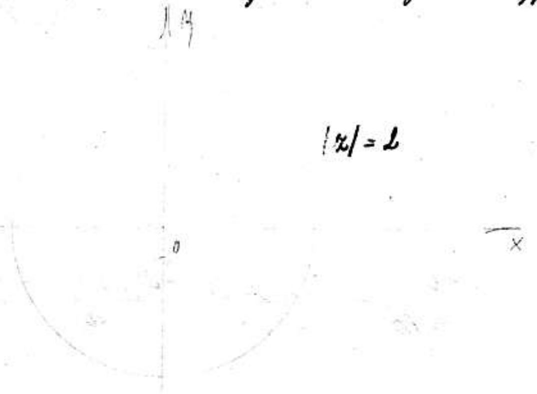
$$(1+i)^4-(1-i)^4=(1+i)^2\cdot(1+i)^2-(1-i)^2\cdot(1-i)^2=$$
$$=(1+2i-1)\cdot(1+2i-1)-(1-2i-1)(1-2i-1)=$$
$$2i\cdot2i-4i^2=0$$

Ábrázoljuk a köv. kompl. számokat

$z_1 = 1+i$
$z_2 = 2-i$
$z_3 = -2+3/2\,i$
$z_4 = -1-2i$



Ábr. mindazon komplex számot melyre érvényes hogy $|z| = 2$



Hat. meg a köv. kompl. sz. absz. értékét:

$z_1 = 1+i \qquad |1+i| = \sqrt{(1)^2+(i)^2} = \sqrt{1+1} = \sqrt{2}$
$z_2 = 3-4i \qquad |3-4i| = \sqrt{9+16} = 5$
$z_3 = -5+12i \qquad |-5+12i| = \sqrt{25+144} = \sqrt{169} = 13$
$z_4 = -1-i\sqrt{3} \qquad |-1-i\sqrt{3}| = \sqrt{1+3} = 2.$

Szám. ki:

$i^8 = 1$
$i^{12} = 1$
$i^{16} = 1$
$i^{20} = 1$
$i^{27} = -i$

33. óra 1966. XI. 14.

Gyökvonás:

$\sqrt{a+bi} = x+yi \quad /^2$

$(\sqrt{a+bi})^2 = (x+yi)^2$

$a+bi = x^2+2xyi-y^2$

$a+bi = x^2-y^2+2xyi$

Komplex számok négyzetgyöke általában ismét komplex szám.

$a = x^2-y^2 \quad /^2$

$b = 2xy \quad /^2$

$a^2 = (x^2-y^2)^2$

$b^2 = 4x^2y^2$

$a^2 = x^4-2x^2y^2+y^4$

$b^2 = 4x^2y^2 \quad /+$

$a^2+b^2 = x^4-2x^2y^2+y^4+4x^2y^2$

$a^2+b^2 = x^4+2x^2y^2+y^2$

$a^2+b^2 = (x^2+y^2)^2 \quad /\sqrt{}$

$\sqrt{(x^2+y^2)^2} = \sqrt{a^2+b^2}$

$x^2+y^2 = \sqrt{a^2+b^2} \quad /1,+ \quad /2,-$

$x^2-y^2 = a$

$2x^2 = a+\sqrt{a^2+b^2}$

$x^2 = (a+\sqrt{a^2+b^2}):2$

$x = \sqrt{\dfrac{a+\sqrt{a^2+b^2}}{2}}$

$2y^2 = -a+\sqrt{a^2+b^2}$

$y^2 = \dfrac{-a+\sqrt{a^2+b^2}}{2}$

$y = \sqrt{\dfrac{-a+\sqrt{a^2+b^2}}{2}}$

$$\sqrt{a+bi} = \pm\sqrt{\frac{a+\sqrt{a^2+b^2}}{2}} \pm \sqrt{\frac{-a+\sqrt{a^2+b^2}}{2}}\cdot i$$

33. Hf. 1966. XI. 14.

$\sqrt{12\pm 5i}$

$\sqrt{12+5i} = \pm\sqrt{\dfrac{12+\sqrt{144+25}}{2}} \pm i\sqrt{\dfrac{-12+\sqrt{144+25}}{2}} = \pm\sqrt{\dfrac{25}{2}} \pm i\sqrt{\dfrac{1}{2}} = \pm\dfrac{5\sqrt{2}}{2} \pm i\dfrac{\sqrt{2}}{2}$

$\sqrt{12-5i} = \pm\sqrt{\dfrac{12+\sqrt{144+25}}{2}} \pm i\sqrt{\dfrac{-12\pm\sqrt{144+25}}{2}} = \pm\dfrac{5\sqrt{2}}{2} \pm i\dfrac{\sqrt{2}}{2}$

1966. XI. 17.

34. óra. Gyakorlat

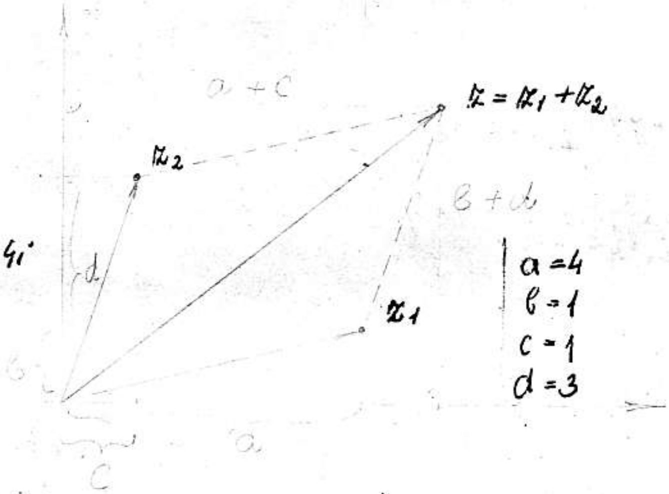
$$\frac{3+i}{2-3i} = \frac{(3+i)(2+3i)}{13} = \frac{3+11i}{13} = \frac{3}{13} + \frac{11i}{13}$$

$$\left|\frac{1+i}{1-2i}\right| \qquad \frac{1+i}{1-2i} = \frac{(1+i)(1+2i)}{5} = \frac{-1+3i}{5} = -\frac{1}{5} + \frac{3i}{5}$$

$$\left|-\frac{1}{5}+\frac{3}{5}i\right| = \sqrt{\frac{1}{25}+\frac{9}{25}} = \sqrt{\frac{10}{25}} = \frac{\sqrt{10}}{5}$$

$z_1 = 4+i$
$z_2 = 1+3i$

$z = z_1 + z_2 = 5+4i$



$z_1 = a+bi$
$z_2 = c+di$

$z = (a+c)+(b+d)i$

$a=4$
$b=1$
$c=1$
$d=3$

Kivonásnál ismét z_1 és z.

1966. XI. 17.

34. H.

$$\frac{50}{3+4i} \qquad \frac{i}{1+i} \qquad \frac{a+b}{1-ai} \qquad (4+3i)\cdot 2 \qquad (3+5i)i \qquad (2+3i)(4+5i)$$

$$(2-3i)(2+i) \qquad (-3\cdot\sqrt{3}+i)\cdot(4+i\sqrt{7})$$

$$(2-3i)^2 \qquad (-4{,}8+3i)^2 \qquad (1+i)(2+i)+(1+i)(1+2i)$$

$(2+3i)(1-4i)-(2-3i)(1+4i)$
$(5+i)(i-3)\cdot(8+i)$
$(1+i)(1+2i)(1+3i)$
$(-1+i\sqrt{3})^3$
$(1-i)^4$

$$\sqrt{1+2i} \qquad \sqrt{6-8i} \qquad \sqrt{5+12i} \qquad \sqrt{9+40i}$$

$$\frac{50}{3+4i} = \frac{50(3+4i)}{25} = \frac{150}{25} + \frac{200}{25}i = 6+8i \qquad \frac{i}{1+i} = \frac{i(1-i)}{2} = \frac{1}{2} + \frac{i}{2}$$

$$\frac{a+i}{1-ai} = \frac{(a+i)(1+ai)}{1+a^2} = \frac{a+i+a^2i-a}{a^2+1} = \frac{i+a^2i}{a^2+1} = \frac{(i+a^2i)(a^2-1)}{a^4-1} = \frac{a^2i+a^4i-i-a^2i}{a^4-1} = \frac{a^4i-i}{a^4-1} = i$$

$$(4+3i)\cdot 2 = 8+6i \qquad (3+5i)\,i = 3i-5 \qquad (2+3i)(4+5i) = -7+22i \qquad (2-3i)(2+i) = 7-4i$$

$$(-3\sqrt{3}+i)(4+i\sqrt{7}) = -\sqrt{7} - 12\sqrt{3} + 4i - 3\sqrt{21}\,i \qquad (2-3i)^2 = 4-12i-9 = -(9+12i)$$

$$(-4{,}8+3i)^2 = 23{,}04 - 28{,}8i - 9 = 14{,}04 - 28{,}8i$$

$$(1+i)(2+i) + (1+i)(1+2i) = 1+3i + 1 + 3i = 6i$$

$$(2+3i)(1-4i) - (2-3i)(1+4i) = 14-5i - 14 - 5i = -10i$$

$$(5+i)(i-3)(8+i) = (-16+2i)(8+i) = -126 = -126$$

$$(1+i)(1+2i)(1+3i) = (-1+3i)(1+3i) = -10$$

$$(-1+i\sqrt{3})^3 = -1 + 3i\sqrt{3} + 9 - i\sqrt{3^3} = 8 + 3i\sqrt{3} - 3i\sqrt{3} = 8$$

$$(1-i)^4 = \left[(1-i)^2\right]^2 = (-2i)^2 = -4$$

$$\sqrt{1+2i} = \pm\sqrt{\frac{a+\sqrt{a^2+b^2}}{2}} \pm \sqrt{\frac{-a+\sqrt{a^2+b^2}}{2}} = \pm\sqrt{\frac{1+\sqrt{5}}{2}} \pm \sqrt{\frac{-1+\sqrt{5}}{2}} =$$

$$= \pm\sqrt{\frac{3{,}24}{2}} \pm \sqrt{\frac{1{,}24}{2}} = \pm 1{,}28 \pm 0{,}79 \; ; \; (-0;\ 2{,}56;\ -2{,}56)$$

$$\sqrt{6-8i} = \sqrt{\frac{1+\sqrt{100}}{2}} \pm \sqrt{\frac{-1+\sqrt{100}}{2}} = \sqrt{5{,}5} \pm \sqrt{4{,}5} = \pm 2{,}34 \pm i\,2{,}12$$

$$\sqrt{5+12i} = \sqrt{\frac{5+\sqrt{169}}{2}} \pm \sqrt{\frac{-5+\sqrt{169}}{2}} = \pm\sqrt{9} \pm \sqrt{4} = \pm 3 \pm i2$$

$$\sqrt{9+40i} = \sqrt{\frac{9+\sqrt{1681}}{2}} \pm \sqrt{\frac{-9+\sqrt{1681}}{2}} = \sqrt{25} \pm \sqrt{16} = \pm 5 \pm i4$$

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35. óra

$$\left|\frac{1}{2} + \frac{1}{2}i\sqrt{3}\right| = \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{\frac{4}{4}} = 1. \Rightarrow \frac{1}{2} + \frac{1}{2}i\sqrt{3}$$ kompl. egység.

$$\frac{1}{2} + \frac{1}{2}i\sqrt{3}$$

$z_1 = 5 + 2i$
$z_2 = 3 + i$

$z = z_1 - z_2 = (5+2i-3-i)$
$= 2+i$

A vektor fogalma

A vektor olyan mennyiség, melynek nagysága és iránya is van. A vektort irányított szakasszal ábrázoljuk.
Jele: $\vec{a}$, $\overline{AB}$ (sebesség, gyorsulás, erő)
A skaláris mennyiségnek csak nagysága van.
(tömeg, hőmérséklet)

A B - irányított szakasz $\rightarrow$ $AB + BA = 0$

Vektorok összeadása:

$F_1 = 30 kp.$
$F_2 = 40 kp.$
$\alpha = 60°$
$R = ?$

$F_{1,2}$ = összetevő
R = eredő

Vektorok összeadására érvényes a kommutatív törvény.

$\vec{a} + \vec{b} = \vec{b} + \vec{a}$

V. össz. érv. az asszociatív törvény.
$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$

1966. XI. 21.

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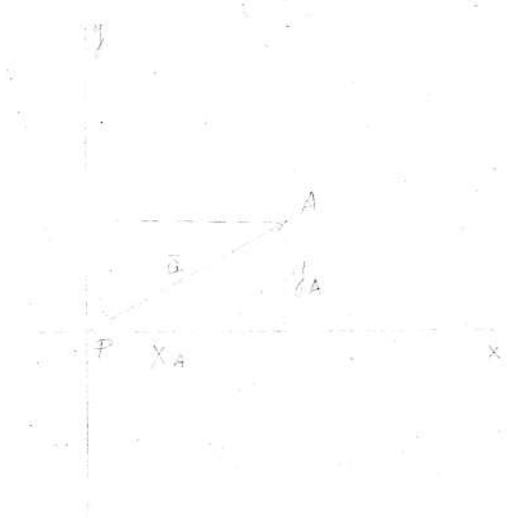
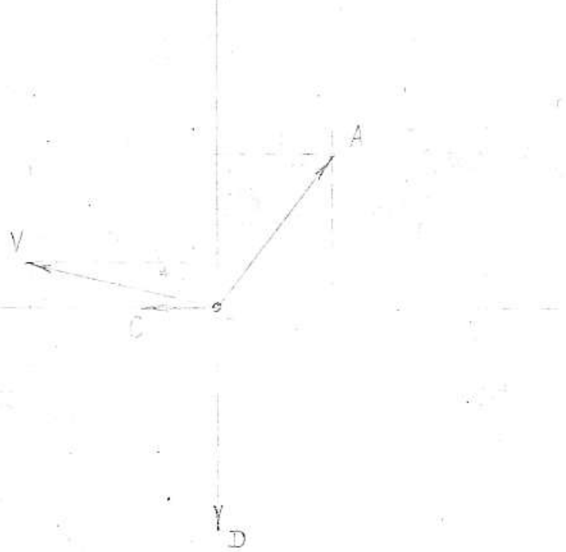
Vektorok a koordináta rendszerben

P(x, y)



PA (3; 4)
PV (-5; 1,2)
PC (-2; 0)
PD (0; -6

minden x_M ; y_M mint rendezett számpárhoz (koordinátákhoz) egy vektor tartozik.



PA ; PB

$PA = PB \rightarrow x_A = x_B \quad y_A = y_B$

$x_A = x_B \quad y_A = y_B \longrightarrow PA = PB$

$\vec{a}\,(x_A\ y_A) \quad \vec{b}\,(x_B\ y_B) \qquad k$ - valós szám

$\vec{c} = \vec{a} + \vec{b} \Rightarrow \vec{c}\ (\underbrace{x_A + x_B}_{x_c})(\underbrace{y_A + y_B}_{y_c})$

$\vec{a}\,(x_A\ y_A) \ldots\ldots k$

$k \cdot \vec{a} = (k x_A\ k y_A)$

1966. XI. 25. 37. óra

$x_1 = 2+3i \quad x_1 + x_2 = -p \quad 2+3i+2-3i = -p \quad p = -4 \quad x^2 - 4x + 13 = 0$

$x_2 = 2-3i \quad x_1 \cdot x_2 = q \quad (2+3i)(2-3i) = q \quad q = 13$

$x_{12} = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{4 \pm \sqrt{-36}}{2}; \quad x_1 = 2+3i \quad x_2 = 2-3i$

$x_1 = \frac{3}{4} - \frac{5}{2}i \quad -p = x_1 + x_2 = \frac{6}{4} \quad p = -\frac{6}{4} = -\frac{3}{2} \qquad x^2 - \frac{3}{2}x + \frac{109}{16} = 0$

$x_2 = \frac{3}{4} + \frac{5}{2}i \quad q = x_1 \cdot x_2 = \frac{9}{16} + \frac{25}{4} = \frac{9+100}{16} = \frac{109}{16}$

$x_{12} = \frac{\frac{3}{2} \pm \sqrt{\frac{9}{4} - \frac{109}{16} \cdot 4}}{2} = \frac{\frac{3}{2} \pm \sqrt{\cancel{\frac{18.9}{16}}}}{2} = \frac{\frac{3}{2} \pm \sqrt{\frac{9}{4} - \frac{436}{16}}}{2} = \frac{\frac{3}{2} \pm \sqrt{\frac{-400}{16}}}{2} = \frac{\frac{3}{2} \pm \frac{20}{4}i}{2}$

$x_1 = \frac{3}{4} - \frac{5}{2}i \qquad x_2 = \frac{3}{4} + \frac{5}{2}i$

$x_1 = 1-i \quad -p = x_1 + x_2 \quad p = -2 \qquad x^2 - 2x + 2 = 0$

$x_2 = 1+i \quad q = x_1 \cdot x_2 \quad q = 2$

$x_{12} = \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm 2i}{2} \qquad x_1 = 1+i \qquad x_2 = 1-i$

$x_1 = \frac{1}{2} - \frac{1}{2}i \quad p = -1$

$x_2 = \frac{1}{2} + \frac{1}{2}i \quad q = \frac{1}{2} \qquad x_2 - x + \frac{1}{2} = 0$

$x_{12} = \frac{1 \pm \sqrt{1-2}}{2} = \frac{1 \pm 1i}{2} \qquad x_1 = \frac{1}{2} + \frac{1}{2}i \quad x_2 = \frac{1}{2} - \frac{1}{2}i$

 1966. XI. 25.

5 példa

$a=3-2i$ válasszuk meg a $b=6+b_2 i$, $c=c_1+10i$, $d=d_1-\frac{2}{11}i$, $e=-\frac{7}{8}+e_2 i$ úgy hogy b c d e komplex számok ugyanazon egyenesen (OA) feküdjenek.

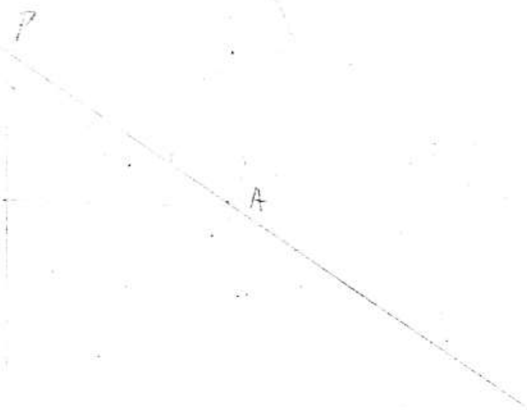
$x_1=2-i \quad -p=4 \quad x^2-4x+5=0$
$x_2=2+i \quad q=5 \quad x_{12}=\frac{4\pm\sqrt{16-20}}{2}=\frac{4\pm 2i}{2} \quad x_1=2+i \quad x_2=2-i$

$x_1=3+i \quad -p=6 \quad x^2-6x+10=0$
$x_2=3-i \quad q=10 \quad x_{12}=\frac{6\pm\sqrt{36-40}}{2}=3\pm 1i$

$x_1=2+2i \quad -p=4 \quad x^2-4x+8=0$
$x_2=2-2i \quad q=8 \quad x_{12}=\frac{4\pm\sqrt{16-32}}{2}=\frac{4\pm 4i}{2}=2\pm 2i$

$x_1=1+2i \quad -p=2 \quad x^2-2x+5=0$
$x_2=1-2i \quad q=5 \quad x_{12}=\frac{2\pm\sqrt{4-20}}{2}=1\pm 2i$

$x_1=3+4i \quad -p=6 \quad x^2-6x+25=0$
$x_2=3-4i \quad q=25 \quad x_{12}=\frac{6\pm\sqrt{36-100}}{2}=3\pm 4i$



a $\triangle\sim$ alapján:

$6:3=b_2:-2 \rightarrow b_2=-4 \quad b=6-4i$
$3:c_1=-2:10 \rightarrow c_1=-15 \quad c=-15+10i$
$3:d_1=-2:-\frac{3}{10} \rightarrow d_1=\frac{9}{20} \quad d=\frac{9}{20}-\frac{2}{11}i$
$3:-\frac{7}{8}=-2:e_2 \rightarrow e_2=\frac{7}{12} \quad e=-\frac{7}{8}+\frac{7}{12}i$

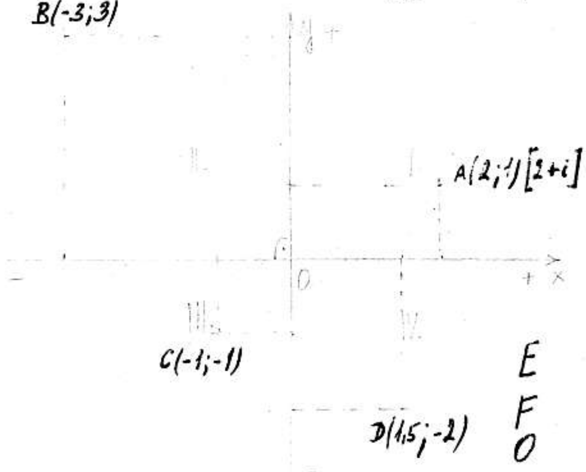
1966. XI. 28. 37. óra.

A komplex szám, mint rendezett számpár.

$$\left|\frac{2+i}{1-i}\right| + \left|\frac{1-i}{2+2i}\right| = \left|\frac{(2+i)(1+i)}{2}\right| + \left|\frac{(1-i)(2+2i)}{8}\right| = \left|\frac{1+3i}{2}\right| + \left|\frac{-4i}{8}\right| =$$

$$= \left|\frac{1}{2}+\frac{3}{2}i\right| + \left|-\frac{1}{2}i\right| = \sqrt{\frac{1}{4}+\frac{9}{4}} + \sqrt{\frac{1}{4}+\frac{1}{4}} = \sqrt{\frac{10}{4}} + \sqrt{\frac{7}{4}} = \frac{\sqrt{10}+\sqrt{4}}{2}$$

B(-3;3)



A(2;1) [2+i]

C(-1;-1)

D(1,5;-2)

I. (+ x + y) A(2;1)
II. (- x + y) B(-3;3)
III. (- x - y) C(-1;-1)
IV. (+ x - y) D(1,5;-2)

E (x;0) ... x tengelyen
F (0;y) .... y tengelyen
O (0;0) .... kezdőpont

Minden komplex számhoz tartozik a sík egyik pontja.

A komplex szám meghatározott sorrendben adott valós számpár (rendezett valós számpár).

Általános alak: $a = (a_1; a_2)$ $\qquad a = [a_1; a_2]$
$b = (b_1; b_2)$

$c = (c_1; 0) \longrightarrow$ c valós szám
$d = (0; d_2) \longrightarrow$ tiszta képzetes szám
$0 = (0; 0) \longrightarrow$ kezdőpont
$a = (a_1; a_2) \longrightarrow$ komplex szám, képzetes szám

Két komplex szám akkor egyenlő, ha $a_1 = b_1 \quad a_2 = b_2$

Ha $a = (a_1; a_2)$; $b = (b_1; b_2)$ $\quad a = b \rightarrow a_1 = b_1$; $a_2 = b_2$

Ha $a_1 = b_1$; $a_2 = b_2 \longrightarrow a = b$

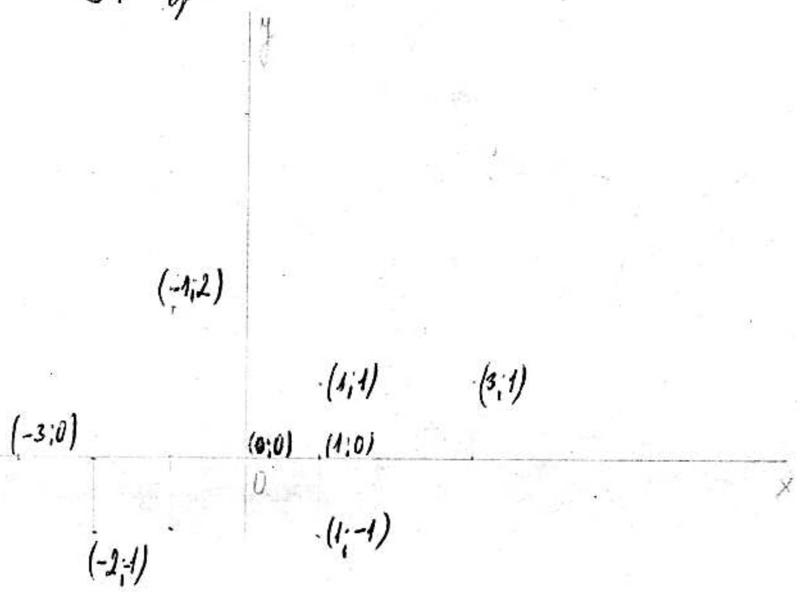
Ábr. a köv. kompl. számokat:

(0;0) (1;0) (1;1)
(3;1) (1;-1) (-1;2)
(-2;-1) (-3;0)

Van-e olyan kompl. sz., mely egyúttal: 1. valós, 2. tiszta képzetes, 3. képzetes szám.
Van-e. o. v. sz., mely e. 1. komplex, 2. tiszta képzetes,

Mely k. sz. ábr. azon □ csúcsait, amely egyik csúcsa a kezdőpont, és egységnyi hosszúságú oldala a valós tengelyen fekszik.
Milyen számok tartoznak a □ csúcsaihoz.

 1966. XI. 28.



I. $A_1(0;0)$ $A_2(1;0)$ $A_3(1;1)$ $A_4(0;1)$
II. $B_1(0;0)$ $B_2(0;1)$ $B_3(-1;1)$ $B_4(-1;0)$
III. $C_1(0;0)$ $C_2(-1;0)$ $C_3(-1;-1)$ $C_4(0;-1)$
IV. $D_1(0;0)$ $D_2(0;-1)$ $D_3(1;-1)$ $D_4(1;0)$

38. óra 1966. XII. 1.

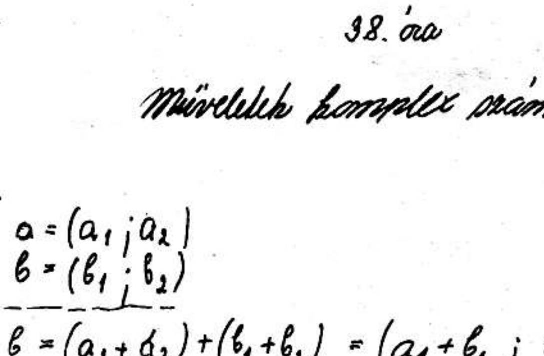
Műveletek komplex számokkal

Összeadás.

$$a = (a_1; a_2)$$
$$b = (b_1; b_2)$$
$$a + b = (a_1 + a_2) + (b_1 + b_2) = (a_1 + b_1; a_2 + b_2)$$

Komplex számokat úgy adunk össze, hogy összeadjuk a megfelelő részeket.

$$(3;4) + (5;2) = (3+5; 4+2) = (8;6)$$



Komplex számok összeadására érvényes a kommutatív és az asszociatív törvény.

$$a+b = b+a \quad K \qquad (a+b)+c = a+(b+c) \quad A$$

$$a(a_1;a_2) \qquad (a_1;a_2)+(b_1;b_2) = (a_1+b_1;\ a_2+b_2)$$
$$b(b_1;b_2) \qquad (b_1;b_2)+(a_1;a_2) = (b_1+a_1;\ b_2+a_2) = (a_1+b_1;\ a_2+b_2)$$

$$a+0 = a$$

H Bebizonyítani az ass. törvényt

$$a=(a_1;a_2) \qquad (a+b)+c = [(a_1;a_2)+(b_1;b_2)]+(c_1;c_2)$$
$$b=(b_1;b_2) \qquad (a_1+b_1;a_2+b_2)+(c_1;c_2)$$
$$c=(c_1;c_2) \qquad (a_1+b_1+c_1;a_2+b_2+c_2)$$

$$a+(b+c) = (a_1;a_2)+[(b_1;b_2)+(c_1;c_2)]$$
$$(a_1;a_2)+(b_1+c_1;b_2+c_2)$$
$$(a_1+b_1+c_1;\ a_2+b_2+c_2)$$
$$(a_1+b_1+c_1;a_2+b_2+c_2) = (a_1+b_1+c_1;a_2+b_2+c_2)$$

Szám. ki: $a(2;3)$
$b(8;5)$

$$a+b = (2;3)+(8;5) = [(2+8);(3+5)] = (10;8)$$

$$(2;3)+(3;1)=(5;4)$$
$$(8;5)+(2;0)=(10;5)$$
$$(3;-2)+(-1;3)=(2;1)$$
$$(2;1)+(-3;-1)=(-1;0)$$

 1966. XII. 5.

Kivonás:

$a=(a_1;a_2)$ $x=(x_1;x_2)$
$b=(b_1;b_2)$ $a+x=b$

$$(a_1;a_2)+(x_1;x_2)=(b_1;b_2) \quad / +(-a_1;-a_2)$$
$$(a_1;a_2)+(-a_1;-a_2)+(x_1;x_2)=(b_1;b_2)+(-a_1;-a_2)$$
$$(a_1-a_1;a_2-a_2)+(x_1;x_2)=(b_1-a_1;b_2-a_2)$$
$$(0;0)+(x_1;x_2)=(b_1-a_1;b_2-a_2)$$
$$\underline{(x_1;x_2)=(b_1-a_1;b_2-a_2)}$$

$$(2;3)-(-1;5)=(2+1;3-5)=(3;-2)$$
$$(2;1)-(0;2)=(2-0;1-2)=(2;-1)$$
$$(3;1)-(3;2)=(3-3;1-2)=(0;-1)$$
$$(-3;5,2)-(-2;4)=(-3+2;5,2-4)=(-1;1,2)$$
$$(-3;-\sqrt{2})-(-3;-2)=(-3+3;-\sqrt{2}+2)=(0;2-\sqrt{2})\doteq(0;59)$$

Szorzás:

$a=(a_1;a_2)$
$b=(b_1;b_2)$ A meghatározás nincs bizonyítva

$$a\cdot b=(a_1;a_2)(b_1;b_2)\ \underline{=(a_1\cdot b_1-a_2b_2;a_1b_2+a_2b_1)}$$

$$(3;2)(5;4)=(3.5-2.4;3.4+2.5)=(15-8;12+10)=(7;22).$$
$$(4;-1)(3;-2)=(12+2;-\cancel{8}-3)=(10;-11)$$
$$(2;5)(3;-4)=(2.3-5.-4;2.-4+5.3)=(26;7)$$
$$(-3;2)(1;-2)=(-3+4;+6+2)=(1;8)$$

Komplex számok szorzására érvényes a kommutatív, asszociatív és disztributív törvény:

$$a\cdot b=b.a \qquad (a.b).c=a(b.c) \qquad (a+b).c=a.c+b.c$$

Biz. $\underline{a.b}=(a_1;a_2)(b_1;b_2)=(a_1b_1-a_2\cdot b_2;a_1b_2+a_2b_1)=(b_1a_1-b_2a_2;b_1a_2+b_2a_1)=$
$=(b_1;b_2)(a_1;a_2)=\underline{b.a}$

$$b\cdot a=(b_1;a_2)(b_2;a_2)=(b_1a_1-b_2a_2;b_1a_2+b_2a_1)$$

$$(a.b)c=[(a_1;a_2)(b_1;b_2)].(c_1;c_2)=(\overbrace{a_1b_1-a_2b_2};\overbrace{a_2b_1+a_1b_2})(c_1;c_2)=$$
$$=[(a_1b_1-a_2b_2).c_1-(a_1b_2+a_2b_1).c_2;(a_1b_1-a_2b_2).c_2+(a_1b_2+a_2b_1).c_1]=$$
$$=(a_1b_1c_1-a_2b_2c_1-a_1b_2c_2-a_2b_1c_2;a_1b_1c_2-a_2b_2c_2+a_1b_2c_1+a_2b_1c_1).$$

$$a\cdot(b\cdot c) = (a_1;a_2)\cdot[(b_1;b_2)\cdot(c_1;c_2)] = (a_1;a_2)\cdot(b_1c_1 - b_2c_2\,;\,b_1c_2 + b_2c_1) =$$

$$= [a_1(b_1c_1 - b_2c_2) - a_2(b_1c_2 + b_2c_1)\,;\,a_1(b_1c_2 + b_2c_1) + a_2(b_1c_1 - b_2c_2)] =$$

$$= (a_1b_1c_1 - a_1b_2c_2 - a_2b_1c_2 - a_2b_2c_1\,;\,a_1b_1c_2 + a_1b_2c_1 + a_2b_1c_1 - a_2b_2c_2) =$$

$$= (a_1b_1c_1 - a_2b_1c_1 - a_1b_2c_2 - a_2b_1c_2\,;\,a_1b_1c_2 - a_2b_2c_2 + a_1b_2c_1 + a_1b_2c_1)$$

$$(a+b)\cdot c = ac + bc$$

$$(a+b)\cdot c = [(a_1;a_2)+(b_1;b_2)]\cdot(c_1;c_2) = (a_1+b_1;a_2+b_2)\cdot(c_1;c_2) =$$

$$= [(a_1+b_1)c_1 - (a_2+b_2)\cdot c_2\,;\,(a_1+b_1)c_2 + (a_2+b_2)\cdot c_1] =$$

$$= (a_1c_1 + b_1c_1 - a_2c_2 - b_2c_2\,;\,a_1c_2 + b_1c_2 + a_2c_1 + b_2c_1)$$

$$a\cdot c + bc = (a_1;a_2)(c_1;c_2) + (b_1;b_2)(c_1;c_2) =$$

$$= (a_1c_1 - a_2c_2\,;\,a_1c_2 + a_2c_1) + (b_1c_1 - b_2c_2\,;\,c_1b_2 + c_2b_1) =$$

$$= (a_1c_1 - a_2c_2 + b_1c_1 - b_2c_2\,;\,a_1c_2 + a_2c_1 + c_1b_2 + c_2b_1) =$$

$$= (a_1c_1 - a_2c_2 + b_1c_1 - b_2c_2\,;\,a_1c_2 + b_1c_2 + a_2c_1 + b_2c_1).$$

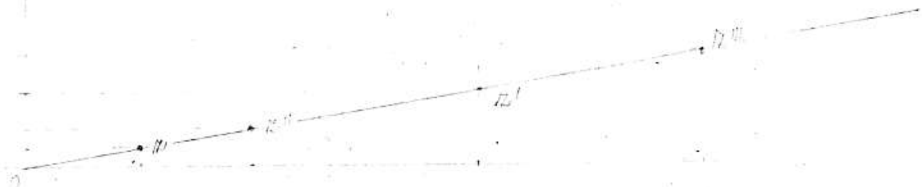
1966. XI. 8. 40. óra.

$z = (3 + \frac{1}{2}i)$

$z' = (12 + 2i)$

$z'' = (6 + i)$

$z''' = (18 + 3i)$



$$(2+3i)^3 = (2+3i)^2(2+3i) = (-5;12i)\cdot(2+3i) = -10 + 24i - 15i - 36 = \underline{-46+9i}$$

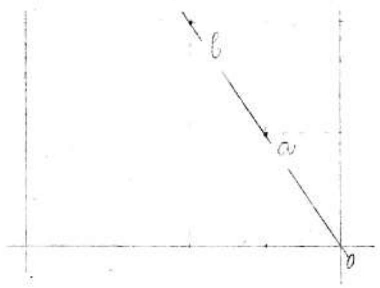
$$(3;2)\cdot(2;4) = (6-8\,;\,6+8) = (-2;14)$$

$$(2+3i)^3 = 8 + 36i - 54 - 27i = -46 + 9i$$

Ha egy komplex számot megszorzunk valamely valós számmal akkor az így kapott komplex szám képe hasonlósági helyzetben van az eredeti komplex szám képével és a hasonlósági középpont a 0 kezdőpont, a hasonlósági együttható pedig a valós szám.

Állapítsuk meg, hogy az a; b számok A; B képe a kezdőponttal egy egyenesen v. nem egy egyenesen fekszik, ha:

$a = (-2 + 3i)$ $\quad$ $b = (-4 + 6i)$. $\quad$ $[b = a \cdot 2]$



$a = (-\sqrt{2} + \frac{1}{2}i)$

$b = (6 + i)$

$(3;2) + (2;1) = (5;3)$

$(3;2) \cdot (2;1) = (6 - 2;\ 6 + 2) = (4;7) = (4;7)$

Legyen adott az $a = (3; -2i)$ szám. Úgy válasszátok meg a $b = (6 + b_2 i)$ $c = (c_1 + 10i)$ $\quad$ $d = (d_1 - 2/11\, i)$ $\quad$ $e = (-7/2 + e_2 i)$ számokat, hogy a b, c, d, e számok képe ugyanazon az egyenesen feküdjön mint az a szám képe.

$k_1 = \frac{6}{3} = 2$

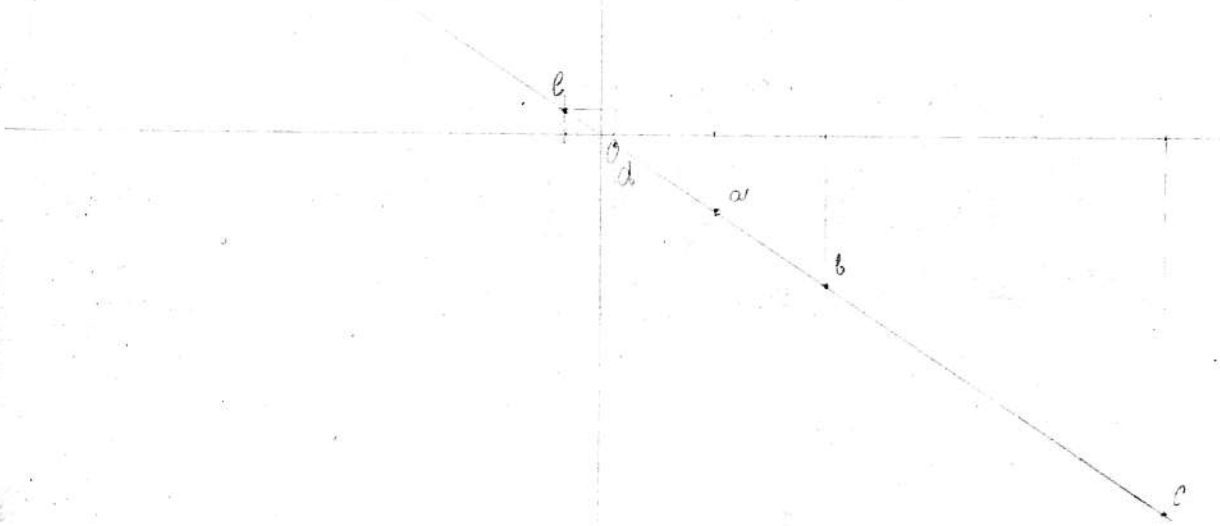
$\Downarrow$

$b_2 = 2 \cdot 2 = 4$

$c_1 = 15$

$d_1 = 3/11$

$e_2 \doteq \frac{2}{3}$



1966. XII. 12. 41. óra.

$(a_1;a_2)$ $(3;5)\cdot(2;-3) = (6+15;-9+10) = (21;1)$

$(2;-3);(a;b)$ $(2;-3)=(a;b)$ ha $a=2$; $b=-3$.

$x^2+1=0$ $(a_1;a_2)$

$x^2=-1$

$(a_1;a_2)^2 = (a_1;a_2)\cdot(a_1;a_2) = (a_1\cdot a_1 - a_2\cdot a_2;\ a_1\cdot a_2 + a_2\cdot a_1) =$
$\underline{(a_1^2-a_2^2;2a_1a_2)} = \underline{-1 = (-1;0)}$

$(a_1^2-a_2^2;2a_1a_2)=(-1;0)$

$a_1^2-a_2^2=-1$ $2.a_1a_2=0$ 1, e. $a_1=0$
2, v. $a_2=0$ a_2 - nem lehet 0 mert különben $(a_1;a_2)$ nem képz. sz.

$0-a_2^2=-1$

$a_2^2=1$

$a_2^2=\pm\sqrt{1}$

$a_2^1=\pm 1$ $a_1=0$ $(a_1;a_2)=(0;1)=i$ [Euler]
$(0;-1)=-i$

a_1+a_2i - algebrai alak $(a_1;a_2)$ - aritmetikai alak

$(a_1+a_2i)=(a_1;a_2)$

$(a_1;a_2) = (a_1;0)+(0;a_2)\ [(a_1;0)+(0;a_2)]$

$(a_2;0)\underline{(0;1)} = (0;-0;a_2+0) = (0;a_2)$

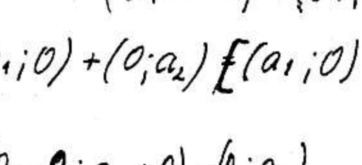
(a_1+a_2i)

1966. XII. 17. 42. óra **Komlex szám goniometriai alakja**

$[(3;4)+(2;\frac{1}{2})]\cdot(3;-1) = (5;4{,}5)\cdot(3;-1) = (15+4{,}5;-5+13{,}5) = 19{,}5 ; 8{,}5$

$\underline{z = a_1+a_2i = |z|(\cos\varphi + i\sin\varphi)}$

φ - argumentum
amplitudo



$\sin\varphi = \frac{a_2}{|z|}$

$\underline{a_2 = \sin\varphi\cdot|z|}$

$\cos\varphi = \frac{a_1}{|z|}$

$\underline{a_1 = \sin\varphi\cdot|z|}$

$z = a_1 + a_2i =$
$= |z|\cdot\cos\varphi + |z|i\sin\varphi =$
$= \underline{|z|(\cos\varphi + i\sin\varphi)}$

$|z|$ - a komplex szám abszolut értéke
φ - argumentum, amplitudo, irányszög

$|z|(\cos\varphi + i\cdot\sin\varphi)$ - komplex szám goniometriai alakja.

$$tg\,\varphi = \frac{a_2}{a_1}$$

$(3;4)\cdot(-2;5) = (-26;7)$
$(-2;3)\cdot(-3;-1) = (9;-7)$
$(-2;3)\cdot(-5;-2) = (16;-11)$
$(2;-2)\cdot[(3;-1)+(2;-1)] = (6;-14)$

$(3+2i)^2 = 5+12i$
$(2+6i)^2 = -32+24i$
$(2+4i)^2 = -12+16i$
$(2+3i)^3 = -46+9i$

$z = 2+3i$ $\qquad |z| = \sqrt{2^2+3^2} \doteq 3{,}6$

$tg\,\varphi = \frac{a_2}{a_1} = 1{,}5 \qquad \varphi \doteq 56°$

$z = |z|(\cos 56 + i\sin 56) \doteq 3{,}6(\cos 56° + i\sin 56°)$

$z = 5+2i \qquad z = 5{,}4(\cos 24{,}5° + i\sin 24{,}5°)$

$z = 6+2i \qquad z = 6{,}3(\cos 18{,}5° + i\cdot\sin 18{,}5°)$

$z = 8+2i \qquad z = 8{,}2(\cos 14° + i\cdot\sin 14°)$

$z = 1+8i \qquad z = 8{,}1(\cos 83° + i\sin 83°)$

43. óra. Ismétlés 1966. I. 12.

$i + i^2 + i^3 + i^4 + i^5 = i - 1 - i + 1 + i = i$

$z = \frac{\sqrt{ab}}{c^2} \qquad \log z = \frac{1}{2}\log a + \frac{1}{2}\log b - 2\cdot\log c$

$\log a\cdot b$

$$\frac{5+4i}{3-4i} = \frac{(5+4i)(3+4i)}{(3-4i)(3+4i)} = \frac{15+12i+20i-16}{9-12i+12i+16} = \frac{32i-1}{25} = -\frac{1}{25} + \frac{32i}{25}$$

$i^{77} = i$

$\log \frac{a}{b}$ $\quad c^x = a$, $c^y = b$

$\log \frac{c^x}{c^y} = \log c^{x-y} = x-y \cdot \log c = x \cdot \log c - y \cdot \log c$

$(1+i)(1-i)(2+3i) = 2 \cdot (2+3i) = 4+6i$

$\frac{2+5i}{3+i} = \frac{(2+5i)(3-i)}{9+1} = \frac{6+15i-2i+5}{10} = \frac{11+13i}{10} = \frac{11}{10} + \frac{13i}{10}$

$a^x = b \;/^m$ $\quad x = \log_a b \;/\cdot m$ $\quad m \cdot \log_a b = \log_a b^m$

$a^{xm} = b^m$ $\quad xm = \log_a b^m$

1967. 1. 14.

44.–45. óra
Dolgozatírás, feleltetés.

$\log(x+6) - \frac{1}{2}\log(2x-3) = 2 - \log 25$

$\log \frac{(x+6)}{\sqrt{2x-3}} = \log \frac{100}{25}$

$(x+6) = 4 \cdot \sqrt{2x-3} \;/^2$

$x^2 + 12x + 36 = 32x - 48$

$x^2 - 20x + 84 = 0$

$x_{12} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q} =$

$= 10 \pm \sqrt{100-84} =$

$= 10 \pm \sqrt{16}$.

$\underline{x_1 = 14}$ $\quad \underline{x_2 = 6}$

$i^{19} = -i$

$\frac{\log \frac{5}{3}(x-2)}{\log (x+2)} = 2$

$\log \frac{5}{3}(x+2) = \log (x+2)^2$

$\frac{5x+10}{3} = x^2 + 4x + 4$

$5x + 10 = 3x^2 + 12x + 12$

$0 = 3x^2 + 7x + 22$

$\left(x_{12} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-7 \pm \sqrt{49-12}}{6} \right.$

$\left. x_{12} = -7 \pm \sqrt{37} \right)$

$\log {}^5/_3 (x-2) = 2 \cdot \log (x+2)$

$\log \frac{5x-10}{3} = \log (x+2)^2$

$5x - 10 = 3x^2 + 12x + 12$

$0 = 3x^2 + 7x + 22$

$x_{12} = \frac{-7 \pm \sqrt{49-264}}{6} = \frac{-7 \pm \sqrt{-215}}{6}$

$$\frac{\log \frac{5}{3}(x-2)}{\log (x-2)} = (\log) = 2$$

$$x_{12} = \frac{+17 \pm \sqrt{256-264}}{6} =$$

$$= \frac{17 \pm \sqrt{-8}}{6}$$

$$\log \frac{5x-10}{3} = 2 \log (x-2)^{\#}$$

$$\frac{5x-10}{3} = x^2 - 4x + 4$$

$$5x - 10 = 3x^2 - 12x + 12$$

$$0 = 3x^2 - 17x + 22$$

$$\log \frac{5}{3}(x-2) = 2 \cdot \log (x-2)$$

$$\frac{5}{3}(x-2) = (x-2)^2 \quad / \cdot \frac{1}{x-2}$$

$$\frac{5}{3} = x - 2$$

$$5 = 3x - 6$$

$$11 = 3x$$

$$x = \frac{11}{3}$$

46. óra

1966. I. 18.

$$\frac{2+i}{2-i} - \frac{2-i}{2+i} = \frac{(2+i)(2+i)}{(2+i)(2-i)} - \frac{(2-i)(2-i)}{(2+i)(2-i)} = \frac{(2+i)(2+i) - (2-i)(2-i)}{5} =$$

$$= \frac{4+4i-1-4+4i+1}{5} = \frac{8i(\cancel{-2})}{5} = \frac{8i}{5}$$

47. óra

1967. I. 19.

$ax^2 + bx + c = 0 \quad a \neq 0$

általános alak

ax^2 - másodf. tag

c - abszolút tag

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a, b, c - ismert sz.

x - ismeretlen

$$D = b^2 - 4ac$$

1, $D > 0$ két különböző valós gyök.

2, $D = 0$ kettősgyök.

3, $D < 0$ konjugált komplex szám.

$x^2 + px + q = 0$ - normál alak

$\frac{b}{a} \quad \frac{c}{a}$

$$x_{12} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$$

$ax^2+bx+c=0$

1, $a=0 \quad b\neq 0 \quad c\neq 0 \longrightarrow bx+c=0 \longrightarrow x=-c/b$ elsőfokú

2, $b=0 \quad a\neq 0 \quad c\neq 0 \longrightarrow ax^2+c=0 \longrightarrow x_{1,2}=\pm\sqrt{-c/a}$ tiszta másodf. egy.

3, $c=0 \quad a\neq 0 \quad b\neq 0 \longrightarrow ax^2+bx=0$ abszolút tag nélküli

$x(ax+b)=0 \longrightarrow x_1=0$

$ax+b=0 \quad x_2=-\frac{b}{a}$

$x_1+x_2=-\frac{b}{a}=-p$ ~ |elsőfokú tag együtthatója|

$x_1\cdot x_2=\frac{c}{a}=q$

$a.(x-x_1).(x-x_2)=0$ ~ 2f.e. gyöktényezős alakja.

$3x^2-4x+6=0 \qquad x_{12}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{4\pm\sqrt{16-72}}{6}=\frac{4\pm\sqrt{-56}}{6}$

$x_1=\frac{4+\sqrt{56}}{6}\doteq\left(\frac{11,4}{6}\right)\doteq\frac{4}{6}+\frac{7,4}{6}i \qquad x_2=\frac{4}{6}-\frac{7,4i}{6}$

$x^2=64 \quad /\sqrt{\ }$

$x_{12}=\pm\sqrt{64}$

$x_1=8 \quad x_2=-8$

$5x^2-12x+64=0$

$x^2-\frac{12}{5}x+\frac{64}{5}=0$

$5x^2+12x=0$

$x(5x+12)=0$

$x_1=0$

$5x=-12$

$x_2=-\frac{12}{5}$

1967. II. 8. 48. óra 49. óra

$x_1=\frac{2}{3}-i \qquad x_1+x_2=-p=-\frac{b}{a} \qquad x^2+px+q=0$

$x_2=\frac{2}{3}+i \qquad x_1\cdot x_2=q=\frac{c}{a}$

$-p=x_1+x_2=\left(\frac{2}{3}-i\right)+\left(\frac{2}{3}+i\right)=\frac{4}{3}$

$q=\left(\frac{2}{3}-i\right)\left(\frac{2}{3}+i\right)=\frac{4}{9}+1=\frac{13}{9}$

$x^2+\frac{4}{3}x+\frac{13}{9}=0$

$x_{12}=-\frac{p}{2}\pm\sqrt{\frac{p^2}{4}-q}=\frac{4}{6}\pm\sqrt{\frac{16}{36}-\frac{13}{9}}$

$x_{12}=\frac{4}{6}\pm\sqrt{\frac{16-52}{36}}=\frac{4}{6}\pm\sqrt{\frac{36}{36}}$

$x_1=\frac{2}{3}+i \qquad x_2=\frac{2}{3}-i$

$x^2+10x+25=10x$

$x^2=-25$

$x_{1,2}=i\sqrt{25}$

$x_{1,2}=\pm i5$

$x_1=5i$

$x_2=-5i$

$5x^2+4x+8=0$

$x_{1,2}=\frac{-4\pm\sqrt{16-160}}{10}=\frac{-4\pm\sqrt{-144}}{10}=\frac{-4\pm 12i}{10}$

$x_1=-\frac{2}{5}+\frac{6}{5}i$

$x_2=-\frac{2}{5}-\frac{6}{5}i$

167. ferrelés

$5x^2+4x+8=0$

$\frac{x-3}{x}-\frac{x-2}{4}=\frac{1}{4x}$

$\frac{4(x-3)-x(x-2)}{4x}=\frac{1}{4x}$

$4x-12-x^2+2x=1$

$0=x^2-6x+13$

$x_{1,2}=\frac{+6\pm\sqrt{36-52}}{2}=\frac{6\pm\sqrt{-16}}{2}$;

$x_1=3+2i$

$x_2=3-2i$

49. ó.

1967. II. 8.

168/ 16, 17

169/ 18, 19, 20, 21, 22, 23

$x^2=2{,}5$

$x=\pm\sqrt{2{,}5}$

$x_1=\sqrt{2{,}5}$

$x_2=-\sqrt{2{,}5}$

$x^2=-25$

$x=i\sqrt{25}$

$x_1=-i5=-5i$

$x_2=5i$

$4x^2=49$

$2x=7$

$x=3{,}5$

$4x^2=-49$

$2x=7i$

$x=3{,}5i$

$x^2=-0{,}09$

$x=i\sqrt{0{,}09}$

$x_1=0{,}3i$

$x_2=-0{,}3i$

$x^2+0{,}0064=0$

$x^2=-0{,}0064$

$x=i\sqrt{0{,}0064}$

$x_1=0{,}08i$

$x_2=0{,}08i$

$x^2=-2{,}85$

$x=i\sqrt{2{,}85}$

$x_1=1{,}69i$

$x_2=-1{,}69i$

$x^2+0{,}8762=0$

$x^2=-0{,}8762$

$x_1=0{,}935i$

$x_2=-0{,}935i$

$0{,}25x^2+1600=0$

$0{,}25x^2=-1600$

$x^2=-400$

$x_1=20i$

$x_2=-20i$

$0{,}04x^2=900$

$x^2=\frac{90000}{4}$

$x^2=22500$

$x_1=150$

$x_2=-150$

$0{,}635x^2-84{,}5=0$

$0{,}635x^2=84{,}5$

$x^2\doteq 131{,}5$

$x_1\doteq \cancel{3{,}1}\ 11{,}5$

$x_2\doteq -11{,}5$

$9{,}32x^2+0{,}0462=0$

$x^2=-\frac{0{,}0462}{9{,}32}$

$x^2=-0{,}002$

$x_1\doteq 0{,}045i$

$x_2\doteq -0{,}045i$

$$\frac{52}{x-10} + 10 + x = \frac{52}{10-x}$$

$$\frac{52+10x-100+x^2-10x}{x-10} = \frac{52}{10-x}$$

$$\frac{x^2-48}{x-10} = \frac{52}{10-x}$$

$$(x^2-48)(10-x) = 52(x-10)$$

$$10x^2-480-x^3+48x = 52x-520$$

$$-x^3+10x^2-4x+40=0$$

$$\frac{-6}{x-4} - \frac{x+1}{3} = 11$$

$$\frac{-18-(x+1)(x-4)}{3\cdot(x-4)} = 11$$

$$-18-x^2-x+4x+4 = 33x-132$$

$$-x^2-30x+118=0$$

$$x^2+30x-118=0$$

$$x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4}-q} = -15 \pm \sqrt{225+118}$$

$$x_{1,2} = -15 \pm \sqrt{343} = -15 \pm 18{,}5$$

$$x_1 \doteq 3{,}5 \quad x_2 \doteq -33{,}5$$

$$\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{x}$$

$$\frac{(x+b)+(x+a)}{(x+a)(x+b)} = \frac{1}{x}$$

$$x(2x+a+b) = x^2+ax+bx+ab$$

$$2x^2+\underline{ax}+\underline{bx} = x^2+\underline{ax}+\underline{bx}+ab$$

$$x^2 = a.b$$

$$x_{1,2} = \pm\sqrt{a.b}$$

$$2x^2+8x+40=0$$

$$x_{1,2} = \frac{-8 \pm \sqrt{64-320}}{4} = \frac{-8 \pm \sqrt{256}}{4}$$

$$x_1 = \frac{-8+\sqrt{384}\,i}{4} \quad x_2 = \frac{-8-\sqrt{386}\,i}{4}$$

$$x_1 = -2+4i \quad x_2 = -2-4i$$

$$4x^2-4x+4=0$$

$$x_{1,2} = \frac{4 \pm \sqrt{16-64}}{8} = \frac{4 \pm \sqrt{-48}}{8}$$

$$x_1 = \frac{1}{2} + \frac{i\sqrt{48}}{8} \quad x_2 = \frac{1}{2} - \frac{i\sqrt{48}}{8}$$

$$5{,}63x^2-6{,}05x+1{,}92=0$$

$$x_{1,2} \doteq \frac{6{,}05 \pm \sqrt{36{,}5-43}}{11{,}26} = \frac{6{,}05 \pm \sqrt{-6{,}5}}{11{,}26} = \frac{6{,}05 \pm i.2{,}54}{11{,}26}$$

$$x_1 \doteq \frac{6{,}05}{11{,}26} + \frac{2{,}54i}{11{,}26} \quad x_2 \doteq \frac{6{,}05}{11{,}26} - \frac{2{,}54i}{11{,}26}$$

$$x^2+x+1=0$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1-4}}{2}$$

$$x_1 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$x_2 = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

$$x_2-6x+7=0$$

$$x_{1,2} = \frac{6 \pm \sqrt{36-28}}{2} = \frac{6 \pm \sqrt{8}}{2}$$

$$x_1 = 3+\sqrt{2}$$

$$x_2 = 3-\sqrt{2}$$

$ax^2 + bx + c = 0 \qquad D < 0$

$4a^2x^2 + 4abx + b^2 = b^2 - 4ac$

$(2ax + b)^2 + (-D) = 0$

$(2ax + b)^2 + |D| = 0$

$$x_{12} = \frac{-b \pm i\sqrt{|D|}}{2a}$$

1967. I. 9.

$x_1 = -3 \qquad x_1 \cdot x_2 = q \qquad (-3)\cdot\left(-\frac{1}{3}\right) = 1 = q$

$x_2 = -\frac{1}{3} \qquad x_1 + x_2 = -p \qquad (-3) + \left(-\frac{1}{3}\right) = -\frac{10}{3} = -p$

$x^2 + \frac{10}{3}x + 1 = 0 \qquad x_{12} = \frac{-10 \pm \sqrt{100 - 36}}{6} = \frac{-10 \pm 8}{6};$

$3x^2 + 10x + 3 = 0 \qquad x_1 = -3 \qquad x_2 = -\frac{2}{6} = -\frac{1}{3}$

$(x - x_1)(x - x_2) = 0$

$(x + 3)\left(x + \frac{1}{3}\right) = 0 = x^2 + 3x + \frac{1}{3}x + 1 = x^2 + \frac{10}{3}x + 1 = 3x^2 + 10x + 3$

$x^2 + 14x - 15 = 0 \qquad x_1 = 1 \rightarrow x_2 = -15$

$x^2 - 3x + 3 = 0 = \left[x - \left(1{,}5 + i\sqrt{3/4}\right)\right] \cdot \left[x - \left(1{,}5 - i\sqrt{3/4}\right)\right].$

50. Hf.

1967. I. 9.

174/32 $a \sim i$

33 $a \sim e$

34 $a \sim f$.

$x^2 - 1{,}5x - 13{,}5 = 0 \qquad -4{,}5 \quad 3$

$x^2 + \frac{29}{12}x + \frac{5}{4} = 0 \qquad 3/4 \quad 5/3$

$x^2 - 9{,}5x + 23{,}26 = 0 \qquad -5{,}3 \quad -4{,}2$

$x^2 + 6x + 13 = 0 \qquad 3 - 2i \quad 3 + 2i$

$x^2 + (t + u)x + tu = 0 \qquad t \quad u$

$x^2 + 2\sin 45^\circ \cdot \cos -15^\circ + \sin 60^\circ \cdot \sin 30^\circ = 0 \qquad \sin 30^\circ \quad \sin 60^\circ$

$x^2 + 16x + 64 = 0 \qquad 8, \ 8$

$x^2 - 0{,}5x + 6{,}25 = 0 \qquad 0{,}25 \quad 0{,}25$

$x^2 - 0{,}645x = 0 \qquad 0 \quad -0{,}645$

$x^2+2x-63=0 \quad (x=-9)$

$x_1 = \frac{-63}{-9} = 7$

$x^2-2{,}4x-0{,}81=0 \quad (x=-0{,}3)$

$x_1 = \frac{-0{,}81}{-0{,}3} = 2{,}7$

$x^2+4=0 \quad (x=4i)$

$x_1 = \frac{4}{4i} = \frac{1}{i} = \frac{i}{-1} = -i$

$x^2+6x+10=0 \quad (x=-3-i)$

$x_1 = -3+i$

$x^2-24{,}8x+153{,}76=0 \quad (x=12{,}4)$

$x_1 = -p-x = 24{,}8-12{,}4 = 12{,}4$

$x^2-11x+24=0$

$x_{12} = \frac{11}{2} \pm \sqrt{\frac{121}{4}-24} = \frac{11}{2} \pm \sqrt{\frac{25}{4}} = \frac{11}{2} \pm \frac{5}{2} \qquad x_1=8 \qquad x_2=3$

$(x-8)(x-3)=0$

$x^2+2{,}8x-5{,}88=0$

$x_{12} = -1{,}4 \pm \sqrt{1{,}56+5{,}88} = -1{,}4 \pm \sqrt{8{,}44} \doteq -1{,}4 \pm 2{,}9 \qquad x_1 \doteq 1{,}5 \qquad x_2 \doteq -4{,}3$

$(x-1{,}5)(x+4{,}3) \doteq 0$

$x^2-8{,}2x+16{,}81=0$

$x_{12} = 4{,}1 \pm \sqrt{16{,}81-16{,}81} = 4{,}1$

$(x-4{,}1)(x-4{,}1)=0$

$x^2-\frac{10}{3}x+1=0$

$x_{12} = \frac{10}{6} \pm \sqrt{\frac{100}{36}-\frac{36}{36}} = \frac{10}{6} \pm \sqrt{\frac{64}{36}} = \frac{10}{6} \pm \frac{8}{6} \qquad x_1=3 \qquad x_2=\frac{1}{3}$

$(x-3)(x-\frac{1}{3})=0$

$x^2-10x+29=0$

$x_{12} = 5 \pm \sqrt{25-29} = 5 \pm 2i$

$[x-(5+2i)] \cdot [x-(5-2i)] = 0$

$3x^2-0{,}72x=0 \qquad x^2-0{,}24x=0 \qquad x-0{,}24=0 \qquad x=0{,}24$

$(x-0)(x-0{,}24)=0$

51. - 2. óra 1967. I. 11.

$11x^2 = 85184$

$(2x+3)(3x-4) + (4x-5)(5x+6) = 0$

$(3-2x)^2 + (x+6)^2 = 5$

$$11x^2 = 85184$$
$$x^2 = 7744$$
$$x_{1,2} = \pm\sqrt{7744}$$
$$x_{1,2} = \pm 88$$

$$(2x+3)(3x-4) + (4x-5)(5x+6) = 0$$
$$6x^2 + x - 12 + 20x^2 - x - 30 = 0$$
$$26x^2 - 42 = 0$$
$$13x^2 - 21 = 0$$
$$x_{1,2} = \pm\sqrt{\frac{21}{13}}$$

$$(3-2x)^2 + (x+6)^2 = 5$$
$$9 - 12x + 4x^2 + x^2 + 12x + 36 = 5$$
$$5x^2 + 45 = 5$$
$$5x^2 = -40$$
$$x_{1,2} = \pm i\sqrt{8}$$

$$\frac{x^2}{15} - \frac{5}{2} = \frac{20 - x^2}{30} \quad /\cdot 30$$
$$2x^2 - 75 = 600 - x^2$$
$$3x^2 = 675$$
$$x^2 = 225$$
$$x_{12} = \pm\sqrt{225}$$
$$x_{12} = \pm 15$$

$$3 + x = 1 + \frac{4}{2-x} \quad /\cdot 2-x$$
$$\left(6 - 2x + 2x - x^2 = 2 - x + 4\right.$$
$$\left.0 = x^2 - x\right)$$

$$6 - 3x + 2x - x^2 = 2 - x + 4$$
$$-x^2 = 0$$
$$x_{12} = 0$$

$$\frac{(15+2x)(x-5)}{5+4x} = x$$

$$15x + 2x^2 - 75 - 10x = 5x + 4x^2$$

$$-75 = 2x^2$$

$$x_{1,2} = \pm i\sqrt{\frac{75}{2}} = \pm i\sqrt{37,5} \doteq 6,1$$

$$\left(\begin{array}{l} x_{1,2} = \pm i \frac{5}{\sqrt{2}} \\ x_{1,2} = \pm i \frac{5 \cdot \sqrt{2}}{2} \end{array} \right)$$

$$x^2 : (x^2+1) = 441 : 841$$

$$\frac{x-\sqrt{3}}{2} + \frac{x \cdot \sqrt{3}}{x+\sqrt{3}} = \sqrt{3} \quad / \cdot 2 \cdot (x+\sqrt{3})$$

$$\sqrt{5+x} - \sqrt{5-x} = 2$$

$$841x^2 = 441x^2 + 441$$

$$400x^2 = 441$$

$$x^2 = \frac{441}{400}$$

$$x_{12} = \pm \frac{\sqrt{441}}{20} = \pm \frac{21}{20} \qquad x_1 = \frac{21}{20} \qquad x_2 = -\frac{21}{20}$$

$$(x-\sqrt{3})(x+\sqrt{3}) + 2 \cdot x \cdot \sqrt{3} = 2 \cdot \sqrt{3}(x+\sqrt{3})$$

$$x^2 - 3 + 2x\sqrt{3} - 2 \cdot \sqrt{3} \cdot x - 6 = 0$$

$$x^2 = 9$$

$$x_{12} = \pm 3 \qquad x_1 = 3 \qquad x_2 = -3$$

$$5 + x + 2 \cdot \sqrt{5+x} \cdot \sqrt{5-x} + 5 - x = 4$$

$$-2\sqrt{5+x} \cdot \sqrt{5-x} = -6 \quad /^2$$

$$4 \cdot (5+x)(5-x) = 36$$

$$4 \cdot (25 - x^2) = 36$$

$$25 - x^2 = 9$$

$$16 = x^2$$

$$x_{12} = 4 \qquad x_1 = 4 \qquad x_2 = -4$$

$x^2-4x-7=0$

$\underline{x^2-12x-63=0}$ – a gyökei az 1. 3x-osai.

174/35

$$x^2-4x+7=0$$

a.) 5x. $\quad \underline{x^2-20x+175=0}$

b.) +5 $\quad \underline{x^2+6x+102=0}$

$$\left(q = (x_1+5)(x_2+5) = x_1 x_2 + 10q \right) \qquad q^2+15q+25$$

$$7+70+25=102$$

$$q^* = (x_1+5)(x_2+5) = x_1 \cdot x_2 + $$
$$+5x_1+5x_2+25=$$
$$= q+5\cdot p+25$$
$$7+(-20)+25$$

174/36 $\quad x^2+0{,}2x-0{,}8=0$

a.) $-x_1 x_2$ $\quad \underline{x^2+0{,}2x+0{,}8=0}$

b.) recipr. $\quad \underline{x^2+0{,}25x-\frac{1}{0{,}8}=0}$

178/42

$$-p^* = \frac{1}{x_1}+\frac{1}{x_2} = \frac{x_2+x_1}{x_1 x_2} = \frac{-p}{q}$$

52. Hf

1967. II. 12.

178/42 $\quad 2x^2-8x-40=0$

$x^2-4x-20=0$

$$x_{1,2} = 2\pm\sqrt{4+20} = 2\pm\sqrt{24}$$
$$x_1 = 2+\sqrt{24} \qquad x_2 = 2-\sqrt{24}$$

$x^2+x+1=0$

$$x_{1,2} = -\frac{1}{2}\pm\sqrt{\frac{1}{4}-1} = -\frac{1}{2}\pm\sqrt{-\frac{3}{4}}$$
$$x_1 = -\frac{1}{2}+i\frac{\sqrt{3}}{2} \qquad x_2 = -\frac{1}{2}-i\frac{\sqrt{3}}{2}$$

$x^2-6x+7=0$

$$x_{1,2} = 3\pm\sqrt{9-7} = 3\pm\sqrt{2}$$
$$x_1 = 3+\sqrt{2} \qquad x_2 = 3-\sqrt{2}$$

$5{,}63x^2-6{,}05x+1{,}92=0$

$$x_{1,2} = \frac{-b\pm\sqrt{b^2-4ac}}{2a} \doteq \frac{6{,}05\pm\sqrt{36-43}}{11{,}2} \doteq$$
$$= \frac{6\pm\sqrt{-7}}{11} \; i$$

$$x_1 \doteq \frac{6+i\sqrt{7}}{11} \qquad x_2 \doteq \frac{6-i\sqrt{7}}{11}$$