1967. II. 15. 53. óra

Binomikus egyenletek –

két tagból állnak. Az első tag az ismeretlen valamely hatványát tartalmazza, a második tag pedig az abszolút tag.

Általános alakja:

$$ax^n \pm b = 0 \quad \rightarrow \quad y^n \pm 1 = 0$$

$$y^n + 1 = 0 \qquad y^n - 1 = 0$$

$$y^n = -1 \qquad y^n = 1$$

$$y = \sqrt[n]{-1} \qquad y = \sqrt[n]{1}$$

Másodfokú binomikus egyenletek:

$$y^2 - 1 = 0$$

$$(y-1)(y+1) = 0$$

$$y - 1 = 0 \rightarrow y_1 = 1$$

$$y + 1 = 0 \rightarrow y_2 = -1$$

$$y^2 - 1 = 0$$

$$y^2 = 1$$

$$y = \sqrt{1}$$

$$y_{1,2} = \pm 1$$

$$y^2 + 1 = 0$$

$$(y+i)(y-i) = 0$$

$$y + i = 0 \rightarrow y_1 = -i$$

$$y - i = 0 \rightarrow y_2 = i$$

$$y^2 + 1 = 0$$

$$y^2 = -1$$

$$y = \sqrt{-1}$$

$$y_{1,2} = \pm i$$

$$x^2 + a = 0$$

$$x^2 = -a$$

$$x = \sqrt{-a}$$

$$x = \sqrt{-1} \cdot \sqrt{a}$$

$$x_{1,2} = \pm i\sqrt{a}$$

$$x^2 - a = 0$$

$$x^2 = a$$

$$x = \sqrt{a}$$

$$x = \sqrt{1} \cdot \sqrt{a}$$

$$x_{1,2} = \pm \sqrt{a}$$

Harmadfokú binomikus egyenletek:

$$A^3 - B^3 = (A-B)(A^2 + AB + B^2)$$

$$y^3 - 1 = 0$$

$$(y-1)(y^2 + y + 1) = 0$$

$$y - 1 = 0 \rightarrow y_1 = 1$$

$$y^2 + y + 1 = 0 \rightarrow y_{2;3} = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$$

$$y_2 = \frac{-1 + i\sqrt{3}}{2} = \varepsilon_1 \qquad y_3 = \frac{-1 - i\sqrt{3}}{2} = \varepsilon_2$$

$y^3 - 1 = 0$

$y^3 = 1$

$y_{123} = \sqrt[3]{1}$

$\sqrt[3]{1} = 1, \quad \frac{-1+i\sqrt{3}}{2}, \quad \frac{-1-i\sqrt{3}}{2}$

$x^3 - a = 0$

$x^3 = a$

$x = \sqrt[3]{a}$

$x = \sqrt[3]{a} \cdot \sqrt[3]{1}$

$x_1 = \sqrt[3]{a} \cdot 1$

$x_2 = \sqrt[3]{a} \cdot \frac{-1+i\sqrt{3}}{2}$

$x_3 = \sqrt[3]{a} \cdot \frac{-1-i\sqrt{3}}{2}$

$x^3 - 27 = 0$

$x^3 = 27$

$x_1 = \sqrt[3]{27} \cdot \sqrt[3]{1} = 3 \cdot 1 = 3$

$x_2 = \sqrt[3]{27} \cdot \frac{-1+i\sqrt{3}}{2} = 3 \cdot \frac{-1+i\sqrt{3}}{2} =$

$= \frac{-3+3i\sqrt{3}}{2} = \frac{-3+i\sqrt{27}}{2}$

$x_3 = \sqrt[3]{27} \cdot \frac{-1-i\sqrt{3}}{2} = \frac{-3-i\sqrt{27}}{2}$

$y^3 + 1 = 0$

$A^3 + B^3 = (A+B)(A^2 - AB + B^2)$

$y^3 + 1 = (y+1)(y^2 - y + 1) = 0$

$y + 1 = 0 \rightarrow y_1 = -1$

$y^2 - y + 1 = 0$

$y_{2,3} = \frac{1 \pm \sqrt{1-4}}{2}$

$y_{2,3} = \frac{1 \pm i\sqrt{3}}{2}$

$y_2 = \frac{1+i\sqrt{3}}{2}$

$y_3 = \frac{1-i\sqrt{3}}{2}$

$x^3 + a = 0$

$x^3 = -a$

$x = \sqrt[3]{-a}$

$x_{1,2,3} = \sqrt[3]{a} \cdot \sqrt[3]{-1}$

53. Hf.

1967.11.15.

$x^3 + 27 = 0$

$x^3 = -27$

$x = \sqrt[3]{-27}$

$x = \sqrt[3]{27} \cdot \sqrt[3]{-1}$

$x_1 = -3$

$x_2 = 3 \cdot \frac{1+i\sqrt{3}}{2}$

$x_3 = 3 \frac{1-i\sqrt{3}}{2}$

1967. I. 18. 54., 55. óra

Gyakorlat

$$(x-3)^3-(27\cdot(x+7))+9x^2=0$$

$$x^3-9x^2+27x-27-27x-189+9x^2=0$$

$$x^3-216=0$$

$$x^3=\sqrt[3]{216}\cdot\sqrt[3]{1}$$

$$x_1=6$$

$$x_2=6\cdot\frac{-1+i\sqrt{3}}{2}=3\cdot(-1+i\sqrt{3})=-3+3i\sqrt{3}$$

$$x_3=6\frac{-1-i\sqrt{3}}{2}=3\cdot(-1-i\sqrt{3})=-3-3i\sqrt{3}$$

$$x^3-216=0$$

$$(x^3-6^3)=0=(x-6)(x^2-6x+36)$$

$$x-6=0\rightarrow x_1=6$$

$$x^2-6x+36$$

$$x_{2,3}=\frac{-6\pm\sqrt{36-144}}{2}=\frac{-6\pm\sqrt{108}}{2}=\frac{-6\pm\sqrt{36.3}}{2}=\frac{-6\pm6\cdot\sqrt{3}}{2}$$

$$x_{2,3}=3\pm3i\sqrt{3} \qquad x_2=-3+3i\sqrt{3} \qquad x_3=-3-3i\sqrt{3}$$

$$x^3+125=0$$

$$x^3=-5^3$$

$$\left.\begin{array}{l} x_1=-5 \\ x_2=+5\frac{1+i\sqrt{3}}{2} \\ x_3=+5\frac{1-i\sqrt{3}}{2} \end{array}\right\}$$

$$x^3-729=0$$

$$x^3=729$$

$$x^3=\sqrt[3]{729}\cdot\sqrt[3]{1}$$

$$x_1=9.$$

$$x_2=9\cdot\frac{-1+i\sqrt{3}}{2}=\frac{-9+9i\sqrt{3}}{2}$$

$$x_3=9\cdot\frac{-1-i\sqrt{3}}{2}=\frac{-9-9i\sqrt{3}}{2}$$

$$(x+5)^3 = 15x(x+5)$$

$$x^3 + \underline{15x^2} + \underline{75x} + 125 = \underline{15x^2} + \underline{75x}$$

$$x^3 = -125 \qquad\qquad x_1 = -5$$

$$x^3 = -1 \cdot 5^3 \qquad\qquad x_2 = 5 \cdot \frac{1+i\sqrt{3}}{2}$$

$$x_{1,2,3} = \sqrt[3]{-1} \cdot 5 \qquad\qquad x_3 = 5 \cdot \frac{1-i\sqrt{3}}{2}$$

Negyedfokú binom-egyenletek

$$y^4 - 1 = 0 \qquad y^4 = 1 \qquad y = \sqrt[4]{1}$$

$$y^4 - 1^4 = 0$$

$$(y^2)^2 - (1^2)^2 = 0 = (y^2-1)(y^2+1)$$

$$(y^2 - 1) = 0 \qquad y^2 + 1 = 0$$

$$y^2 = 1 \qquad y^2 = -1$$

$$y = \sqrt{1} \qquad y = \sqrt{-1}$$

$$y_1 = 1 \qquad y_3 = i$$

$$y_2 = -1 \qquad y_4 = -i$$

$$\sqrt[4]{1} = 1$$

$$\sqrt[4]{1} = -1$$

$$\sqrt[4]{1} = i$$

$$\sqrt[4]{1} = -i$$

$$x^4 - 16 = 0$$

$$x^4 = 16 \quad \longrightarrow \quad x_1 = 2 \quad x_2 = -2 \quad x_3 = 2i \quad x_4 = -2i$$

$$x = \sqrt[4]{16} \cdot \sqrt[4]{1}$$

$$y^4 + 1 = 0 \qquad y^4 = -1 \qquad y = \sqrt[4]{-1}$$

$$y^4 + 1 = (y^2 + i)(y^2 - i) = 0$$

$$(y^2 + i) = 0 \qquad y^2 - i = 0$$

$$y^2 = -i \qquad y^2 = i$$

$$y = \sqrt{-i} \qquad y = \sqrt{i}$$

$$y_1 = \sqrt{-i} \qquad y_3 = \sqrt{i}$$

$$y_2 = -\sqrt{-i} \qquad y_4 = -\sqrt{i}$$

$$\sqrt[4]{-1} = \sqrt{i}$$

$$\sqrt[4]{-1} = -\sqrt{i}$$

$$\sqrt[4]{-1} = \sqrt{-i}$$

$$\sqrt[4]{-1} = -\sqrt{-i}$$

A $\sqrt{i}$ és $\sqrt{-i}$ értékét a következőképpen számítjuk ki.

$(1+i)^2 = 1 + 2i + i^2 = 1 + 2i - 1$

$(1+i)^2 = 2i$

$i = \frac{(1+i)^2}{2} \quad /\sqrt{\ }$

$\sqrt{i} = \sqrt{\frac{(1+i)^2}{2}} = \frac{1+i}{\sqrt{2}} = \frac{(1+i)\cdot\sqrt{2}}{2}$

$\sqrt{i} = \frac{(1+i)\sqrt{2}}{2}$

$(1-i)^2 = 1 - 2i + i^2 = 1 - 2i - 1 = -2i$

$(1+i)^2 = -2i$

$-i = \frac{(1-i)^2}{2} \quad /\sqrt{\ }$

$\sqrt{-i} = \sqrt{\frac{(1-i)^2}{2}} = \frac{1-i}{\sqrt{2}} = \frac{(1-i)\sqrt{2}}{2}$

$\sqrt{-i} = \frac{(1-i)\sqrt{2}}{2}$

$x^4 + 16 = 0$

$x^4 = -16$

$x_{1,2,3,4} = \sqrt[4]{-1} \cdot \sqrt[4]{16}$

$x_1 = 2\sqrt{i} \quad x_2 = -2\sqrt{i} \quad x_3 = 2\sqrt{-i} \quad x_4 = 2\sqrt{-i}$

$x^4 + 16 = 0$

$(x^2 + 4i)(x^2 - 4i)$

1967. V. 18. 55. H.

$x^4 + 81 = 0$

$x_1 = 3 \cdot \sqrt{i}$

$x_2 = 3 \cdot -\sqrt{i}$

$x_3 = 3 \cdot \sqrt{-i}$

$x_4 = 3 \cdot -\sqrt{-i}$

$x^4 - 81 = 0$

$x_1 = 3$

$x_2 = -3$

$x_3 = 3i$

$x_4 = -3i$

56. óra 1967. II. 20.

A negyedfokú binom egyenletet gyakran helyettesítéssel is megoldják.

$$y^4+1=0 \quad \Big/ \cdot \frac{1}{y^2}$$

$$y^2+\frac{1}{y^2}=0 \qquad \Big| \quad y+\frac{1}{y}=z \quad \Big/^2$$

$$y^2+2+\frac{1}{y^2}=z^2$$

$$y^2+\frac{1}{y^2}=z^2-2$$

$$z^2-2=0$$

$$z=\pm\sqrt{2}$$

$$z_1=\sqrt{2}$$

$$z_2=-\sqrt{2}$$

$$y+\frac{1}{y}=z_1 \qquad\qquad y+\frac{1}{y}=z_2$$

$$y+\frac{1}{y}=\sqrt{2} \qquad\qquad y+\frac{1}{y}=-\sqrt{2}$$

$$y^2+1=y\sqrt{2} \qquad\qquad y^2+1=-y\sqrt{2}$$

$$y^2-y\sqrt{2}+1=0 \qquad\qquad y^2+y\sqrt{2}+1=0$$

$$y_{12}=\frac{\sqrt{2}\pm\sqrt{2-4}}{2}=\frac{\sqrt{2}\pm\sqrt{-2}}{2}=\frac{\sqrt{2}\pm i\sqrt{2}}{2}=\frac{(1\pm i)\sqrt{2}}{2}$$

$$y_{34}=\frac{-\sqrt{2}\pm\sqrt{2-4}}{2}=\frac{-\sqrt{2}\pm\sqrt{-2}}{2}=\frac{\sqrt{2}\pm i\sqrt{2}}{2}=\frac{(-1\pm i)\sqrt{2}}{2}$$

$$y_1=\frac{(1+i)\sqrt{2}}{2} \qquad y_2=\frac{(1-i)\sqrt{2}}{2}$$

$$y=\frac{(-1+i)\cdot\sqrt{2}}{2} \qquad y_4=\frac{(-1-i)\sqrt{2}}{2}$$

Ötödfokú binom egyenletek

$$y^5-1=0 \qquad y^5=1 \qquad y=\sqrt[5]{1}$$

$$y^5-1=(y-1)(y^4+y^3+y^2+y+1)=0$$

$$y-1=0 \rightarrow \underline{\underline{y_1=1}} \qquad y^4+y^3+y^2+y+1=0 \quad \Big/ \cdot\frac{1}{y^2}$$

$$y^2+y+1+\frac{1}{y}+\frac{1}{y^2}=0$$

$$\left(y^2+\frac{1}{y^2}\right)+\left(y+\frac{1}{y}\right)+1=0 \qquad \left\}\begin{array}{l} y+\frac{1}{y}=z \\ y^2+\frac{1}{y^2}=z^2-2\end{array}\right.$$

$$z^2-2+z+1=0$$

$$z^2+z-1=0$$

$$z_{12}=\frac{-1\pm\sqrt{1+4}}{2}=\frac{-1\pm\sqrt{5}}{2}$$

$$z_1=\frac{-1+\sqrt{5}}{2} \qquad z_2=\frac{-1-\sqrt{5}}{2}$$

$$y+\frac{1}{y}=\frac{-1+\sqrt{5}}{2} \quad \Big/ \cdot 2y$$

$$2y^2+2=-y+y\sqrt{5}$$

$$2y^2-(-1+\sqrt{5})y+2=0$$

$$y_{2,3}=\frac{-1+\sqrt{5}\pm\sqrt{(-1+\sqrt{5})^2-16}}{4}=$$

$$=\frac{-1+\sqrt{5}\pm\sqrt{1-2\sqrt{5}+5-16}}{4}=\frac{-1+\sqrt{5}\pm\sqrt{-2\sqrt{5}-10}}{4}=\frac{-1+\sqrt{5}\pm i\sqrt{2(\sqrt{5}+5)}}{4}$$

$$\underline{\underline{y_{2,3}=\frac{\sqrt{5}-1}{4}\pm i\,\frac{\sqrt{2(5+\sqrt{5})}}{4}}}$$

$$y+\frac{1}{y}=z_2$$

$$y+\frac{1}{y}=\frac{-1-\sqrt{5}}{2}$$

$$y+\frac{1}{y}=-\frac{1+\sqrt{5}}{2} \quad /\cdot 2y$$

$$2y^2+2=-(1+\sqrt{5})\cdot y$$

$$2y^2+(1+\sqrt{5})y+2=0$$

$$y_{4,5}=\frac{-(1+\sqrt{5})\pm\sqrt{(1+\sqrt{5})^2-16}}{4}=$$

$$=\frac{-(1+\sqrt{5})\pm\sqrt{(1+2\sqrt{5}+5-16)}}{4}=$$

$$=\frac{-(1+\sqrt{5})\pm\sqrt{2\sqrt{5}-10}}{4}=$$

$$=\frac{-(1+\sqrt{5})\pm\sqrt{2(\sqrt{5}-5)}}{4};$$

$$\underline{\underline{y_4=\frac{-1-\sqrt{5}+\sqrt{2(\sqrt{5}-5)}}{4}}}$$

$$\underline{\underline{y_5=\frac{-1-\sqrt{5}-\sqrt{2(\sqrt{5}-5)}}{4}}}$$

$$y^5+1=0 \qquad y^5=-1 \qquad y=\sqrt[5]{-1}$$

$$y^5+1=(y+1)(y^4-y^3+y^2-y+1)=0$$

$$\underline{\underline{y_1=-1}}$$

$$y^2-y+1-\frac{1}{y}+\frac{1}{y^2}=0 \qquad z=y+\frac{1}{y}$$

$$\left(y^2+\frac{1}{y^2}\right)-\left(y+\frac{1}{y}\right)+1=0$$

$$z^2+2-z+1=0$$

$$z^2-z+3=0$$

$$z_{1,2}=\frac{1\pm\sqrt{1-12}}{2}=\frac{1\pm i\sqrt{11}}{2}$$

$$z_1=y+\frac{1}{y}=\frac{1+i\sqrt{11}}{2} \quad /\cdot 2y \qquad 2y^2+2=y(1+i\sqrt{11})$$

$$y_{2,3}=\frac{-1-i\sqrt{11}\pm\sqrt{(1+i\sqrt{11})^2-16}}{4}=\frac{-1-i\sqrt{11}\pm\sqrt{2(i\sqrt{11}-13)}}{4}$$

$$z_2=y+\frac{1}{y}=\frac{1-i\sqrt{11}}{2} \quad /\cdot 2y \qquad 2y^2+2=y(1-i\sqrt{11})$$

$$y_{4,5}=\frac{1-i\sqrt{11}\pm\sqrt{(1-i\sqrt{11})^2-16}}{4}=\frac{1-i\sqrt{11}}{4}\pm\frac{\sqrt{-2i\sqrt{11}-4}}{4}$$

$$y^5+1=0$$

$$(y+1)(y^4-y^3+y^2-y+1)=0$$

$$\underline{\underline{y=-1}}$$

$$y^2-y+1-\frac{1}{y}+\frac{1}{y^2}=0$$

$$\left(y^2+\frac{1}{y^2}\right)-\left(y+\frac{1}{y}\right)+1=0$$

$$x^2-2\quad-x\quad+1=0$$

$$x^2-x-1=0$$

$$x_{12}=\frac{+1\pm\sqrt{1+4}}{2}$$

$$x_1=\frac{1+\sqrt{5}}{2}$$

$$x_2=\frac{1-\sqrt{5}}{2}$$

$$y+\frac{1}{y}=\frac{1+\sqrt{5}}{2}\quad/\cdot 2y$$

$$2y^2+2=y(1+\sqrt{5})$$

$$2y^2-y(1+\sqrt{5})+2=0$$

$$y_{2,3}=\frac{1+\sqrt{5}}{4}\pm\frac{\sqrt{(1+\sqrt{5})^2-16}}{4}$$

$$=\frac{1+\sqrt{5}}{4}\pm\frac{\sqrt{2(\sqrt{5}-5)}}{4}$$

$$y_2=\frac{1+\sqrt{5}}{4}+\frac{\sqrt{2(\sqrt{5}-5)}}{4}$$

$$y_3=\frac{1+\sqrt{5}}{4}-\frac{\sqrt{2(\sqrt{5}-5)}}{4}$$

$$y+\frac{1}{y}=\frac{1-\sqrt{5}}{2}\quad/\cdot 2y$$

$$2y^2+2=y(1-\sqrt{5})$$

$$2y^2-y(1-\sqrt{5})+2=0$$

$$y_{4,5}=\frac{+(1-\sqrt{5})}{4}\pm\frac{\sqrt{(1-\sqrt{5})^2-16}}{4}$$

$$=\frac{1-\sqrt{5}}{4}\pm i\frac{\sqrt{2(\sqrt{5}+5)}}{4}$$

$$y_4=\frac{1-\sqrt{5}}{4}+i\frac{\sqrt{2(\sqrt{5}+5)}}{4}$$

$$y_5=\frac{1-\sqrt{5}}{4}-i\frac{\sqrt{2(\sqrt{5}+5)}}{4}$$

A hatodfokú binom-egyenlet

Ötödfokú b.e. :

$x^5 - a = 0$
$x^5 = a$
$x_{1\ldots5} = \sqrt[5]{a} \cdot \sqrt[5]{1}$

$x^5 + a = 0$
$x^5 = -a$
$x_{1\ldots5} = \sqrt[5]{-a} = \sqrt[5]{a} \cdot \sqrt[5]{-1}$

$y^6 - 1 = 0 \quad y^6 = 1 \quad y = \sqrt[6]{1}$

$y^6 - 1 = y^6 - 1^6 = (y^3)^2 - (1^3)^2 = (y^3+1)(y^3-1) = 0$

a, $y^3 - 1 = 0$
$y^3 = 1$
$y = \sqrt[3]{1}$

$y_1 = 1$

$y_2 = \dfrac{-1+i\sqrt{3}}{2}$

$y_3 = \dfrac{-1-i\sqrt{3}}{2}$

b, $y^3 + 1 = 0$
$y^3 = -1$
$y^3 = \sqrt[3]{-1}$

$y_4 = -1$

$y_5 = \dfrac{1+i\sqrt{3}}{2}$

$y_6 = \dfrac{1-i\sqrt{3}}{2}$

$x^6 - a = 0$
$x^6 = a$
$x = \sqrt[6]{a}$
$x_{1\ldots6} = \sqrt[6]{a} \cdot \sqrt[6]{1}$

$y^6+1=0 \qquad y^6=-1 \qquad y=\sqrt[6]{-1}$

$y^6+1=0 \;/\cdot\frac{1}{y^2} \qquad y^6+1=0 \;\int\frac{1}{y^3}$

$y^4+\frac{1}{y^2}=0 \qquad y^3+\frac{1}{y^3}=0 \qquad y+\frac{1}{y}=z$

$y^3+\frac{1}{y^3}=\left(y+\frac{1}{y}\right)\cdot\left(y^2+\frac{1}{y^2}\right)=0 \qquad z^3=\left(y+\frac{1}{y}\right)^3=y^3+\frac{1}{y^3}+3\left(y+\frac{1}{y}\right)$

$y+\frac{1}{y}=z \quad /\cdot \quad y^2+\frac{1}{y^2}=z^2-2$

$y^3+\frac{1}{y^3}=z^3-3\left(y+\frac{1}{y}\right)=0=z^3-3z$

$z(z^2-3)=0$

$z=0 \qquad z_1=0$

$z^2-3=0 \qquad z_2=\sqrt{3}$

$z_3=-\sqrt{3}$

$y+\frac{1}{y}=0 \;/\cdot y$

$y^2+1=0$

$y=\sqrt{-1}$

$y_{1,2}=\pm i$

$y+\frac{1}{y}=\sqrt{3} \;/\cdot y$

$y^2+1=y\cdot\sqrt{3}$

$y^2-y\cdot\sqrt{3}+1=0$

$y_{3,4}=\frac{\sqrt{3}\pm\sqrt{3-4}}{2}$

$y_{3,4}=\frac{\sqrt{3}\pm i\sqrt{1}}{2}$

$y_3=\frac{\sqrt{3}+i}{2}$

$y_4=\frac{\sqrt{3}-i}{2}$

$y+\frac{1}{y}=-\sqrt{3} \;/\cdot y$

$y^2+1=-y\sqrt{3}$

$y^2+y\sqrt{3}+1=0$

$y_{5,6}=\frac{-\sqrt{3}\pm\sqrt{3-4}}{2}$

$y_5=\frac{-\sqrt{3}+i}{2}$

$y_6=\frac{-\sqrt{3}-i}{2}$

Trinomikus egyenletek -

olyan egyenletek, amelyek az ismeretlen $2n$ és n-edik hatványát tartalmazzák valamint abszolut tagot.
Általános alakjuk: $ax^{2n} + bx^n + c = 0$

$$ax^{2n} + bx^n + c = 0 \qquad y = x^n$$
$$ay^2 + by + c = 0$$

$$x^6 + 23x^3 - 108 = 0 \qquad y = x^3$$
$$y^2 + 23y - 108 = 0$$

$$y_{1,2} = -\frac{23}{2} \pm \sqrt{\frac{529}{4} + \frac{432}{4}} = -\frac{23}{2} \pm \sqrt{\frac{961}{4}} = -\frac{23}{2} \pm \frac{31}{2}$$

$$y_1 = \frac{8}{2} = 4 \qquad y_2 = -\frac{54}{2} = -27$$

$x^3 = 4$

$$\left.\begin{aligned} x_4 &= \sqrt[3]{4} \\ x_5 &= \sqrt[3]{4} \cdot \frac{-1 + i\sqrt{3}}{2} \\ x_6 &= \sqrt[3]{4} \cdot \frac{-1 - i\sqrt{3}}{2} \end{aligned}\right\}$$

$x^3 = -27 = 27 \cdot -1$

$$\left.\begin{aligned} x_1 &= -3 \\ x_2 &= 3 \cdot \frac{1 + i\sqrt{3}}{2} \\ x_3 &= 3 \cdot \frac{1 - i\sqrt{3}}{2} \end{aligned}\right\}$$

Összefoglalás

A binomikus egyenletek gyökei

$y^2-1=0$	$y_1=1$	$y_2=-1$			
$y^2+1=0$	$y_1=i$	$y_2=-i$			A másodfokú egyenletnek 2 gyöke van.
$y^3-1=0$	$y_1=1$	$y_2=\frac{-1+i\sqrt{3}}{2}$	$y_3=\frac{-1-i\sqrt{3}}{2}$		
$y^3+1=0$	$y_1=-1$	$y_2=\frac{1+i\sqrt{3}}{2}$	$y_3=\frac{1-i\sqrt{3}}{2}$		Harmadfokúnak 3 gyöke van.
$y^4-1=0$	$y_1=1$	$y_2=-1$	$y_3=i$	$y_4=-i$	
$y^4+1=0$	$y_1=\sqrt{i}$	$y_2=-\sqrt{i}$	$y_3=\sqrt{-i}$	$y_4=-\sqrt{-i}$	4 gyök
$y^5-1=0$	$y_1=1$	$y_2=\frac{\sqrt{5}-1}{4}+i\frac{\sqrt{2(\sqrt{5}+5)}}{4}$	$y_3=\frac{\sqrt{5}-1}{4}-i\frac{\sqrt{2(\sqrt{5}+5)}}{4}$		
		$y_4=\frac{-1-\sqrt{5}}{4}+\frac{\sqrt{2(\sqrt{5}-5)}}{4}$	$y_5=\frac{-1-\sqrt{5}}{4}-\frac{\sqrt{2(\sqrt{5}-5)}}{4}$		

$y^5+1=0$	$y_1 = -1$ $y_2 = \frac{1+\sqrt{5}}{4} + i\frac{\sqrt{2(\sqrt{5}+5)}}{4}$ $y_3 = \frac{1+\sqrt{5}}{4} - \frac{\sqrt{2(\sqrt{5}+5)}}{4}$ $y_4 = \frac{1-\sqrt{5}}{4} + i\frac{\sqrt{2(\sqrt{5}+5)}}{4}$ $y_5 = \frac{1-\sqrt{5}}{4} - i\frac{\sqrt{2(\sqrt{5}+5)}}{4}$
$y^6-1=0$	$y_1 = 1$ $y_2 = \frac{-1+i\sqrt{3}}{2}$ $y_3 = \frac{1-i\sqrt{3}}{2}$ $y_4 = -1$ $y_5 = \frac{-1-i\sqrt{3}}{2}$ $y_6 = \frac{1+i\sqrt{3}}{2}$ A hatodfokú egyenletnek 6 gyöke van.
$y^6+1=0$	$y_1 = i$ $y_2 = -i$ $y_4 = \frac{\sqrt{3}+i}{2}$ $y_4 = \frac{\sqrt{3}-i}{2}$ $y_5 = \frac{-\sqrt{3}+i}{2}$ $y_6 = \frac{-\sqrt{3}-i}{2}$
$\sqrt{i}$	$\sqrt{i} = \frac{(1+i)\cdot\sqrt{2}}{2}$ $\sqrt{-i} = \frac{(1-i)\sqrt{2}}{2}$

1967. III. 13.

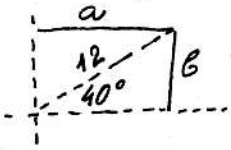
$z = 3 + 2i$ $\qquad tg\,\varphi = \frac{2}{3} = 6{,}6\ldots$

$\varphi \doteq 33°40'$

$|z| = \sqrt{9+4} = \sqrt{13} \doteq 3{,}6$ $\qquad z = |z|(\cos 33°40' + i \sin 33°40')$

$z = 12(\cos 40° + i \sin 40°)$ $\qquad \sin 40° \rightarrow \varphi = 40°$

$z = a + b_i$



$\sin 40° = \frac{b}{12}$ $\qquad \cancel{tg\,60° =}$

$b = 12 \cdot \sin 40°$ $\qquad b = \sqrt{144 - 59{,}2}$

$b = 12 \cdot 0{,}643$ $\qquad b \doteq 9{,}12$

$b \doteq 7{,}72$

$z = 9{,}12 + 7{,}72i$

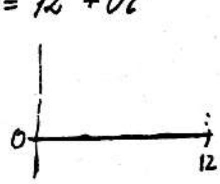
F.

$z = 5(\cos 240° + i \cdot \sin 240°)$
$z = 2 - 2i$
$z = -2 - i$
$z = 3(\cos 480° + i \sin 480°)$

Határozzuk meg az 1, -1, i, $-i$ komplex számok trigonometrikus alakját.

$z = 12 + 0i$ $\qquad |z| = 12$ $\qquad z = 12(\cos 0° + i \cdot \sin 0°)$

$z = 12(1 + i \cdot 0) = 12$



$z = -6 + 0i$ $\qquad |z| = \sqrt{36 + 0} = 6$ $\qquad z = 6(\cos 180° + i \sin 180°)$

$z = 6(-1 + i \cdot 0) = -6$

Szorzás   Trigonometrikus alakban adott komplex számok szorzata olyan trigonometrikus a. komplex szám, mely abszolut értéke a tényezők abszolut értékeinek szorzata és argumentuma a tényezők argumentumainak összege.

$$z_1 = r_1(\cos\varphi_1 + i\sin\varphi_1) \qquad r_1 = |z_1|$$
$$z_2 = r_2(\cos\varphi_2 + i\sin\varphi_2) \qquad r_2 = |z_2|$$

$$z = z_1 \cdot z_2 = r_1 \cdot r_2 \left[\cos(\varphi_1 + \varphi_2) + i\sin(\varphi_1 + \varphi_2)\right]$$

Pl:
$$z_1 = 2(\cos 30° + i\sin 30°)$$
$$z_2 = 5(\cos 40° + i\sin 40°)$$

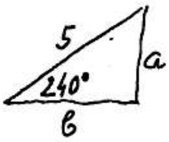
$$z = z_1 \cdot z_2 = 2 . 5 [\cos 70° + i\sin 70°]$$

$$= [2(\cos 30° + i\sin 30°) \; . \; 5(\cos 40° + i\sin 40°)] =$$
$$= 10(\cos 30° \cdot \cos 40° + i\sin 30° \cdot \cos 40° + \cos 30° \cdot i\sin 40° - \sin 30° \cdot \sin 40°) =$$
$$= 10(\cos 30° \cdot \cos 40° - \sin 30° \cdot \sin 40° + i\sin 30° \cdot \cos 40° + i\cos 30° \cdot \sin 40°) =$$
$$= \underline{10(\cos(30° + 40°) + i \cdot \sin(30° + 40°))}$$

$$z = z_1 \cdot z_2 = r_1(\cos\varphi_1 + i\sin\varphi_1) \cdot r_2(\cos\varphi_2 + i\sin\varphi_2) =$$
$$= r_1 \cdot r_2 \cdot (\cos\varphi_1 \cdot \cos\varphi_2 + i\sin\varphi_1 \cdot \cos\varphi_2 + i\sin\varphi_2 \cdot \cos\varphi_1 - \sin\varphi_1 \cdot \sin\varphi_2) =$$
$$= r_1 \cdot r_2 (\cos\varphi_1 \cdot \cos\varphi_2 - \sin\varphi_1 \cdot \sin\varphi_2 + i\sin\varphi_1 \cdot \cos\varphi_2 + i\sin\varphi_2 \cdot \cos\varphi_1) =$$
$$= r_1 \cdot r_2 \left[\cos(\varphi_1 + \varphi_2) + i\sin(\varphi_1 + \varphi_2)\right].$$

1967. III. 14.

$z = 5(\cos 240° + i \sin 240°)$



$\sin 240° = -\sin 60° = \frac{a}{5}$ $\quad\quad$ $\sin 30° = \frac{b}{5}$

$a = -5 \cdot \sin 60°$ $\quad\quad$ $b = 5 \cdot \sin 30°$

$a = -0{,}577$ $\quad\quad$ $b = -0{,}25$

$z = -0{,}25i - 0{,}577$

$2 - 2i$ $\quad\quad$ $z = \sqrt{8}\,(\cos 45° + i \sin 45°)$

$-2 - i$ $\quad\quad$ $z \doteq \sqrt{5}\,(\cos 206{,}5° + i \sin 206{,}5°)$ $\quad\quad$ $\operatorname{tg}\alpha = \frac{1}{2}$ $\quad$ $\alpha \doteq 26°33'$

$3(\cos 480° + i \sin 480°)$

$\sin 480° = \sin 120° = \sin 60°$

$\sin 60° = \frac{a}{3}$ $\quad\quad$ $\sin 30° = \frac{b}{3}$

$a = 3 \cdot \sin 60°$ $\quad\quad$ $b = 3 \sin 30°$

$a = 2{,}6$ $\quad\quad$ $b \doteq 1{,}5$

$z = 2{,}6 - 1{,}5i$

$1 \quad = 1 + 0i$

$z = 1 \cdot (\cos 0° + i \sin 0°)$

$z = 1 + i \cdot 0$

$-1 \quad = -1 + i \cdot 0$

$z = 1(\cos 180° + i \sin 180°)$

$z = -1 + i \cdot 0$

$i \quad = 0 + 1i \quad\quad z = 1(\cos 90° + i \sin 90°)$

$-i \quad = 0 - 1i \quad\quad z = 1(\cos 270° + i \sin 270°)$

$$z_1 = 5(\cos 80° + i \sin 80°)$$
$$z_2 = 12(\cos 50° + i \sin 50°)$$

$$z = z_1 \cdot z_2 = 60(\cos 130° + i \sin 130°) = 60(-\cos 50° + i \sin 50°)$$

$$z_1 = 3(\cos 160° + i \sin 160°)$$
$$z_2 = 15(\cos 140° + i \sin 140°)$$

$$z = z_1 \cdot z_2 = 45(\cos 300° + i \sin 300°) = 45(60° - i \sin 60°)$$

$$z_1 = 5(\cos 250° + i \sin 250°)$$
$$z_2 = 18(\cos 300° + i \sin 300°)$$

$$z = 90(\cos 550° + i \sin 550°) = 90(\cos 190° + i \sin 190°).$$

Osztás

Trigonometrikus alakú komplex számok hányadosa olyan komplex szám, amelynek abszolút értéke az adott számok abszolút értékeinek hányadosa, argumentuma pedig a számláló és nevező argumentumainak különbsége:

$$z_1 = r_1(\cos \varphi_1 + i \sin \varphi_1)$$
$$z_2 = r_2(\cos \varphi_2 + i \sin \varphi_2)$$

$$z = \frac{z_1}{z_2} = \frac{r_1 \cdot (\cos \varphi_1 + i \sin \varphi_1)}{r_2 \cdot (\cos \varphi_2 + i \sin \varphi_2)} = \frac{r_1}{r_2}\left[\cos(\varphi_1 - \varphi_2) + i \sin(\varphi_1 - \varphi_2)\right]$$

$$z = \frac{r_1}{r_2} \cdot \frac{\cos \varphi_1 + i \sin \varphi_1}{\cos \varphi_2 + i \sin \varphi_2} = \frac{r_1}{r_2} \cdot \frac{(\cos \varphi_1 + i \sin \varphi_1)(\cos \varphi_2 - i \sin \varphi_2)}{(\cos \varphi_2 + i \sin \varphi_2)(\cos \varphi_2 - i \sin \varphi_2)} =$$

$$= \frac{r_1}{r_2} \cdot \frac{\cos \varphi_1 \cdot \cos \varphi_2 + i \sin \varphi_1 \cdot \cos \varphi_2 - i \sin \varphi_2 \cdot \cos \varphi_1 + \sin \varphi_1 \cdot \sin \varphi_2}{\cos^2 \varphi_2 + \sin^2 \varphi_2} =$$

$$= \frac{r_1}{r_2}\left[\cos(\varphi_1 - \varphi_2) + i \sin(\varphi_1 - \varphi_2)\right]$$

$$z_1 = 6(\cos 70^\circ + i \sin 70^\circ) \qquad z_1 : z_2 = 3(\cos 30^\circ + i \sin 30^\circ)$$
$$z_2 = 2(\cos 40^\circ + i \sin 40^\circ)$$

$$z_1 = 8(\cos 50^\circ + i \sin 50^\circ)$$
$$z_2 = 4(\cos 10^\circ + i \sin 10^\circ) \qquad z_1 : z_2 = 2(\cos 40^\circ + i \sin 40^\circ)$$

$$z_1 = 5(\cos 230^\circ + i \sin 230^\circ)$$
$$z_2 = 7(\cos 140^\circ + i \sin 140^\circ) \qquad z_1 : z_2 = \frac{5}{7}(\cos 90^\circ + i \sin 90^\circ)$$

1967. III. 16.

61. óra.

$$z_1 = 5(\cos 220^\circ + i \sin 220^\circ) \qquad z = \frac{z_1}{z_2} = 0{,}5(\cos 180^\circ + i \sin 180^\circ) =$$
$$z_2 = 10(\cos 40^\circ + i \sin 40^\circ) \qquad = 0{,}5(-1 + i \cdot 0) = -0{,}5$$

$$z_1 = 6(\cos 50^\circ + i \sin 50^\circ) \qquad z = \frac{6}{7}(\cos -50^\circ + i \sin -50^\circ) =$$
$$z_2 = 7(\cos 100 + i \sin 100) \qquad = \frac{6}{7}(\cos 310^\circ + i \sin 310^\circ) =$$
$$= \frac{6}{7}(\cos 50^\circ - i \sin 50^\circ).$$

$$z_1 = 8(\cos 110^\circ + i \sin 110^\circ)$$
$$z_2 = 5(\cos 170^\circ + i \sin 170^\circ) \qquad z = \frac{z_1}{z_2} = \frac{8}{5}(\cos -60^\circ + i \sin -60^\circ) =$$
$$= 1{,}6(\cos 300^\circ + i \sin 300^\circ).$$

$$z_1 = 12(\cos 320^\circ + i \sin 320^\circ) \qquad z = \frac{z_1}{z_2} = \frac{12}{7}(\cos 150^\circ + i \sin 150^\circ)$$
$$z_2 = 7(\cos 170^\circ + i \sin 170^\circ)$$
$$= 1{,}71428(\cos 150^\circ + i \sin 150^\circ)$$

Hatványozás

Trigonometrikus alakú komplex szám n-edik hatványa olyan t.a. komplex szám, mely abszolút értéke az alap abszolút értékének n-edik hatványa, argumentuma pedig az alap argumentumának n-szerese.

$$z = r \cdot (\cos \varphi + i \sin \varphi)$$
$$z^n = [r(\cos \varphi + i \sin \varphi)]^n = r^n \cdot [\cos n\varphi + i \sin n\varphi]$$

Ez Moivre tétele (moávr).

$z = 2(\cos 30° + i \sin 30°)$

$z^5 = 32 (\cos 150° + i \sin 150°)$

$z = 4(\cos 70° + i \sin 70°)$ $z^3 = 64 (\cos 210° + i \sin 210°) =$

$z^3 = 64 (- \cos 30° + \sin 30°).$

$z = 3(\cos 140° + i \sin 140°)$ $z^4 = 81(\cos 560° + i \sin 560°) =$

$= 81(\cos 200° + i \sin 200°) =$

$= 81(- \cos 20° + i \sin 20°).$

1967. VI. 17.

61 7.

$z = 2(\cos 40° + i \sin 40°)$ $z^9 = 512(\cos 360° + i \sin 360°)$

$z = \sqrt{3} + i$ $z^5 = 32(\cos 150° + i \sin 150°)$

$2(\cos 30° + i \sin 30°)$

$z = 3 - 3i$ $z^4 = 324(\cos 900° + i \sin 900°)$

$3\sqrt{2}(\cos 315° + i \sin 315°)$ $= 324 (\cos 180° + i \sin 180°)$

$z^4 = 324(\cos 1260° + i \sin 1260°)$

$= 324(\cos 180° + i \sin 180°)$

$z = -3 - 5i$ $z^5 = 7776(\cos 1824° + i \sin 1824°)$

$6(\cos 304° + i \sin 304°)$ $= \quad (\cos 24° + i \sin 24°)$

1967. II. 18. 32. óra

$z = 3 + 5i \qquad z^4 = ?$

$\operatorname{tg} \varphi = \frac{5}{3} \qquad \sin\varphi = \frac{3}{6} = \frac{6}{2}$

$z = \sqrt{34}(\cos 59^\circ + i \sin 59^\circ)$

$z^4 = (\sqrt{34})^4 \cdot (\cos 236^\circ + i \sin 236^\circ)$

$z^4 = 1156 \cdot (\cos 236^\circ + i \sin 236^\circ)$

Gyökvonás

Minden trigonometrikus alakú komplex számnak n darab n-edik gyöke van, melynek abszolút értéke a komlex szám abszolút értékének n-edik gyöke és argumentuma $(\varphi + k \cdot 360) : n$, ahol k a 0; 1; 2; 3 .... n-1 egész értékeket veheti fel. Az egyes gyököket $z_1; z_2 \dots$ -vel jelöljük.

$$z = r \cdot (\cos\varphi + i \sin\varphi)$$

$$\sqrt[n]{z} = \sqrt[n]{r} \cdot \left(\cos\frac{\varphi + k \cdot 360}{n} + i \sin\frac{\varphi + k \cdot 360}{n}\right)$$

Ha az n-edik gyök n-edik hatványát képezzük, akkor mindegyik abszolut értéke r. és az argumentumok φ-től csak $k \cdot 360^\circ$-kal különböznek, ami azt jelenti, hogy az n-edik hatványok egyenlők.

$z = 32 \cdot (\cos 60^\circ + i \sin 60^\circ) \qquad \sqrt[5]{z} = ?$

$\sqrt[5]{z} = 2 (\cos 12^\circ + i \sin 12^\circ) - 1., = z_1 \qquad k = 0$

$2 (\cos 84^\circ + i \sin 84^\circ) - 2., = z_2 \qquad k = 1$

$2 (\cos 156^\circ + i \sin 156^\circ) - 3., = z_3 \qquad k = 2$

$2 (\cos 216^\circ + i \sin 216^\circ) - 4., = z_4 \qquad k = 3$

$2 (\cos 300^\circ + i \sin 300^\circ) - 5., = z_5 \qquad k = 4$

$z_1 = 2(\cos 12^\circ + i \sin 12^\circ)$

$z_2 = 2(\cos 84^\circ + i \sin 84^\circ)$

$z_3 = 2(\cos 156^\circ + i \sin 156^\circ)$

$z_4 = 2(\cos 216^\circ + i \sin 216^\circ)$

$z_5 = 2(\cos 300^\circ + i \sin 300^\circ)$

$$\sqrt[3]{1} = ?$$

$$z = 1+0i \qquad z = (\cos 0^\circ + i\sin 0^\circ)$$

$$z_1 = (\cos 0^\circ + i\sin 0^\circ) = 1$$

$$z_2 = \left(\cos \frac{360^\circ}{3} + i\sin\frac{360}{3}\right) = (\cos 120^\circ + i\sin 120^\circ) = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$

$$z_3 = \left(\cos\frac{720}{3} + i\sin\frac{720}{3}\right) = (\cos 240^\circ + i\sin 240^\circ) = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$$

63. óra

1967. III. 20.

$$\log\sqrt[3]{20} = \frac{1}{3}\cdot\log 20 = \frac{1}{3}\cdot 1{,}301$$

$$\log x = 0{,}434$$

$$x \doteq 2{,}71$$

$$\sqrt[3]{20} = \left.\begin{aligned} &= 2{,}71 \\ &= 2{,}71\cdot\frac{-1+i\sqrt{3}}{2} \\ &= 2{,}71\cdot\frac{-1-i\sqrt{3}}{2} \end{aligned}\right\}$$

$$z = 16\cdot(\cos 50^\circ + i\sin 50^\circ)$$

$$\sqrt[4]{z} = \sqrt[4]{16}\left(\cos\frac{50+k360^\circ}{n} + i\sin\frac{50+k360^\circ}{n}\right)$$

$$z_1 = 2(\cos 12{,}5^\circ + i\sin 12{,}5^\circ)$$

$$z_2 = 2(\cos 102{,}5^\circ + i\sin 102{,}5^\circ)$$

$$z_3 = 2(\cos 192{,}5^\circ + i\sin 192{,}5^\circ)$$

$$z_4 = 2(\cos 282{,}5^\circ + i\sin 282{,}5^\circ)$$

$$z = 64(\cos 120^\circ + i\sin 120^\circ)$$

$$\sqrt[3]{z} = \quad z_1 = 4(\cos 40^\circ + i\sin 40^\circ)$$

$$z_2 = 4(\cos 160^\circ + i\sin 160^\circ)$$

$$z_3 = 4(\cos 280^\circ + i\sin 280^\circ)$$

$\sqrt[3]{-1} = ?$ $\qquad -1 = -1 + 0i$

$z = 1(\cos 180° + i\sin 180°)$

$z_1 = 1.(\cos 60° + i\sin 60°) = \frac{1}{2} + i\frac{\sqrt{3}}{2} = \frac{1+i\sqrt{3}}{2}$

$z_2 = (\cos 180° + i\sin 180°) = -1$

$z_3 = (\cos 300° + i\sin 300°) = \frac{1-i\sqrt{3}}{2}$

$\sqrt[6]{729} = ?$ $\qquad 729 = 729 + 0i$

$z = 729(\cos 0° + i\sin 0°)$

$z_1 = \sqrt[6]{729}\left(\cos\frac{0°}{6} + i\sin\frac{0°}{6}\right) = 3(1+0i) = 3$

$z_2 = 3.(\cos 60° + i\sin 60°) = 3\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = \frac{3+3i\sqrt{3}}{2}$

$z_3 = 3(\cos 120° + i\sin 120°) = 3.\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = \frac{-3+3i\sqrt{3}}{2}$

$z_4 = 3(\cos 180° + i\sin 180°) = 3(-1+0) = -3$

$z_5 = 3(\cos 240° + i\sin 240°) = 3\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}\right) = -\frac{3+3\sqrt{3}}{2}$

$z_6 = 3(\cos 300° + i\sin 300°) = 3\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right) = \frac{3-3.\sqrt{3}}{2}$

$\sqrt[3]{-8} = ?$ $\qquad -8 = -8 + 0i$

$= 8(\cos 180° + \sin 180°)$

$z_1 = 2(\cos 60° + i\sin 60°) = 2\cdot\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = 1 + i\sqrt{3}$

$z_2 = 2(\cos 180° + i\sin 180°) = 2\cdot(-1 + i0) = -2$

$z_3 = 2(\cos 300° + i\sin 300°) = 2\left(\frac{1}{2} + \left(-i\frac{\sqrt{3}}{2}\right)\right) = 1 - i\sqrt{3}$

$x^6 - 729 = 0$
$x^6 = 729$
$x = \sqrt[6]{729}$

$25x^{12} - 348 = 0$
$25x^{12} = 348$
$x^{12} = \cancel{27,00}\ 13{,}96$

63. ? 1967. III. 21.

$z_{1-12} \doteq 1{,}245 \cdot \sqrt[12]{1}$

$z_1 = 1{,}25$ $z_7 = 1{,}25i$

$z_2 = -1{,}25$ $z_8 = -1{,}25i$

$z_3 = \frac{-1+i\sqrt{3}}{2} \cdot 1{,}25$ $z_9 = 1{,}25 \cdot \frac{\sqrt{3}+i}{2}$

$z_4 = \frac{-1-i\sqrt{3}}{2} \cdot 1{,}25$ $z_{10} = 1{,}25 \cdot \frac{\sqrt{3}-i}{2}$

$z_5 = \frac{1+i\sqrt{3}}{2} \cdot 1{,}25$ $z_{11} = 1{,}25 \cdot \frac{-\sqrt{3}+i}{2}$

$z_6 = \frac{1-i\sqrt{3}}{2} \cdot 1{,}25$ $z_{12} = 1{,}25 \cdot \frac{-\sqrt{3}-i}{2}$

$x^5 - 5 = 0$
$x^5 = 5$
$x_{1-5} = \sqrt[5]{5}$

$-x_1 =$
$\log \sqrt[5]{5} = \frac{1}{5} \cdot \log 5 = \frac{1}{5} \cdot 0{,}716 =$
$(\doteq 0{,}0632)$
$\doteq 0{,}14$
$x \doteq 1{,}38$

$x^5 - 5 = 0$
$x^6 = 64$

$x_1 = 1{,}38 \cdot (\cos 0° + i \sin 0°) = \underline{1{,}38}$
$x_2 = 1{,}38 \cdot (\cos 72° + i \sin 72°) = 0{,}426 + i\,0{,}132 = \underline{0{,}43 + 1{,}3i}$
$x_3 = 1{,}38 \cdot (\cos 144° + i \sin 144°) = -0{,}81 +$
$= \underline{-1{,}12 + 0{,}81i}$
$x_4 = 1{,}38 \cdot (\cos 216° + i \sin 216°) = -1{,}12 - 0{,}81i$
$x_5 = 1{,}38 (\cos 288° + i \sin 288°) = \underline{0{,}248 - 1{,}3i}$

$x^6 = 64$ $x = \sqrt[6]{64}$

$x_1 = 2 \cdot 1 = \underline{2}$
$x_2 = 2 \cdot \frac{-1+i\sqrt{3}}{2} = \underline{-1+i\sqrt{3}}$
$x_3 = 2 \cdot \frac{-1-i\sqrt{3}}{2} = \underline{-1-i\sqrt{3}}$
$x_4 = 2 \cdot \frac{1+i\sqrt{3}}{2} = \underline{1+i\sqrt{3}}$
$x_5 = 2 \cdot \frac{1-i\sqrt{3}}{2} = \underline{1-i\sqrt{3}}$
$x_6 = \underline{-2}$

1967. III. 22.

64. óra

Reciprok egyenletek

Azokat az egyenleteket, amelyek minden egyes gyökének reciprok értéke szintén gyöke az egyenletnek, reciprok egyenletnek nevezzük.

(A) $$a_m x^m + a_{m-1} x^{m-1} + a_{m-2} x^{m-2} + \dots + a_2 x^2 + a_1 x + a_0 = 0$$

$x \neq 0$

$$\frac{a_m}{x^m} + \frac{a_{m-1}}{x^{m-1}} + \dots + \frac{a_2}{x^2} + \frac{a_1}{x} + a_0 = 0$$ / ha (A) recipr. e.

(B) $$a_0 x^m + a_1 x^{m-1} + a_2 x^{m-2} + \dots + a_{m-1} x + a_m = 0$$

A és B összehasonlításából látjuk, hogy a B egyenlet úgy módon származott az A egyenletből, hogy ebben az egyenlet elejétől és végétől számított ugyanannyiadik tagok együtthatóját felcseréltük.

Minthogy ilyen formán is helyes egyenlethez jutottunk ennélfogva ez csak úgy lehetséges, ha a reciprok egyenletben az egyenlet elejétől és végétől számított ugyanannyiadik tagok együtthatói egymással egyenlők:

$a_m = a_0$

$a_{m-1} = a_1$

A reciprok egyenlet általános alakja.

$$ax^m + bx^{m-1} + cx^{m-2} + \dots + \dots + cx^2 + bx^1 + ax^0 = 0$$

Harmadfokú reciprok egyenletek:

$$7x^3 - 43x^2 - 43x + 7 = 0$$
$$(7x^3 + 7) - (43x^2 + 43x) = 0$$
$$7(x^3 + 1) - 43x(x + 1) = 0$$
$$7(x+1)(x^2 - x + 1) - 43x(x + 1) = 0$$
$$(x + 1) \cdot [7(x^2 - x + 1) - 43x] = 0$$

$$A^3 + B^3 = (A+B)(A^2 - AB + B^2)$$
$$A^3 - B^3 = (A-B)(A^2 + AB + B^2)$$

I. $(x + 1) = 0 \longrightarrow \underline{x_1 = -1}$

II. $7(x^2 - x + 1) - 43x = 0$

$7x^2 - 7x + 7 - 43x = 0$

$7x^2 - 50x + 7 = 0$

$x_{12} = \dfrac{50 \pm \sqrt{50^2 - 176}}{14} \ldots$

$\underline{x_2 = 7}$

$\underline{x_3 = \frac{1}{7}}$

$$5x^3 - 11x^2 + 11x - 5 = 0$$
$$5(x^3 - 1) - 11(x^2 - x) = 0$$
$$5(x-1)(x^2 + x + 1) - 11x(x - 1) = 0$$
$$(x-1) \cdot [5(x^2 + x + 1) - 11x] = 0$$

$5x^2 + 5x + 5 - 11x = 0$

$5x^2 - 6x + 5 = 0$

$x_{12} = \dfrac{6 \pm \sqrt{36 - 100}}{10} = \dfrac{6 \pm 8i}{10}$

$\underline{x_1 = 1}$

$\underline{x_2 = \frac{3 + 4i}{5}}$

$\underline{x_3 = \frac{3 - 4i}{5}}$

$$ax^3 + bx^2 \pm bx \pm a = 0$$
$$(ax^3 \pm a) + (bx^2 \pm bx) = 0$$
$$a(x^3 \pm 1) + bx(x \pm 1) = 0$$
$$a(x \pm 1)(x^2 \pm x + 1) + bx(x \pm 1) = 0$$
$$a(x \pm 1)(x^2 \mp x + 1) + (x \pm 1) \cdot bx = 0$$
$$(x \pm 1) \cdot [a(x^2 \mp x + 1) + bx] = 0$$
$$(x \pm 1)(ax^2 \pm ax + a + bx) = 0$$
$$(x \pm 1)(ax^2 \pm (a \mp b)x + a) = 0$$

$\downarrow$ x_1 $\quad$ $\downarrow$ $x_{2,3}$

1967. III. 22.

64. H.

$8x^3 - 57x^2 - 57x + 8 = 0$

$3x^3 - 13x^2 + 13x - 3 = 0$

$x^3 - 5x^2 + 5 - 1 = 0$

$2x^3 + 7x^2 + 7x + 2 = 0$

$4x^3 - 13x^2 + 13x - 4 = 0$

$x^3 + x^2 + x + 1 = 0$

$8x^3 - 57x^2 + 57x + 8 = 0$

$8(x^3+1) - 57(x^2+x) = 0$

$8(x+1)(x^2-x+1) - 57x(x+1) = 0$

$8x^2 - 8x + 8 - 57x = 0$

$8x^2 - 65x + 8 = 0$

$x_1 = -1$

$x_{12} = \frac{65 \pm \sqrt{4240-256}}{16} = \frac{65 \pm 63}{16} ; \left(\frac{132}{16} \quad \frac{45}{16}\right)$

$\frac{128}{16} = 8 \quad \frac{1}{8}$

$3x^3 - 13x^2 + 13x - 3 = 0$

$3(x^3 - 1) - 13x(x-1) = 0$

$3x^2 + 3x + 3 - 13x = 0$

$3x^2 - 10x + 3 = 0$

$x_1 = 1$

$x_{12} = \frac{10 \pm \sqrt{100-36}}{6} = \frac{10 \pm 8}{6} ; 3 ; \frac{1}{3}$

$x^3 - 5x^2 + 5 - 1 = 0$

$x^3 - 1 - 5x(x-1) = 0$

$x^2 + x + 1 - 5x = 0$

$x^2 - 4x + 1 = 0$

$x_1 = 1$

$x_{12} = \frac{4 \pm \sqrt{16-4}}{2} = \frac{4 \pm 3{,}47}{2} \quad 3{,}74 \quad 0{,}27$

$2x^3 + 7x^2 + 7x + 2 = 0$

$2x^2 - 2x + 2 + 7x = 0$

$2x^2 + 5x + 2 = 0$

$x_1 = -1$

$x_{12} = \frac{-5 \pm \sqrt{25-16}}{4} = \frac{-5 \pm 3}{4} \quad -\frac{1}{2} \quad -2$

$4x^3 - 13x^2 + 13 - 4 = 0$

$4x^2 + 4x + 4 - 13x = 0$

$4x^2 - 9x + 4 = 0$

$x_1 = 1$

$x_{12} = \frac{9 \pm \sqrt{81-64}}{8} = \frac{9 \pm 3{,}86}{8} \quad 1{,}6 \quad 0{,}64$

$x^3 + x^2 + x + 1 = 0$

$x^2 - x + 1 + x = 0$

$x^2 + 1 = 0$

$x_1 = -1$

$x_{2,3} = \sqrt{-1} = i \quad -i$

65 óra

1967. III. 23.

$$x^3 + \frac{7}{6}x^2 - \frac{7}{6}x - 1 = 0$$

$$x^3 - 1 + \frac{7}{6}x^2 - \frac{7}{6}x = 0$$

$$(x-1)(x^2+x+1) + \frac{7}{6}x(x-1) = 0$$

$$x^2 + x + 1 + \frac{7}{6}x = 0$$

$$x^2 + \frac{13}{6}x + 1 = 0$$

$$x_1 = 1$$

$$x_{23} = \frac{-\frac{13}{6} \pm \sqrt{\frac{169}{36} - \frac{144}{36}}}{2} = \frac{-\frac{13}{6} \pm \frac{5}{6}}{2}$$

$$x_1 = -\frac{2}{3} \qquad x_2 = -\frac{3}{2}$$

$$x^3 + \frac{3}{2}x^2 - \frac{3}{2}x - 1 = 0$$

$$(x-1)(x^2+x+1) + \frac{3}{2}x(x-1) = 0$$

$$x^2 + \frac{5}{2}x + 1 = 0$$

$$x_1 = x_1 = 1$$

$$x_{23} = \frac{-\frac{5}{2} \pm \sqrt{\frac{25}{4} - \frac{16}{4}}}{2} = \frac{-\frac{5}{2} \pm \frac{3}{2}}{2}$$

$$x_1 = -\frac{1}{2} \qquad x_3 = -2$$

$$27x^3 - 9x^2 - 3x + 1 = 0$$

$$(27x^3+1) - (9x^2+3x) = 0$$

$$\cancel{(27+1)(x-1)}$$

$$(3x+1)(9x^2-3x+1) - 3x(3x+1) = 0$$

$$9x^2 - 6x + 1 = 0$$

$$3x+1 = 0$$

$$3x = -1$$

$$x_1 = -\frac{1}{3}$$

$$x_{23} = \frac{6 \pm \sqrt{36-36}}{18} \qquad x_{23} = \frac{1}{3}$$

65. f

$$3x^3 - 43/2 \cdot x^2 + 43/2 \cdot x - 3 = 0$$

Negyedfokú reciprok e.

$$ax^4+bx^3+cx^2\pm bx+a=0 \quad /\cdot\frac{1}{x^2}$$

$$ax^2+bx+c\pm\frac{b}{x}+\frac{a}{x^2}=0$$

$$\left(ax^2+\frac{a}{x^2}\right)+\left(\pm\frac{b}{x}+bx\right)+c=0$$

$$a\left(x^2+\frac{1}{x^2}\right)+b\left(x\pm\frac{1}{x}\right)+c=0$$

$$x+\frac{1}{x}=y$$

$$\left(x\pm\frac{1}{x}\right)^2=x^2\pm2+\frac{1}{x^2}$$

$$x^2+\frac{1}{x^2}=y^2\pm2$$

$$a\left(y^2\pm2\right)+b\cdot y+c=0$$

$$ay^2\pm2a+by+c=0$$

Kapunk egy másodfokú egyenletet y-ra mely megoldása adja az y_1, y_2 gyököket.

Ezután behelyettesítünk az $x+\frac{1}{x}=y_{1,2}$ egyenletbe.

Így megkapjuk az x_{1-4} egyenleteket.

$$6x^4-35x^3+62x^2-35x+6=0 \quad /\cdot\frac{1}{x^2}$$

$$6x^2-35x+62-35\cdot\frac{1}{x}+6\cdot\frac{1}{x^2}=0$$

$$\left(6x^2+\frac{6}{x^2}\right)-\left(35x+\frac{35}{x}\right)+62=0$$

$$6\left(x^2+\frac{1}{x^2}\right)-35\left(x+\frac{1}{x}\right)+62=0$$

$$x+\frac{1}{x}=u \quad /^2$$

$$x^2+2+\frac{1}{x^2}=u^2$$

$$x^2+\frac{1}{x^2}=u^2-2$$

$$6\left(u^2-2\right)-35\cdot u+62=0$$

$$6u^2-12-35u+62=0$$

$$6u^2-35u+50=0$$

$$u_{1,2}=\frac{35\pm\sqrt{1225-1200}}{12}=\frac{35\pm5}{12}$$

$$u_1=\frac{40}{12}=\frac{10}{3} \qquad u_2=\frac{30}{12}=\frac{5}{2}$$

$$u_1=x+\frac{1}{x} \quad /\cdot x$$

$$\frac{10}{3}x+x^2+1=0$$

$$3x^2+10x+3=0$$

$$x_{1,2}=\frac{10\pm\sqrt{100+36}}{6}=\frac{10\pm8}{6}$$

$$x+\frac{1}{x}=\frac{5}{2}$$

$$2x^2+2=5x$$

$$2x^2-5x+2=0$$

$$x_{3,4}=\frac{5\pm\sqrt{25-16}}{4}=\frac{5\pm3}{4}$$

$$x_1=3 \qquad x_2=\frac{1}{3}$$

$$x_3=2 \qquad x_4=\frac{1}{2}$$

Egyszerűbb a következő alakú 4. fokú reciprok e. megoldása.

$$ax^4 + bx^3 - bx - a = 0$$
$$(ax^4 - a) + (bx^3 - bx) = 0$$
$$a(x^4 - 1) + bx(x^2 - 1) = 0$$
$$a(x^2+1)(x^2-1) + bx(x^2-1) = 0$$
$$(x^2-1)[a(x^2+1) + bx] = 0$$

$$x^2 - 1 = 0$$
$$x^2 = 1$$
$$\underline{x_1 = 1} \qquad \underline{x_2 = -1}$$

$$ax^2 + a + bx = 0$$
$$ax^2 + bx + a = 0$$
$$\Downarrow$$
$$x_{3,4}$$

– az egyenlet megoldása adja $x_{3,4}$-et.

$$2x^4 + 5x^3 - 5x - 2 = 0$$
$$(2x^4 - 2) + (5x^3 - 5x) = 0$$
$$2(x^2+1)(x^2-1) + 5x(x^2-1) = 0$$

$$x^2 - 1 = 0 \rightarrow \underline{x_{1,2} = \pm 1}$$

$$2x^2 + 2 + 5x = 0$$
$$2x^2 + 5x + 2 = 0$$

$$x_{3,4} = \frac{-5 \pm \sqrt{25-16}}{4} = \frac{-5 \pm 3}{4}$$

$$\underline{x_3 = -\frac{1}{2}} \qquad \underline{x_4 = -2}$$

$$x^4 - 10x^3 + 26x^2 - 10x + 1 = 0 \quad / \cdot \frac{1}{x^2}$$
$$x^2 - 10x + 26 - \frac{10}{x} + \frac{1}{x^2} = 0$$
$$\left(x^2 + \frac{1}{x^2}\right) + \left(10x + \frac{10}{x}\right) + 26 = 0$$

$$u^2 - 10u + 24 = 0 \qquad u = x + \frac{1}{x}$$
$$u_1 = 6$$
$$u_2 = 4$$

$$x_1 = \frac{4 - \sqrt{12}}{2}$$
$$x_2 = \frac{6 + \sqrt{32}}{2}$$

4.

$$x^4+8x^3+2x^2+8x+1=0$$
$$x^4+4x^3-10x^2+4x+1=0$$
$$x^4+2x^3-22x^2+2x+1=0$$
$$x^4+x^3-x-1=0$$

$$x^4-10/3\cdot x^3+\frac{10}{3}x-1=0$$
$$x^4-26/5\,x^3+26/5\,x-1=0$$

$$x^4+8x^3+2x^2+8x+1=0$$
$$(x^4+1)+(8x^3+8x)+2x^2=0 \quad /\cdot\frac{1}{x^2}$$
$$x^2+\frac{1}{x^2}+8x+\frac{8}{x}+2=0$$
$$y^2-2+8y+2=0$$
$$y^2+8y=0$$
$$y_1=0 \quad y_2=-8$$

$$x+\frac{1}{x}=0 \quad /\cdot x$$
$$x^2+1=0$$
$$x_{1,2}=\pm i$$

$$x+\frac{1}{x}+8=0 \quad /\cdot x$$
$$x^2+8x+1=0$$
$$x_{3,4}=\frac{-8\pm\sqrt{64-4}}{2}=\frac{-8\pm\sqrt{60}}{2}$$

$$x_1=i \qquad x_2=-i \qquad x_3=-4+\sqrt{15} \qquad x_4=-4-\sqrt{15}$$

$$x^4+4x^3-10x^2+4x+1=0 \quad /\cdot\frac{1}{x^2}$$
$$\left(x^2+\frac{1}{x^2}\right)+4\left(x+\frac{1}{x}\right)-10=0$$
$$y^2-2+4y-10=0$$
$$y^2+4y-12=0$$
$$y_{1,2}=\frac{-4\pm\sqrt{16+48}}{2}=\frac{-4\pm8}{2}$$
$$y_1=-6 \quad y_2=2$$

$$x+\frac{1}{x}=-6$$
$$x^2+6x+1=0$$
$$x_{1,2}=\frac{-6\pm\sqrt{36-4}}{2}=\frac{-6\pm\sqrt{32}}{2}$$
$$x^2-2x+1=0$$
$$x_{3,4}=\frac{2\pm\sqrt{4-4}}{2}$$

$$x_1=-3+\sqrt{8} \qquad x_2=-3-\sqrt{8} \qquad x_3=1 \qquad x_4=1$$

$$x^4+x^3-x-1=0$$
$$(x^4-1)+(x^3-x)=0$$
$$(x^2+1)(x^2-1)+x(x^2-1)=0$$
$$(x^2-1)(x^2+1+x)=0$$
$$x_{1,2}=\pm1$$

$$x^2+x+1=0$$
$$x_{3,4}=\frac{-1\pm\sqrt{1-4}}{2}=\frac{-1\pm\sqrt{-3}}{2}=\sqrt[3]{1}$$

$$x_{1-3}=\sqrt[3]{1} \qquad x_4=-1$$

$$x^4+2x^3-22x^2+2x+1=0 \quad / \cdot \frac{1}{x^2} \qquad x+\frac{1}{x}=4 \qquad x_{1,2}=\frac{4\pm\sqrt{16-4}}{2}$$

$$\left(x^2+\frac{1}{x^2}\right)+2\left(x+\frac{1}{x}\right)-22=0 \qquad x^2-4x+1=0 \qquad =\frac{4\pm\sqrt{12}}{2}$$

$$y^2-2+2y-22=0 \qquad x+\frac{1}{x}=-6 \qquad x_{3,4}=\frac{-6\pm\sqrt{36-4}}{2}$$

$$y^2+2y-24=0 \qquad x^2+6x+1=0$$

$$y_{12}=-1\pm\sqrt{1+24}=-1\pm5$$

$$x_1=2+\sqrt{3} \qquad x_2=2-\sqrt{3} \qquad x_3=-3+\sqrt{8} \qquad x_4=-3-\sqrt{8}$$

$$x^4-\frac{10}{3}x^3+\frac{10}{3}x-1=0 \qquad x_{1,2}=\pm1$$

$$(x^4-1)-\left(\frac{10}{3}x^3-\frac{10}{3}x\right)=0= \qquad x^3-\frac{10}{3}x+1=0$$

$$(x^2+1)(x^2-1)-\frac{10}{3}x(x^2-1)=0 \qquad x_{3,4}=\frac{10}{6}\pm\sqrt{\frac{100}{36}-\frac{36}{36}}=\frac{10}{6}\pm\frac{8}{6}$$

$$(x^2-1)\left(x^2+1-\frac{10}{3}x\right)=0$$

$$x_1=1 \qquad x_2=-1 \qquad x_3=3 \qquad x_4=\frac{1}{3}$$

$$x^4-\frac{26}{5}x^3+\frac{26}{5}x-1=0$$

$$(x^2+1)(x^2-1)-\frac{26}{5}x(x^2-1)=0.$$

$$(x^2-1)\left(x^2+1-\frac{26}{5}x\right)=0 \qquad x_{1,2}=\pm1$$

$$x^2-\frac{26}{5}x+1=0 \qquad x_{3,4}=\frac{26}{10}\pm\sqrt{\frac{26^2}{100}-\frac{100}{100}}=\frac{26}{10}\pm\frac{24}{10}$$

$$x_1=1 \qquad x_2=-1 \qquad x_3=5 \qquad x_4=\frac{1}{5}$$

1967. II. 30.

66. óra

Ötödfokú r. e.

$$4x^5+12x^4+11x^3+11x^2+12x+4=0$$

$$(4x^5+4)+(12x^4+12x)+(11x^3+11x^2)=0$$

$$4(x^5+1)+12x(x^3+1)+11x^2(x+1)=0$$

$$4(x+1)(x^4-x^3+x^2-x+1)+12x(x+1)(x^2-x+1)+11x^2(x+1)=0$$

$$(x+1)\left[4x^4-4x^3+4x^2-4x+4+12x^3-12x^2+12x+11x^2\right]=0$$

$$(x+1)(4x^4+8x^3+3x^2+8x+4)=0$$

$$\underline{\underline{x_1=-1}}$$

$$4(x^4+1)+8x(x^3+x)+3x^2=0 \qquad /\cdot\frac{1}{x^2}$$

$$4\left(x^2+\frac{1}{x^2}\right)+8\left(x+\frac{1}{x}\right)+3=0$$

$$4(y^2-2)+8y+3=0 \qquad x+\frac{1}{x}=y$$

$$4y^2-8+8y+3=0$$

$$4y^2+8y-5=0$$

$$y_{1,2}=\frac{-8\pm\sqrt{64+80}}{8}=\frac{-8\pm12}{8} \qquad y_1=\frac{1}{2} \qquad y_2=-\frac{5}{2}$$

$$x+\frac{1}{x}-\frac{1}{2}=0 \quad /\cdot 2x \qquad\qquad x+\frac{1}{x}+\frac{5}{2}=0 \quad /\cdot 2x$$

$$2x^2-x+2=0 \qquad\qquad 2x^2+5x+2=0$$

$$x_{2,3}=\frac{1\pm\sqrt{1-16}}{4} \qquad\qquad x_{4,5}=\frac{-5\pm\sqrt{25-16}}{4}$$

$$x_{2,3}=\frac{1\pm i\sqrt{15}}{4} \qquad\qquad x_{4,5}=\frac{-5\pm3}{4}$$

$$x_1=-1 \quad x_2=\frac{1}{4}+i\cdot\frac{\sqrt{15}}{4} \quad x_3=\frac{1}{4}-i\frac{\sqrt{15}}{4} \quad x_4=-\frac{1}{2} \quad x_5=-2$$

$$ax^5 + bx^4 + cx^3 \pm cx^2 \pm bx \pm a = 0$$

$$(ax^5 \pm a) + (bx^4 \pm bx) + (cx^3 \pm cx^2) = 0$$

$$a(x^5 \pm 1) + bx(x^3 \pm 1) + cx^2(x \pm 1) = 0$$

$$\underline{a(x\pm1)(x^4 \mp x^3 + x^2 \mp x + 1)} + \underline{bx(x\pm1)(x^2 \mp x + 1)} + \underline{cx^2(x\pm1)} = 0$$

$$\underline{(x\pm1)}\cdot\underline{\left[a(x^4 \mp x^3 + x^2 \mp x + 1) + bx(x^2 \mp x + 1) + cx^2\right]} = 0$$

I. $x \pm 1 = 0$

II. $ax^4 \mp ax^3 + ax^2 \mp ax + a + bx^3 \mp bx^2 + bx + cx^2 = 0$

$$ax^4 + x^3(b \mp a) + \underbrace{x^2(a + c \mp b)}_{} + x(b \mp a) + a = 0$$

negyedfokú reciprok egyenlet.

$$6x^5 + 29x^4 + 27x^3 - 27x^2 - 29x - 6 = 0$$

$$6(x^5 - 1) + 29(x^4 - x) + 27(x^3 - x^2) = 0$$

$$6(x-1)(x^4 + x^3 + x^2 + x + 1) + 29(x-1)(x^3 + x^2 + x + 1) + 27(x-1)(x^2 \cancel{+ x + 1}) = 0$$

1. $x - 1 = 0 \rightarrow \underline{\underline{x_1 = 1}}$

$$6x^4 + 6x^3 + 6x^2 + 6x + 6 + 29x^3 + 29x^2 + 29x + 29 + 27x^2 + \cancel{27x + 27} = 0$$

$$6x^4 + 35x^3 + 62x^2 + 35x + 6 = 0$$

$$6x^2 + 35x + 62 + 35\cdot\frac{1}{x} + \frac{6}{x^2} = 0$$

$$6\left(x^2 + \frac{1}{x^2}\right) + 35\left(x + \frac{1}{x}\right) + 62 = 0$$

$$6y^2 - 12 + 35y + 62 = 0$$

$$6y^2 + 35y + 50 = 0$$

$$y_{1,2} = \frac{-35 \pm \sqrt{1225 - 1200}}{12} \qquad y_1 = -\frac{30}{12} = \frac{15}{6} = -\frac{5}{2} \qquad y_2 = -\frac{40}{3} = -\frac{10}{3}$$

$$x + \frac{1}{x} + \frac{5}{2} = 0 \qquad\qquad x + \frac{1}{x} + \frac{10}{3} = 0$$

$$2x^2 + 5x + 2 = 0 \qquad\qquad 3x^2 + 10x + 3 = 0$$

$$x_{2,3} = \frac{-5 \pm \sqrt{25 - 16}}{4} = \frac{-5 \pm 3}{4} \qquad\qquad x_{4,5} = \frac{-10 \pm \sqrt{100 - 36}}{6} = \frac{-10 \pm 8}{6}$$

$$\underline{\underline{x_2 = -\frac{1}{2} \qquad x_3 = -2}} \qquad\qquad \underline{\underline{x_4 = -\frac{1}{3} \qquad x_5 = -3}}$$

$$4x^5 - 11x^3 - 11x^2 + 4 = 0$$
$$x^5 + x^4 + x^3 - x^2 - x - 1 = 0$$
$$x^5 + x^4 + x^3 + x^2 + x + 1 = 0$$
$$15x^5 - 49x^4 + 34x^3 + 34x^2 - 49x + 15 = 0$$
$$x^5 - \frac{7}{2}x^2 + \frac{5}{2}x^3 + \frac{5}{2}x^2 - \frac{7}{2}x + 1 = 0$$
$$x^5 + x^4 - x - 1 = 0$$
$$x^6 - 7x^5 + 15x^4 - 14x^3 + 15x^2 - 7x + 1 = 0 \qquad / \cdot \frac{1}{x^3} \qquad x^3 + \frac{1}{x^3} = u^3 - 3u$$
$$u^3 - 7u^2 + 12u = 0$$

17o. kis könyv. — összes példa —

1967. IV. 2. 66. gy.

$$4x^5 - 11x^3 - 11x^2 + 4 = 0$$
$$4(x^5+1) - 11x^2(x^3+1) = 0$$
$$(x+1)\cdot\left[4\cdot(x^4 - x^3 + x^2 - x + 1) - 11x^2\right] = 0 \qquad \underline{\underline{x_1 = -1}}$$
$$4x^4 - 4x^3 - 7x^2 - 4x + 4 = 0$$
$$4y^2 - 8 - 4y - 7 = 0 \qquad y = x + \frac{1}{x}$$
$$4y^2 - 4y - 15 = 0$$
$$y_{12} = \frac{4 \pm \sqrt{16+240}}{8} = \frac{4 \pm 16}{8} \qquad y_1 = \frac{5}{2} \quad y_2 = -\frac{3}{2}$$

$$\frac{5}{2} = x + \frac{1}{x} \quad /\cdot 2x \qquad\qquad -\frac{3}{2} = x + \frac{1}{x} \quad /\cdot 2x$$
$$5x = 2x^2 + 2 \qquad 2x^2 - 5x + 2 = 0 \qquad\qquad -3x = 2x^2 + 2 \neq \varphi$$
$$2x^2 + 3x + 2 = 0$$
$$x_{12} = \frac{5 \pm \sqrt{25-16}}{4} = \frac{5 \pm 3}{4}$$
$$x_{12} = \frac{-3 \pm \sqrt{9-16}}{4} = \frac{-3 \pm i\sqrt{7}}{4}$$

$$\underline{\underline{x_2 = 2}} \qquad \underline{\underline{x_3 = \frac{1}{2}}} \qquad \underline{\underline{x_4 = \frac{-3+i\sqrt{7}}{4}}} \qquad \underline{\underline{x_5 = \frac{-3-i\sqrt{7}}{4}}}$$

$x^5+x^4+x^3-x^2-x-1=0$ $\qquad$ $\underline{\underline{x_1=1}}$

$x^5-1 \quad + \quad x^4-x \quad +x^3-x^2=0$

$(x-1)(x^4+x^3+x^2+x+1) \quad +(x-1)(x^3+x^2+x) \quad + \quad x^2(x-1)=0$

$x^4+x^3+x^2+x+1 \quad +x^3+x^2+x+x^2=0$

$x^4+2x^3+3x^2+2x+1=0$ $\qquad$ $-1=x+\frac{1}{x} \quad /\cdot x$

$y^2-2 \quad +2y+3 \quad =0$ $\qquad$ $x^2+x+1=0$

$y^2+2y+1=0$ $\qquad$ $x_{1234}=\frac{-1\pm\sqrt{1-4}}{2}=\frac{-1\pm i\sqrt{3}}{2}$

$y_{12}=\frac{-2\pm\sqrt{4-4}}{2}=-1$ $\qquad$ $\underline{\underline{x_{1,2}=\frac{-1+i\sqrt{3}}{2}}}$ $\qquad$ $\underline{\underline{x_{3,4}=\frac{-1-i\sqrt{3}}{2}}}$

$x^5+x^4+x^3+x^2+x+1=0$

$(x+1)(x^4-x^3+x^2-x+1)+(x+1)(x^3-x^2+x)+(x+1)x^2=0$ $\qquad$ $\underline{\underline{x_1=-1}}$

$x^4-x^3+x^2-x+1+x^3-x^2+x+x^2=0$

$x^4+x^2+1=0$

$x^2+1+\frac{1}{x^2}=0$ $\qquad$ $1=x+\frac{1}{x} \quad /\cdot x$ $\qquad$ $x_{12}=\frac{1\pm\sqrt{1-4}}{2}=\frac{1\pm i\sqrt{3}}{2}$

$y^2-2 \quad +1 \quad =0$ $\qquad$ $x=x^2+1$ $\qquad$ $x_{34}=\frac{-1\pm\sqrt{1-4}}{2}=\frac{-1\pm i\sqrt{3}}{2}$

$y^2=1$ $\qquad$ $x^2\pm x+1=0$

$y=\pm 1$

$\underline{\underline{x_2=\frac{1+i\sqrt{3}}{2}}}$ $\qquad$ $\underline{\underline{x_3=\frac{1-i\sqrt{3}}{2}}}$ $\qquad$ $\underline{\underline{x_4=\frac{-1+i\sqrt{3}}{2}}}$ $\qquad$ $\underline{\underline{x_5=\frac{-1-i\sqrt{3}}{2}}}$

$15x^5-49x^4+34x^3+34x^2-49x+15=0$

$15x^4-15x^3+15x^2-15x+15 \quad -49x^3+49x^2-49x+34x^2$ $\qquad$ $\underline{\underline{x_1=-1}}$

$15x^4-64x^3+98x^2-64x+15=0$

$15y^2-30-64y+98=0$ $\qquad$ $x+\frac{1}{x}-2{,}27 \doteq 0 \quad /\cdot x$

$15y^2-64y+68=0$ $\qquad$ $x^2-2{,}27x+1=0$

$y_{12}=\frac{64\pm\sqrt{4096-4080}}{30}=\frac{64\pm4}{30}$ $\qquad$ $x_{12}=\frac{2{,}27\pm\sqrt{5{,}15-4}}{2}=\frac{2{,}27\pm1{,}1}{2}$

$y_1=\frac{68}{30}=\frac{34}{15}\doteq 2{,}27$ $\qquad$ $x+\frac{1}{x}-2=0 \quad /\cdot x$

$x^2-2x+1=0$

$y_2=2$ $\qquad$ $x_{3,4}=\frac{2\pm\sqrt{4-4}}{2}=1$

$\underline{\underline{x_2=1}}$ $\qquad$ $\underline{\underline{x_3=1}}$ $\qquad$ $\underline{\underline{x_4\doteq 1{,}7}}$ $\qquad$ $\underline{\underline{x_5\doteq 0{,}58}}$

$$x^5+x^4-x-1=0$$

$$(x-1)(x^4+x^3+x^2+x+1)+(x-1)(x^3+x^2+x)=0 \qquad \underline{\underline{x_1=1}}$$

$$x^4+2x^3+2x^2+2x+1=0$$

$$y^2-2+2y+2=0 \qquad x+\frac{1}{x}=0 \quad /\cdot x$$

$$y^2+2y=0 \qquad x^2+1=0$$

$$x=\pm i$$

$$y_1=0 \qquad x+\frac{1}{x}+2=0 \quad /\cdot x \qquad x=\frac{-2\pm\sqrt{4-4}}{2}=-1$$

$$y_2=-2 \qquad x^2+2x+1=0$$

$$\underline{\underline{x_2=i}} \quad \underline{\underline{x_3=-i}} \quad \underline{\underline{x_4=-1}} \quad \underline{\underline{x_5=-1}}$$

$$x^5-\frac{7}{2}x^4+\frac{5}{2}x^3+\frac{5}{2}x^2-\frac{7}{2}x+1=0$$

$$\left(x^5+\frac{5}{2}x^3-x^2-\frac{7}{2}x+1=0\right) \qquad \underline{\underline{x=-1}}$$

$$(x+1)(x^4-x^3+x^2-x+1)-\frac{7}{2}(x+1)(x^3-x^2+x)+\frac{5}{2}(x+1)x^2$$

$$x^4-x^3+x^2-x+1-\frac{7}{2}x^3+\frac{7}{2}x^2-\frac{7}{2}x+\frac{5}{2}x^2=0$$

$$x^4-\frac{9}{2}x^3+7x^2-\frac{9}{2}x+1=0$$

$$\cancel{(x+1)}x^3$$

$$y^2-2-\frac{9}{2}y+7=0 \qquad x+\frac{1}{x}-2{,}2=0 \quad /\cdot x$$

$$y^2-\frac{9}{2}y+5=0 \qquad x^2-2{,}2x+1=0$$

$$x_{12}=\frac{2{,}2\pm\sqrt{4{,}9-4}}{2}\doteq\frac{2{,}2\pm0{,}95}{2}$$

$$y_{1,2}\doteq\frac{4{,}5\pm\sqrt{20-20}}{2}\doteq 2{,}3\ ;\ 2{,}2$$

$$x_{3,4}=\frac{2{,}3\pm\sqrt{5{,}3-4}}{2}\doteq\frac{2{,}3\pm1{,}1}{2}$$

$$\underline{\underline{x_2\doteq1{,}6}} \quad \underline{\underline{x_3=0{,}6}} \quad \underline{\underline{x_4=1{,}7}} \quad \underline{\underline{x_5=1{,}1}}$$

$$x^6 - 7x^5 + 15x^4 - 14x^3 + 15x^2 - 7x + 1 = 0 \qquad / \cdot \frac{1}{x^3}$$

$$\left(x^3 + \frac{1}{x^3}\right) - 7\left(x^2 + \frac{1}{x^2}\right) + 15\left(x + \frac{1}{x}\right) - 14 = 0$$

$$y^3 - 3y - 7y^2 + 14 + 15y - 14 = 0$$

$$y^3 - 7y^2 + 12y = 0$$

$$y\left(y^2 - 7y + 12\right) = 0$$

$$y_1 = 0 \qquad y_{2,3} = \frac{7 \pm \sqrt{49-48}}{2} = \frac{7 \pm 1}{2}$$

$$y_2 = 4 \qquad y_2 = 3$$

$$x + \frac{1}{x} + 0 = 0 \quad / \cdot x$$

$$x^2 + 1 = 0$$

$$x = \pm i$$

$$x + \frac{1}{x} - 4 = 0 \quad / \cdot x$$

$$x^2 - 4x + 1 = 0$$

$$x_{1,2} = \frac{4 \pm \sqrt{16-4}}{2} = \frac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3}$$

$$x + \frac{1}{x} - 3 = 0 \quad / \cdot x$$

$$x^2 - 3x + 1 = 0$$

$$x_{1,2} = \frac{6 \pm \sqrt{36-4}}{2} = \frac{6 \pm \sqrt{32}}{2} = 3 \pm \sqrt{8}$$

$$x_1 = i \quad x_2 = -i \quad x_3 = 2+\sqrt{3} \quad x_4 = 2-\sqrt{3} \quad x_5 = 3+\sqrt{8} \quad x_6 = 3-\sqrt{8}$$

67 - 68 ó

1967. IV. 1.

A sorozatok

Az olyan számok mely bizonyos törvényszerűség szerint következnek egymás után, sor. alkotnak. Az egyes számok a sor tagjai. Indexel ellátott kis betűvel jelöljük őket.

$$a_1 ; a_2 ; a_3 ; a_4 ; \ \dots \ a_{n-1} ; a_n$$

n - term. szám: 1, 2, 3 ....

P_1 1, 2, 3, 4 ....

P_2 2, 4, 6, 8, 10 ...

P_3 5, 5, 5 ...

P_4 11, 9, 7, 5, 3, 1, -1,

P_5 1, 4, 9, 16; 25 ...

1. növekvő
2. csökkenő
3. egyenlő sz.

} sorozata

$\left\{\frac{n}{n+1}\right\}$ $\frac{1}{2} ; \frac{2}{3} ; \frac{3}{4} \dots$

$\left\{\frac{1}{n^2}\right\}$ $1 ; \frac{1}{4} ; \frac{1}{9} \dots$

A számtani sorozat -

A számoknak az a sora, melyben két szomszédos tag különbsége állandó:

1, 2, 3, 4 ...
2, 4, 6 ... $a_1 ; a_2 ; a_3 ; a_4 \dots$
1, 3, 5, 7 ...

$a_2 - a_1 = d$
$a_n - a_{n-1} = d$

1., Ha $d > 0$ - a sor növekvő
2., Ha $d = 0$ - állandó kül. $= 0$
3., Ha $d < 0$ - csökkenő számtani sor.

$7, 7, 7 \dots$ - $d = 0$
$-1; -3; -5 \dots$ - $d < 0$
$2, 4, 6 \dots$ - $d > 0$

$$\left.\begin{array}{l} a_1 \qquad d \\ a_2 = a_1 + d \\ a_5 = a_1 + 4d \\ a_n = a_{n-1} + d \end{array}\right\} \rightarrow \quad \underline{\underline{a_n = a_1 + (n-1) \cdot d}}$$

A az n. bármely tagját megkapjuk ha az 1. taghoz hozzáadjuk az idl. tagot megelőző tagok számával szorzott különbséget.

A számtani sorozat 5 jellemző adata:

a_1 - első tag
a_n - ennedik tag
n - tagmutató
d - differencia
S_n - tagösszeg

$a_1 = -14$ $n = 6$
$\underline{d = 6}$
$a_6 = ?$ $a_6 = 16$
$a_{10} = ?$ $a_{10} = 40$

$a_1 = -14$ $a_2 = -8$ $a_3 = -2$ $a_4 = 4$ ...

$1, 5, 9, 13, 17, 21, 25, 29$

$1+29=30$
$5+25=30$

$$S_m = a_1 + a_2 + \ldots + a_m$$
$$S_m = a_m + \ldots + a_2 + a_1 \quad / +$$
$$2S_m = (a_1+a_m) + \ldots + (a_m+a_1)$$
$$S_m = \frac{a_1+a_m}{2} \cdot m$$

A sz. s. összege = első + utolsó tag számtani középértéke szorozva a tagok számával.

A számtani sor 3 ismert adata meghatározza a két ismeretlent.

1 utas 8 napig utazott. 1 napon 12 km-t.
A továbbiakban 3-3-mal többet. Hány km-t tett meg?

$$S_m = \frac{a_1+a_m}{2} \cdot m$$
$$a_m = a_1 + (m-1) \cdot d.$$

Hf.

$a_1 = 2$
$a_m = 26$
$m = 9$ $\rightarrow$ $S_n = 126$, $d = 3$

$a_1 = -7$
$a_m = 9$
$d = 2$ $\rightarrow$ $m = 9$, $S_m = 9$

$a_1 = 14$
$a_m = 44$
$S_m = 174$ $\rightarrow$ $m = 6$, $d = 6$

$a_1 = 60$
$m = 17$
$d = 6$ $\rightarrow$ $S_m = -238$, $a_m = 220$

69 óra. 1967. IV. 3.

Számtani sor: d - differencia $\quad d = a_{n+1} - a_n$

$a \gtreqless 0 \rightarrow$ csökkenő, növekvő, állandó sorozat.

$$a_n = a_1 + (n-1)\cdot d$$

$$S_n = \frac{a_1 + a_n}{2}\cdot n$$

Jellemzők: $a_1; a_n; n; d; S_n$

Ha ebből 3 ismert, a 2 meghatározható.

Pl.:

$a_n = -4{,}4 \qquad a_1 = ?$

$d = -\frac{3}{5} \qquad n = ?$

$S_n = -17$

$$a_n = a_1 + (n-1)\cdot d$$

$$S_n = \frac{a_1 + a_n}{2}\cdot n$$

$$-\frac{22}{5} = a_1 + (n-1)\cdot -\frac{3}{5}$$

$$-17 = \frac{a_1 + (-22/5)}{2}\cdot n$$

$$-4{,}4 = a_1 - \frac{3}{5}n + \frac{3}{5} \rightarrow a_1 = \frac{3}{5}n - 5$$

$$-34 = a_1 n - 4{,}4n \rightarrow a_1 = \frac{4{,}4n - 34}{n}$$

$$\frac{3}{5}n - 5 = \frac{4{,}4n - 34}{n} \qquad 0{,}6n^2 - 5n = 4{,}4n - 34$$

$$0{,}6n - 5 = 4{,}4n - 34 \qquad 0{,}6n^2 - 9{,}4n + 34 = 0$$

$$32 = 3{,}8n$$

$$n = \frac{32}{3{,}8} =$$

$$n_{1,2} = \frac{9{,}4 \pm \sqrt{9{,}4^2 - 2{,}4\cdot 34}}{1{,}2} = \frac{9{,}4 \pm \sqrt{88{,}16 - 81{,}6}}{1{,}2} = \frac{9{,}4 \pm \sqrt{6{,}56}}{1{,}2} = \frac{9{,}4 \pm 2{,}5}{1{,}2}$$

$n_1 = \frac{12}{1{,}2} = \underline{\underline{10}} \qquad n_2$ - nem lehet.

$$a_1 = 0{,}6n - 5 = 6 - 5 = \underline{\underline{1}}$$

$m = 20$
$d = -1{,}9 = -\frac{19}{10}$
$S_m = 99$

$a_1 = ?$
$a_m = ?$

$$a_m = a_1 + (m-1) \cdot d$$

$$S_m = \frac{a_1 + a_m}{2} \cdot m$$

$$a_m = a_1 + 19 \cdot d \longrightarrow a_1 = a_m - 19d$$

$$\frac{198}{20} = a_1 + a_m \longrightarrow a_1 = \frac{198}{20} - a_m = 9{,}9 - a_m$$

$$a_m + \frac{19 \cdot 19}{10} = 9{,}9 - a_m$$

$$2a_m = 9{,}9 - 36{,}1$$

$$\underline{\underline{a_m = -13{,}1}}$$

$$a_1 = a_m - 19d$$

$$a_1 = -13{,}1 + 36{,}1$$

$$a_1 = 22{,}9 + 0{,}1 = \underline{\underline{23}}$$

1967. IV. 3.

7.

$a_1 = 3$	$a_1 = 5$	$a_m = 3\frac{9}{10}$	$a_m = -220$
$m = 12$	$d = -2/3$	$m = 9$	$m = 7$
$S_m = 69$	$S_m = 0$	$d = 0{,}8$	$S_m = -196$
$a_m = ?$	$a_m = ?$	$a_1 = ?$	$a_1 = ?$
$d = ?$	$m = ?$	$S_m = ?$	$d = ?$

1.,

$$a_m = a_1 + (m-1)d \qquad a_m = 3 + 11d$$

$$S_m = \frac{a_1 + a_m}{2} \cdot m \qquad 138 = 36 + 12a_m$$

$$3 + 11d = \frac{51}{6} \qquad a_m = 3 + 5{,}5$$

$$18 + 66d = 51 \qquad \underline{\underline{a_m = 8{,}5}}$$

$$66d = 33$$

$$\underline{\underline{d = 0{,}5}}$$

$$a_m = 5 + \left(-\frac{2}{3}d\right) + \frac{2}{3} = \frac{17}{3} - \frac{2}{3}d$$

2.,

$$S_m = 0 = \frac{5 + a_m}{2} \cdot m = 5m + a_m \cdot m$$

$$m(5 + a_m) = 0 \rightarrow \underline{\underline{m = 0}} \quad \underline{\underline{a_m = -5}}$$

3.) $a_n = \frac{39}{10} = a_1 + 8 \cdot 0{,}8 = a_1 + 6{,}4$ $\quad a_n = a_1 + 6{,}4 \rightarrow a_1 = a_n - 6{,}4$

$$S_n = \frac{\cancel{a_n} \cdot \frac{39}{10} + -\left(\frac{25}{10}\right)}{2} \cdot 9 = \frac{7}{10} \cdot 9 = \frac{63}{10} \qquad S_n = \frac{63}{10} \qquad a_1 = -\frac{5}{2}$$

4.) $-220 = a_1 + 6d \rightarrow a_1 = -220 - 6d = d = \frac{-220 - a_1}{6}$

$$-196 = \frac{a_1 - 110}{2} \cdot 7 \rightarrow a_1 = \frac{-392 + 770}{7} = \frac{378}{7} = 54 = a_1$$

$$d = \frac{-220 - 54}{6} = \frac{274}{6} = 45{,}\dot{6}$$

$$\frac{56}{n} = 11 + a_n \rightarrow a_n = \frac{56}{n} - 11$$

$$a_n = \frac{25}{2} - \frac{3}{2}n \rightarrow a_n = \frac{55}{2} - \frac{3}{2}n$$

$$\frac{56 - 11n}{n} = \frac{55 - 3n}{2}$$

$$112 - 22n = 55n - 3n^2$$

$$3n^2 - 77n + 112 = 0$$

$$n_{12} = \frac{77 \pm \sqrt{5929 - 1344}}{6} = \frac{77 \pm 68}{6}$$

$$n_1 = \frac{146}{6} = 24$$

$$a_n = \frac{55}{2} - \frac{72}{2} = -\frac{17}{2}$$

$S_n = 28$

$a_3 = 8$ $\quad d = -3$ $\quad a_n = a_1 + (n-1) \cdot d$

$a_4 = 5$ $\quad S_n = \frac{a_1 + a_n}{2} \cdot n$

$n =$

$a_n = ?$ $\quad a_n = 14 + (n-1)(-3)$

$a_1 =$ $\quad S_n = \frac{14 + a_n}{2} \cdot n$

$$a_n = 14 - 3n + 3$$

$$56 = 14n + a_n n$$

$$3n^2 - 31n + 56 = 0 \qquad n_1 = 8$$

$$a_n = 14 + (-7 \cdot 3) = -7$$

A számtani sorozat tetszőleges 2 tagjára érvényes, hogy:

$$a_s = a_r + (s-r)\cdot d$$

$a_s = a_1 + (s-1)\cdot d$ (1)

$a_r = a_1 + (r-1)\cdot d$ (2)

(1)-(2) $a_s - a_r = a_1 + (s-1)d - a_1 - (r-1)\cdot d =$

$= sd - d - rd + d = d(s-r)$.

$a_s = a_r + (s-r)\cdot d$

Számtani sorozat interpolációja. (közbeiktatás).

2, 4, 6, 8 .... 16, 18, 20, 38, 56

Interpolációnak v. közbeiktatásnak nevezzük azt a műveletet amikor 2 szomszédos tag közé úgy iktatunk számokat, hogy azok egymással és a két szomszédos taggal is számtani sorozatot alkossanak.

$d = 18$

$\delta = 2$ – a közbeiktatás ált. ny. új sorozat diferenciája

$m = 8$ – a közbeiktatott tagok száma.

$$\delta = \frac{d}{m+1}$$

$\delta = \frac{18}{8+1} = \frac{18}{9} = 2$

Egy számtani sorozat második és 4. elemének összege (28) 16, első és ötödik elemének különbsége 28. Állapítsuk meg a számtani sorozatnak 1. elemét és különbségét!

$a_2 + a_4 = 16$ | $a_1 = ?$ | $a_2 = a_1 + d$

$a_1 - a_5 = 28$ | $d = ?$ | $a_3 = \dots$

$a_1 + d + a_1 + 3d = 16$

$a_1 - (a_1 + 4d) = 28$

$2a_1 + 4d = 16$

$-4d = 28$

$\underline{d = -7}$

$\underline{a_1 = 22}$

$2a_1 - 28 = 16$

$2a_1 = 44$

$\underline{a_1 = 22}$

Kör alakú cirkusz 1. sorában 60 személy ülhet. Minden következő sorban 5 - 5 személlyel több. Hányan ülnek a 15. sorban és hányan férnek el a cirkuszban ha a sorok száma 15.

$n = 15$
$a_1 = 60$
$d = 5$

$a_{15} = ?$
$S_n = ?$

$$S_n = \frac{a_1 + a_{15}}{2} \cdot n$$

$$a_n = a_1 + d(n-1)$$

$$S_n = \frac{60 + a_{15}}{2} \cdot 15$$

$$a_{15} = 60 + 5 \cdot 14 \longrightarrow a_{15} = 60 + 70 = \underline{\underline{130}}$$

$$S_n = 95 \cdot 15 = \underline{\underline{1425}}$$

Egy számtani sor 4. eleme 10 ; 7 eleme 19 állapítsuk meg a sorozat 1. elemét és különbségét.

$a_4 = 10$
$a_7 = 19$

$$a_s = a_r + (s - r) \cdot d$$

$$a_1 + 3d = 10 \longrightarrow a_1 = 10 - 3d$$
$$a_1 + 6d = 19 \longrightarrow a_1 = 19 - 6d$$

$$10 - 3d = 19 - 6d \qquad a_1 = 10 - 9$$
$$3d = 9 \qquad \underline{\underline{a_1 = 1}}$$
$$\underline{\underline{d = 3}}$$

Egy számtani sor össz. 28. 3e. 8
4e. 5
Hány eleme van a sorozatnak. Mi az 1. és utolsó eleme.

$S_n = 28$
$a_3 = 8$
$a_4 = 5$

$n = ?$
$a_1 = ?$
$a_n = ?$

$$a_n = a_1 + d(n-1)$$
$$a_n = 11 + \left(-\frac{3}{2}n\right) + \frac{3}{2}$$
$$a_n = \frac{25}{2} - \frac{3}{2}n$$
$$28 = \frac{11 + \frac{25}{2} - \frac{3}{2}n}{2} \cdot n$$
$$56 = \left(\frac{47}{2} \cancel{n} - \frac{3}{2}n\right) \cdot n$$
$$56 = n(11 + a_n)$$

$$a_3 = a_1 + 2d = 8$$
$$a_4 = a_1 + 4d = 5$$

$$8 - 2d = 5 - 4d$$
$$2d = -3$$
$$\underline{\underline{d = -\frac{3}{2}}}$$

$$a_1 = 8 - 2d = 8 + 3 = \underline{\underline{11}} = a_1$$

→ 2 oldallal előbb.

Egy aknába ejtett kő 5 sec alatt ért az akna fenekére.
Milyen mély az akna.

$$s = \frac{g.t^2}{2} \doteq \frac{10.25}{2} \doteq 5.25 \doteq \underline{\underline{\underline{125\,m}}}$$

Oldjuk meg a köv. egyenletet:

$$1+4+7+10\ldots+x = 117$$

$$S_m = \frac{a_1+a_m}{2}\cdot m \qquad a_m = a_1 + (m-1)d$$

$$2.117 = (1+x)\cdot m \qquad x = 1+3(m-1)$$

$$234 = m\cdot(1+x) \qquad x = 1+3m-3 = 3m-2$$

$$m = \frac{234}{1+x} \qquad m = \frac{x+2}{3}$$

$$702 = (1+x)\cdot(x+2) = x + x^2 + 2 + 2x = x^2 + 3x + 2$$

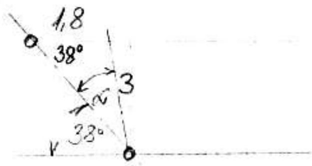
$$x^2 + 3x - 700 = 0$$

$$x_{12} = \frac{-3 \pm \sqrt{9+2800}}{2} = \frac{-3 \pm 53}{2};$$

$$\underline{\underline{\underline{x_1 = 25}}} \qquad x_2 = -28$$

$$m = \frac{234}{26} = \underline{\underline{\underline{9}}}$$

1967. IV. 4. Csónakkal akarunk a folyó túlsó partjára jutni. A cél iránya a folyó partjával 38°-os ∢-et alkot számunkra.
Hogy a víz sodra ellenére is egyenesen a célhoz jussunk, a cél irányától egy bizonyos szöggel eltérő irányban kell eveznünk. Mekkora ez a szög, ha a csónak sebessége állóvízben 3 m/s, a sodrásé 1,8 m/s [21,68°]



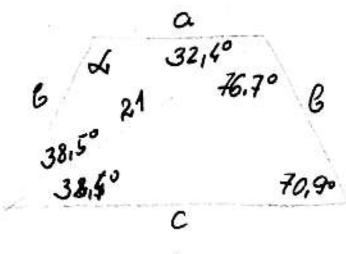
$$\frac{a}{b} = \frac{\sin \alpha}{\sin \beta} \qquad \frac{1{,}8}{3} = \frac{\sin \alpha}{\sin 38°}$$

$$\sin \alpha = \sin 38° \cdot \frac{1{,}8}{3} \doteq 0{,}37$$

$$\alpha \doteq 21°$$

Egy egyenlőszárú trapézból ismerjük az átló hosszát, 21 cm és azt a 2 szöget, amelyre az átló a mellette levő hegyesszöget osztja. A szár felé 38,5°, az alap felé 32,4°.
Szám. ki az egy. sz. trapéz oldalait, szögeit.

[a 21,63 b 11,91 c 13,83 $\alpha = \beta = 70{,}9°$ $\gamma = \delta = 109{,}1°$]



$\alpha = \beta = ?$ $b = ?$ $a = ?$ $c = ?$

$\gamma = \delta = 70{,}9°$

$\alpha = \beta = (180 - \gamma)° = 180° - 70{,}9° = 109{,}1° = \alpha = \beta$

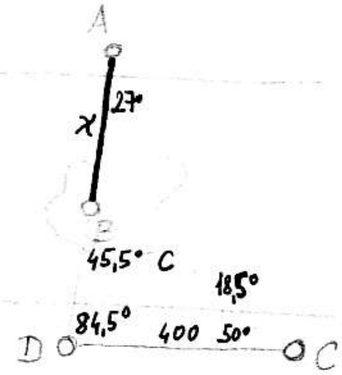
$$\frac{a}{b} = \frac{\sin 38{,}5°}{\sin 32{,}4°} \qquad \frac{c}{b} = \frac{\sin 76{,}7°}{\sin 32{,}4°} \qquad \frac{b}{21} = \frac{\sin 32{,}4°}{\sin 70{,}9°}$$

$a \cdot \sin 32{,}4° = b \cdot \sin 38{,}5°$ $\qquad$ $c \cdot \sin 32{,}4° = b \cdot \sin 76{,}7°$ $\qquad$ $b \doteq 11{,}9$

$$b = \frac{a \cdot \sin 32{,}4°}{\sin 38{,}5°} = \frac{c \cdot \sin 32{,}4°}{\sin 76{,}7°} =$$

$$a \doteq \frac{b \cdot \sin 38{,}5°}{\sin 32{,}4°} \doteq 13{,}8 \qquad c = \frac{b \cdot \sin 76{,}7°}{\sin 32{,}4°} \doteq 21{,}6$$

Egy sziget B pontjának a folyó túlsó partján levő A pontjától való távolságát akarjuk meghatározni. A folyó innenső partján DC = 400 m távolságot mérünk fel úgy, hogy D pont AB egyenesbe essék,
$ADC\sphericalangle = 84{,}5°$ $BCD\sphericalangle = 50°$ $BCA\sphericalangle = 18°35'$
mekkora az AB távolság?



$$\frac{c}{400} = \frac{\sin 84{,}5°}{\sin 45{,}5°}$$

$$c \doteq \frac{400 \cdot \sin 84{,}5°}{\sin 45{,}5°} \doteq 558 \doteq c$$

$$\frac{x}{c} = \frac{\sin 18{,}5°}{\sin 27°}$$

$$x \doteq 558 \cdot 0{,}7 \doteq 390\ m$$

Valamely község artézi kutat fúratott. Az 1. méter fúrásáért 10 Kčs, minden további méteréért pedig a megelőző méter áránál egy állandó összeggel többet fizetett. Az utolsó méter 45,8 Kčs-ba, az egész kút fúrása pedig 5022 Kčs-be került.
Hány m mély az artézi kút? És mekkora összeggel nagyobbodott az egyes lejjebb fekvő méterek fúrásának ára.

$a_1 = 10$
$a_m = 45{,}8$
$S_m = 5022$
$m = ?$
$d = ?$

$$S_m = \frac{a_1 + a_m}{2} \cdot m$$

$$5022 = \frac{55{,}8}{2} \cdot m$$

$$m = \frac{10044}{55{,}8}$$

$$m = 180\ m$$

100440 : 558 = 180
4464

$$a_m = a_1 + d(m-1)$$

$$45{,}8 = 10 + d \cdot 179$$

$$d = \frac{35{,}8}{179}$$

358 : 1790 = 0,2
0
= 0

$$d = 0{,}2\ Kčs$$

6 15 24 33 42....51 60

5. és 6. tag közé 5 tagot ikt.

$\delta = \frac{d}{m+1}$ $\delta = \frac{9}{6} = \frac{3}{2}$

$\underline{\delta = \frac{3}{2}}$ 42 <u>43,5 45 46,5 48 49,5</u> 51

5 17 29 .. Iktassunk 5 számot a szomsz. tagok közé

$\delta = \frac{d}{m+1} = \frac{12}{6} = 2$

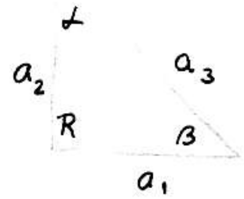
<u>5 7 9 11 13 15 17 19 21 23 25 27 29</u>

9, $17\frac{1}{4}$ közé 10 tagot úgy, hogy azok az adott számokkal számtani sort alkossanak

$\delta = \frac{d}{m+1} = \frac{8,25}{11} = 0,75$.

<u>9 9,75 10,5 11,25 12 12,75 13,5 14,25 15 15,75 16,5 17,25</u>

Valamely R∢ háromszög oldalai számtani sort adnak. Mily nagyok az oldalak ha $T_\triangle = 384\ cm^2$?



$$T = 384 = \frac{a_1 \cdot a_2}{2} = \frac{a_1 \cdot (a_1+d)}{2} = 384$$

$$a_1^2 + a_1 d = 768 \quad \rightarrow \quad d_1 = \frac{768 - a_1^2}{a_1}$$

$$(a_1+2d)^2 = (a_1+d)^2 + a_1^2$$

$$a_1^2 + 4a_1 d + 4d^2 = a_1^2 + 2a_1 d + d^2 + a_1^2$$

$$0 = a_1^2 - 3d^2 - 2a_1 d$$

$$3d^2 + 2da_1 - a_1^2 = 0$$

$$d_{1,2} = \frac{-2a_1 \pm \sqrt{4a_1^2 + 12a_1^2}}{6} = \frac{-2a_1 \pm 4a_1}{6}; \quad d_1 = \frac{a_1}{3} \quad d_2 = -a_1$$

$$\frac{a_1}{3} = \frac{768 - a_1^2}{a_1} \quad a_1^2 = 2304 - 3a_1^2 \quad 4a_1^2 = 2304 \quad a_1^2 = 576 \quad a_1 = 24$$

$$a_2 = 32 \quad a_3 = 40$$

$$x^5 + 2x^4 + 17x^3 - 34x^2 + 16x + 32 = 0$$

$$(x^5 + 32) + (2x^4 + 16x) + (17x^3 - 34x^2) = 0$$

$$(x^5 + 2^5) + 2x(x^3 + 2^3) + 17x^2(+x - 2) = 0 \qquad x_1 = 2$$

$$(x-2)(x^4 + 2x^3 + 4x^2 + 8x + 16) + (x-2)(2x^3 + 4x^2 + 8x) + (x-2) \cdot 17x^2 = 0$$

$$x^4 + 4x^3 + 25x^2 + 16x + 16 = 0 \quad / \cdot \frac{1}{x^2}$$

$$x^2 + 4x + 25 + 16 \cdot \frac{1}{x} + 16 \cdot \frac{1}{x^2} = 0$$

$$\left(x^2 + \frac{16}{x^2}\right) + \left(4x + \frac{16}{x}\right) + 25 = 0 \qquad y^2 = \left(x + \frac{4}{x}\right)^2 = x^2 + 8 + \frac{16}{x^2}$$

$$y^2 - 8 + 4y + 25 = 0$$

$$y^2 + 4y + 17 = 0$$

$$y_{1,1} = \frac{-4 \pm \sqrt{16 - 68}}{2} = \frac{-4 \pm i\sqrt{52}}{2} = -2 \pm i\sqrt{13}$$

$$x + \frac{4}{x} = -2 + i\sqrt{13}$$

$$x^2 + 4 = -2x + x \cdot i\sqrt{13}$$

$$x^2 + x(2 - i\sqrt{13}) + 4 = 0$$

$$x_{2,3} = \frac{i\sqrt{3} - 2 \pm \sqrt{-9 - 4i\sqrt{13} - 16}}{2} = \frac{i\sqrt{3} - 2 \pm i\sqrt{25 + 4i\sqrt{13}}}{2}$$

$$x_{4,5} = \frac{2 + i\sqrt{13} \pm \sqrt{9 + 4i\sqrt{13} - 16}}{2} = \frac{2 + i\sqrt{3} \pm i\sqrt{7 - 4i\sqrt{13}}}{2}$$

$$2x^8 - 3x^7 + 12x^6 + 2x^5 - 2x^3 - 12x^2 + 3x - 2 = 0$$

$$x_2 = 1$$

$$2(x^8 - 1) - 3x(x^6 - 1) + 12x^2(x^4 - 1) + 2x^3(x^2 - 1) = 0$$

$$x_{12} = \pm 1$$

$$2(x^6 + x^4 + x^2 + 1) - 3x(x^4 + x^2 + 1) + 2x^3 = 0$$

$$2x^6 + 2x^4 + 2x^2 + 2 - 3x^5 - 3x^3 - 3x + 2x^3 = 0$$

$$2x^6 - 3x^5 + 2x^4 - 5x^3 + 2x^2 - 3x = 0$$

$$x_3 = 0$$

$$x(2x^5 - 3x^4 + 2x^3 - 5x^2 + 2x - 3) = 0$$

$$2x^5 - 3x^4 + 2x^3 - 5x^2 + 2x - 3 = 0$$

$$4x^4 - 12x^3 - 4x^2 + 18x + 9 = 0 \qquad 4x^2 - 12x - 4 + 18\cdot\frac{1}{x} + \frac{9}{x^2} = 0$$

$$2^2x^4 + 3^2 - 12x^3 + 18x - 4x^2 = 0 \quad / \cdot \frac{1}{x^2}$$

$$2^2x^2 + \frac{3^2}{x^2} - 12x + \frac{18}{x} - 4 = 0$$

$$4(x^2 - 1) \qquad \left(4x^2 + \frac{9}{x^2}\right) - \left(12x - \frac{18}{x}\right) - 4 = 0$$

$$4x^5 - 11x^3 - 11x^2 + 4 = 0$$

$$4(x^5+1) - 11(x^3+x^2) = 0$$

$$4(x^5+1) - 11x^2(x+1) = 0 \qquad \underline{\underline{x_1 = -1}}$$

$$4(x+1)(x^4 - x^3 + x^2 - x + 1) - 11x^2(x+1) = 0$$

$$4x^4 - 4x^3 + 4x^2 - 4x + 4 - 11x^2 = 0$$

$$4x^4 - 4x^3 - 7x^2 - 4x + 4 = 0$$

$$4\left(x^2 + \frac{1}{x^2}\right) - 4\left(x + \frac{1}{x}\right) - 7 = 0$$

$$4y^2 - 8 - 4y - 7 = 0$$

$$4y^2 - 4y - 15 = 0$$

$$y_{12} = \frac{4 \pm \sqrt{16 + 60\cdot 4}}{8} = \frac{4 \pm 16}{8}$$

$$y_1 = \frac{5}{2} \quad y_2 = -\frac{3}{2}$$

$$x + \frac{1}{x} - \frac{5}{2} = 0 \qquad x_{23} = \frac{5 \pm \sqrt{25 - 16}}{4} = \frac{5 \pm 3}{4}$$

$$2x^2 + 2 - 5x = 0 \qquad \underline{\underline{x_2 = 2}} \quad \underline{\underline{x_3 = \frac{1}{2}}}$$

$$x + \frac{1}{x} + \frac{3}{2} = 0 \qquad x_{45} = \frac{-3 \pm \sqrt{9 - 16}}{4} =$$

$$2x^2 + 2 + 3x = 0 \qquad = \frac{-3 \pm i\sqrt{7}}{4}$$

$$\underline{\underline{x_4 = \frac{-3 + i\sqrt{7}}{4}}} \quad \underline{\underline{x_5 = \frac{-3 - i\sqrt{3}}{4}}}$$

1967. IV. 10.

73. Gyakorlat (logarléc)

Szorzás: $2 \times 4 = 8,$ $3 \times 7 = 21,0$ $0,7 \cdot 6 = 4,2$

$0 + 0 + 0 = 0$ $0 + 0 + 1 = 1$ $-1 + 0 + 1 = 0$

$683 \cdot 0,0071 = 4,85$

$2 + (-3) + 1 = 0$

Osztás: $8 : 4 = 2,0$ $12 : 3 = 4,0$ $4,85 : 0,0071 = 683,0$

$0 - 0 = 0$ $1 - 0 - 1 = 0$ $0 - (-3) - 1 = 2$

Hatványozás: $2^2 = 4,0$ $5^2 = 25,0$ $41^2 = 1681,0$

$0.2 = 0$ $0.2 + 1 = 1$ $1.2 + 1 = 3$

$2^3 = 8,0$ $3^3 = 27,0$ $7^3 = 343,$

$3.0 = 0$ $0.3 + 1 = 1$ $0.3 + 2 = 2$

$105^3 = 1160000,0$ $388^3 = 582000000,0$

$2.3 = 6$ $3.2 + 1 = 7$

Gyökvonás: $\sqrt{8} = 2,82$ $\sqrt{14} = 3,74$ $\sqrt{6'28} = 25$

$\sqrt{48'16'36} = 694$

$\sqrt[3]{9} = 2,08$ $\sqrt[3]{36} = 3,3$ $\sqrt[3]{712} = 8,94$

$\sqrt[3]{654\,321\,056} = 869$

Szögfüggvények:

$tg\,30° = 0,577$

$\sin 2° \doteq tg\,2° \doteq 0,0349$

$30' - 5°30' \sim 0,0abc$

$45° \sim 1,abc$

$\sin \alpha \sim 5°30' - 90°$ kikereshető a sin. skálán.

$0, — 1,$

„$\cos \alpha$ = ~~α - sin 90°~~" / $\cos 80° = \sin 10°$

$= \sin (90 - 80)°$

$tg\,\alpha = \sim 5°30' - 45°$ kikeresh. a tg. skálán

$0, — 1,$

$\frac{tg\,\alpha}{\sin \alpha} = \sim 30' - 5°30'$ sin-tg skála.

$0,0xy \sim 0,ab$

$tg\,\alpha \quad 45° \sim 90° = \dfrac{1}{tg(90-\alpha)}$ tg skálán
1, — (300~).

$ctg\,\alpha = tg(90-\alpha)$ pl. $ctg\,60° = tg\,30°$

Pl: $tg\,62° = tg\dfrac{1}{90-62°} = tg\dfrac{1}{28} = 1{,}88$

$tg\,78° = \dfrac{1}{tg\,12°} = 4{,}7$

74. óra

Hány tagja van annak a sz. s-nak., melyben: 1967. IV. 17.

$d = -12 \quad a_n = 15 \quad S_n = 456$

$$a_n = a_1 + (n-1)\cdot d \longrightarrow 15 = a_1 + (n-1)\cdot(-12)$$
$$S_n = \frac{a_1 + a_n}{2}\cdot n \longrightarrow 456 = \frac{a_1 + 15}{2}\cdot n$$

$15 = a_1 - 12n + 12$

$912 = a_1 n + 15n$ $\qquad 15 = \dfrac{912 - 15n}{n} - 12n + 12 \quad /\cdot n$

$a_1 = \dfrac{912 - 15n}{n}$ $\qquad 15n = 912 - 15n - 12n^2 + 12n$

$12n^2 + 18n - 912 = 0 \quad /:6$

$2n^2 + 3n - 152 = 0$

$$n_{12} = \frac{-3 \pm \sqrt{9 + 1216}}{4} = \frac{-3 \pm \sqrt{1225}}{4} = \frac{-3 \pm 35}{4}$$

$\underline{\underline{n_1 = 8}} \qquad n_2 = -9{,}5$

$a_1 = \dfrac{912 - 15\cdot 8}{8} = 99$

23   33   43   két szomsz. tag közé 4 sz.

$$\delta = \frac{d}{n+1} = \frac{10}{5} = 2.$$

$$\Downarrow$$

23 25 27 29 31 **33** 35 37 39 41 43

$a_{10} = 1 \quad d = ?$

$a_{100} = 2 \quad a_n = ?$

$$a_{100} = a_{10} + d(100-10)$$
$$2 = 1 + d(100-10)$$
$$2 = 1 + 90d$$
$$1 = 90d$$
$$d = \frac{1}{90} \doteq 0{,}0\dot{1}$$

$$a_n = a_1 + (n-1)\cdot d \qquad a_{10} = a_1 + 9d \rightarrow a_1 = 1 - \frac{1}{10} = \frac{9}{10} = a_1$$

$$a_n = 0{,}9 + (n-1)\cdot\frac{1}{90}$$

$$a_n = 0{,}9 + \frac{n}{90} - \frac{1}{90} = \frac{n}{90} + \frac{8}{9} = a_n$$

1967. IV. 19.

75. óra

Egy sz. s. kül. 6, össz. 154, . Ha minden 2,2 tag közé még 2 tagot közbeiktatunk, az így kelett. új sor összege 418 lesz. Mennyi az eredeti sor első és utolsó tagja.

$d = 6 \qquad \delta = \frac{d}{n+1} = \frac{6}{3} = 2 \qquad$ o | | o | | o | | o

$S_n = 154 \qquad S_n' = 418$

n - t. $\quad (n-1)\cdot 3 + 1$ - t.

$a_1' = a_1$

$a_n' = a_n$

$$S_n = \frac{(a_1 + a_n)}{2}\cdot n \qquad S_n' = \frac{(a_1' + a_n')}{2}\cdot\left[(n-1)\cdot 3 + 1\right]$$

$$154 = \frac{a_1 + a_n}{2}\cdot n \qquad 418 = \frac{a_1' + a_n'}{2}\cdot(3n-3+1)$$

$$a_n = a_1 + (n-1)\cdot d$$

$$308 = n\cdot(a_1 + a_n) \qquad 836 = (a_1' + a_n')\cdot(3n-2)$$

$$308 = n\cdot\left[a_1 + a_1 + (n-1)\cdot d\right] \qquad 836 = (3n-2)\cdot\left[a_1 + a_1 + (n-1)\cdot d\right]$$

$$\frac{308}{n} = \frac{836}{3n-2}$$

$$\begin{array}{r} 924 \\ -836 \\ \hline =88 \end{array}$$

$$924n - 616 = 836n$$
$$88n = 616 \quad \rightarrow n = 7$$

$S_n = 154$
$d = 6$
$n = 7$

$154 = \frac{a_1 + a_n}{2} \cdot 7$

$\frac{308}{7} = a_1 + a_n$

$44 = a_1 + a_n$

$44 - a_1 = a_n$

$a_n = a_1 + d \cdot 6$

$a_1 + 7d = 44 - a_1$

$a_1 + 36 = 44 - a_1$

$2a_1 = 8$

$\underline{\underline{a_1 = 4}}$

$a_n = 4 + 36$

$\underline{\underline{a_n = 40}} = 40$

Vál. sz. sor. 3. és 4. t. összege −5.
5 és 6 t " 23. mennyi a 10. tag.

$a_3 + a_4 = -5$
$a_5 + a_6 = 23$

$a_4 = a_3 + d$
$a_5 = a_3 + 2d$
$a_6 = a_3 + 3d$

$a_3 + a_3 + d = -5 \longrightarrow 2a_3 + d = -5$
$a_3 + 2d + a_3 + 3d = 23 \longrightarrow 2a_3 + 5d = 23$

$-5 - d = 23 - 5d$
$4d = 28$
$\underline{\underline{d = 7}}$

$2a_3 = -7 - 5$
$\underline{\underline{a_3 = -6}}$

$a_{10} = a_3 + 7d = -6 + 49 = \underline{\underline{43}}$

Mértani sorozat.

Számok azon sorát, melyben 2 szomszédos tag hányadosa állandóan ugyanaz, mértani sorozatnak nevezzük.

$3, 6, 12, 24, 48, 96, 192, 384 \ldots$

$\frac{6}{3} = 2 \quad \frac{12}{6} = 2 \quad \frac{24}{12} = 2 \quad \ldots\ldots$

A mértani sorozatnak 5 jellemző adata van:

1., első tag a_1
2., n-edik tag a_n
3., tagmutató n
4., összeg S_n
5., hányados q (quociens).

a_1
$a_2 = a_1 \cdot q$
$a_3 = a_2 \cdot q = a_1 \cdot q^2$
$a_4 = a_3 \cdot q = a_1 \cdot q^3$
$a_5 = a_4 \cdot q = a_1 \cdot q^4$

$a_{11} = a_1 \cdot q^{10}$
$a_{n-1} = a_1 \cdot q^{n-2}$
$\underline{a_n = a_1 \cdot q^{n-1}}$

A mértani sorozat utolsó (v. bármely) tagját megkapjuk, ha az 1. tagot megszorozzuk az illető tagot megelőző tagok számára hatványozott hányadossal.

A mértani sorozat összege:

$$a_1 \quad a_2 \quad a_4 \quad \ldots \quad a_{n-1} \quad a_n \qquad q = \frac{a_n}{a_{n-1}}$$

$$S_n = a_1 + a_2 + a_3 + \ldots + a_{n-1} + a_n$$

$$S_n = a_1 + a_1 q + a_1 q^2 + \ldots + a_1 q^{n-2} + a_1 q^{n-1} \quad / \cdot q \qquad ① \quad q \neq 0$$

$$S_n q = a_1 q + a_1 q^2 + a_1 q^3 + \ldots a_1 q^{n-1} + a_1 q^n. \qquad ② \quad (2-1)$$

$$S_n q - S_n = a_1 q + a_1 q^2 + a_1 q^3 + \ldots + a_1 q^{n-1} + a_1 q^n - $$
$$- a_1 - a_1 q - a_1 q^2 - \ldots - a_1 q^{n-2} - a_1 q^{n-1}$$

$$S_n(q-1) = -a_1 + a_1 q^n$$

$$S_n(q-1) = a_1(q^n - 1)$$

$$S_n = a_1 \cdot \frac{q^n - 1}{q - 1}$$

1967. IV. 20.

76. ó.

$$\{n^2-1\} \qquad 0, 3, 8, 15, 24, 35 \ldots\ldots$$
$$a_1 \; a_2 \; a_3 \; a_4 \; a_5 \; a_6$$

$a_6 - a_5 = a_n - a_{n-1}$ ⟶ számtani sor pl. 1, 4, 7, 10...

$a_1;\ a_n;\ n;\ d;\ S_n$

Ha: $d > 0 \rightarrow a_n < a_{n+1}$

$d = 0 \rightarrow a_n = a_{n+1}$

$d < 0 \rightarrow a_n > a_{n+1}$

⊕ $a_n = a_1 + (n-1) \cdot d \qquad \delta = \frac{d}{m+1}$

⊕ $S_n = \frac{a_1 + a_n}{2} \cdot n \qquad a_r = a_s + d(r-s)$

$$\frac{a_6}{a_5} = \frac{a_4}{a_3} = \ldots = \frac{a_n}{a_{n-1}} = q \qquad a_1;\ a_n;\ n;\ q;\ S_n$$

$$2, 4, 8, 16, 32, 64 \quad (q = 2)$$

⊕ $a_n = a_1 q^{n-1}$

⊕ $S_n = a_1 \cdot \frac{q^n - 1}{q - 1} \qquad q \neq 1$

A mértani sorozat összegét megkapjuk, ha az 1. tagot megszorozzuk azzal a hányadossal, melynek számlálója a q n-edik hatványának és az egységnek különbsége, nevezője pedig az állandó hányadosnak és egységnek a különbsége.

$a_1 = 2$
$q = 1$ $\quad 2, 2, 2, 2 \ldots$

Ha $q = 1 \rightarrow \underline{S_n = a_1 \cdot n}$

$a_1 = 3 \quad q = 2$

$3, 6, 12, 24, 48, 96, 192, 384 \ldots$

$a_1 = 3 \quad q = \frac{1}{2} \qquad 3, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16} \ldots$

$a_1 = 3 \quad q = -2 \qquad 3, -6, +12, -24, +48 \ldots$

I. 1, $a_1 > 0 \quad q > 1 \longrightarrow a_n < a_{n+1}$ a sor. növekvő

2, $a_1 > 0 \quad q \begin{smallmatrix}\leq 1\\ > 0\end{smallmatrix} \longrightarrow a_n > a_{n+1}$ a sorozat csökkenő.

3, $q < 0 \quad a_1 > 0 \longrightarrow$ a sorozat nem csökkenő, de nem is növekvő. Tagjai váltakozó előjelűek.

II. 1, $a < 0 \quad q \begin{smallmatrix}\geq 0\\ > 1\end{smallmatrix} \longrightarrow$ csökkenő sor. $a_n > a_{n+1}$

2. $a < 0 \quad 1 > q > 0 \longrightarrow$ növekvő sor. $a_n < a_{n+1}$

3 $a < 0 \quad q < 0 \longrightarrow$ tagjai váltakozó előjelűek.

$a_1 = -4 \quad q = 3 \qquad -4, -12, -36, -108, -324, -972, \ldots$

$a_1 = -4 \quad q = \frac{3}{4} \qquad -4, -3, -\frac{9}{4}, -\frac{27}{16},$

$a_1 = -4 \quad q -2 \qquad -4, 8, -16, 32, -64, 128 \ldots$

III. $q = 1.$ Egyenlő számok sorozata

IV. $q = 0$ A sor. tagjai az 1. kivételével 0-val egyenlőek.

A mértani sorozat interpolációja -

- két szomszédos tag közé úgy iktatunk számokat, hogy azok egymással és a két szélső taggal is mértani sorozatot képezzenek.

új hányados: $Q = \sqrt[m+1]{q}$ m - közbeikt. tagok száma

Az új mértani haladvány hányadosát megkapjuk, ha az eredetiből eggyel nagyobb kitevőjű gyököt vonunk, mint ahány tagot akarunk közbeiktatni.

$2, 6, 18, 54, 162$

$18 = \sqrt[2]{6 \cdot 54} = \sqrt{324} = 18$

$18 = \sqrt{2 \cdot 162} = \sqrt{324} = 18.$

A mértani sor. bármely tagja a rá szimmetrikus tagok mértani középaránya.

A m. s: $a_1 = 4$ $q = 3$ számki: $a_5 = ?$

$S_5 = ?$

243

$a_m = a_1 q^{m-1} = 4 \cdot 3^4 = 4 \cdot 81 = \underline{\underline{324}}$

$S_m = a_1 \frac{q^m - 1}{q - 1} = a_1 \cdot \frac{3^5 - 1}{3 - 1} = 4 \cdot \frac{242}{2} = 4 \cdot 121 = \underline{\underline{484}}$

$a_1 = 2$ $m = 7$ $a_7 = 1458$ $q = ?$ $a_m = ?$

$a_m = a_1 \cdot q^{m-1}$

$a_7 = a_1 \cdot q^6$

$1458 = 2 \cdot q^6$

$729 = q^6$

$\underline{\underline{q = \pm 3}}$

$a_1 = \frac{1}{8}$

$a_m = 128$

$m = 6$

$q = ?$

$S_m = ?$

$a_m = a_1 \cdot q^{m-1}$

$128 = \frac{1}{8} \cdot q^5$

$1024 = q^5$

$q = \sqrt[5]{1024}$

$q = 4$

$S_m = a_1 \cdot \frac{q^m - 1}{q - 1} = \frac{1}{8} \cdot \frac{4^6 - 1}{3}$

$S_m = \frac{4095}{24} \doteq 170{,}5 = 170 + \frac{15}{24}.$

$\frac{1}{8};\ \frac{1}{2};\ 2;\ 8;\ 32;\ 128.$

76 H.

1967.IV.22.

$a_1 = 27$

$a_n = \frac{1}{27}$

$q = -\frac{1}{3}$

$a_m = a_1 \cdot q^{m-1}$

$\frac{1}{27} = 27 \cdot \left[-\frac{1}{3}\right]^{m-1}$

$\frac{1}{3^6} = \left(-\frac{1}{3}\right)^{m-1}$

$3^6 = 3^{m-1}$

$6 = m-1$

$m = 7$

$S_m = a_1 \frac{q^m - 1}{q - 1}$

$S_m = 27 \cdot \frac{\left(-\frac{1}{3}\right)^7 - 1}{-\frac{1}{3} - 1} = 27 \cdot \frac{-\frac{1}{2187} - 1}{-\frac{4}{3}}$

$S_m = 27 \cdot \frac{-\frac{2188}{2187}}{-\frac{4}{3}} = 27 \frac{3 \cdot 2188}{4 \cdot 3^7}$

$S_m = \frac{2188}{4 \cdot 27} = \frac{547}{27} \approx 20{,}26$

$a_1 = -4$

a_m

$m = 10$

$q = -2$

$a_n = a_1 \cdot q^{n-1}$

$a_m = -4 \cdot (-2)^9$

$a_m = 2^{11} = 2048$

$S_m = a_1 \frac{q^n - 1}{q - 1} = -4 \cdot \frac{(-2)^{10} - 1}{(-2) - 1} = 4 \cdot \frac{1023}{3}$

$S_m = 4 \cdot 341 = 1364$

$a_1 = 50$
$a_n = 64/125$ $\qquad a_n = a_1 \cdot q^{n-1}$
$S_n = 82\frac{124}{125}$ $\qquad S_n = a_1 \cdot \frac{q^n - 1}{q-1}$

$$\frac{64}{125} = 50 \cdot q^{n-1} \qquad 82\frac{124}{125} = 50 \cdot \frac{q^n - 1}{q-1}$$

$$\frac{60}{50 \cdot 125} = q^{n-1} \qquad q^{n-1} = \frac{10374}{125} \cdot \frac{q-1}{50}$$

$$\frac{60}{50 \cdot 125} = \frac{10374 \cdot (q-1)}{125 \cdot 50}$$

$$60 = 10374(q-1)$$

$$q-1 = \frac{60}{10374} = \frac{15}{\cancel{2}59} = \frac{30}{5187} = \frac{10}{1729}$$

$$q = \frac{1739}{1729} \approx 1{,}005$$

$$\frac{60}{50 \cdot 125} = \left(\frac{1739}{1729}\right)^{n-1}$$

$$\frac{6}{625} = A^{n-1} \qquad \approx \quad 0{,}009 \doteq 1{,}006^{n-1}$$

$$\log 0{,}009 \doteq \log 1{,}006^{n-1}$$

$$\log 0{,}009 \doteq (n-1) \cdot (\log 1{,}006)$$

$$-3{,}955 = (n-1) \cdot 0,$$

$$\log 9 \cdot 10^{-3} = (n-1) \cdot \log 1006 \cdot 10^{-3}$$

$$\log 9 + \log 10^{-3} = (n-1)[\log 1006 + \log 10^{-3}]$$

$$0{,}955 + (-3) = (n-1) \cdot [3{,}0026 + (-3)]$$

$$-2{,}045 = (n-1) \cdot 0{,}0026$$

$$-2{,}045 = 0{,}0026n - 0{,}0026$$

$$0{,}0026n = -2{,}0424$$

$$26n = -20424$$

$$n = -784$$

$$a_n = a_1 \cdot q^{n-1}$$

$$S_n = a_1 \frac{q^n - 1}{q - 1}$$

$$\frac{64}{125} = 50 \cdot q^{n-1}$$

$$82\frac{124}{125} = 50 \cdot \frac{q^n - 1}{q - 1}$$

$$q^{n-1} = \frac{64}{125.50}$$

$$q^{n-1} = 82\frac{124}{125} \cdot \frac{q-1}{50}$$

$$\frac{64}{125.50} = \frac{10374}{125} \cdot \frac{q-1}{50}$$

$$64 = (q-1) \cdot 10374$$

$$\frac{64}{10374} = q - 1$$

$$64 = 10374q - 10374$$

$$10438 = 10374q$$

$$q = \frac{10438}{10374} =$$

1967. IV. 24.

77. óra!

$a_1 = 2 \quad q = 2 \qquad 2 ; 4 ; 8 ; 16$

$a_1 = 5 \quad q = \frac{1}{3} \qquad 5 ; \frac{5}{3} ; \frac{5}{9} ; \frac{5}{27} ; \frac{5}{81} ;$

$a_1 = 7$

$q = -\frac{1}{2}$

$a_1 = \frac{1}{2}$

$q = -\frac{1}{3}$

$m = 5$

$S_5 = ?$

$$S_m = a_1 \frac{q^m - 1}{q - 1}$$

$$S_5 = \frac{1}{2} \cdot \frac{(-\frac{1}{3})^5 - 1}{-\frac{1}{3} - 1} = \frac{1}{2} \cdot \frac{-\frac{1}{243} - 1}{-\frac{1}{3} - 1} = \frac{1}{2} \cdot \frac{-\frac{244}{243}}{-\frac{4}{3}}$$

$$S_5 = \frac{1}{2} \cdot \frac{3 \cdot 244}{4 \cdot 243} = \frac{1}{2} \cdot \frac{61}{81} = \frac{61}{162} \doteq 0{,}3$$

$a_1 = 8$

$q = \frac{1}{2}$

$m = 5$

$S_5 = ?$

$$S_5 = a_1 \cdot \frac{(\frac{1}{2})^5 - 1}{\frac{1}{2} - 1} = 8 \cdot \frac{\frac{1}{32} - 1}{\frac{1}{2} - 1} = 8 \cdot \frac{-\frac{31}{32}}{-\frac{1}{2}}$$

$$S_5 = 8 \cdot \frac{62}{32} = \frac{62}{4} = 15{,}5$$

$a_1 = 5$
$n = 10$
$a_{10} = 200$

$q = ?$

$$a_n = a_1 \cdot q^{n-1}$$
$$a_{10} = 5 \cdot q^9$$
$$200 = 5 \cdot q^9$$
$$40 = q^9 \quad /\sqrt[3]{}$$
$$\sqrt[3]{40} = q^3$$
$$3{,}42 = q^3 \quad /\sqrt[3]{}$$
$$\underline{\underline{q \doteq 1{,}51}}$$

77. Ü

1967. IV. 24.

$q = 4$
$n = 7$
$a_7 = 1024$
$a = ?$

$$a_7 = a_1 \cdot q^{n-1}$$
$$1024 = a_1 \cdot 4^6$$
$$a_1 = \frac{1024}{4096} = \underline{\underline{\frac{1}{4}}}$$

$$S_n = \frac{1}{4} \cdot \frac{4^6 - 1}{3}$$
$$S_n = \frac{1}{4} \cdot \frac{4095}{3} = \frac{1}{4} \cdot 1365$$
$$S_n \doteq 341{,}..$$

$a_1 = 2$
$q = 4$
$a_n = 128$
$n = ?$

$$a_n = a_1 \cdot q^{n-1}$$
$$128 = 2 \cdot 4^{n-1}$$
$$64 = 4^{n-1}$$
$$4^3 = 4^{n-1}$$
$$\underline{\underline{4 = n}}$$

$$S_n = 2 \cdot \frac{4^4 - 1}{3} = \frac{2}{3} \cdot 255$$
$$S_n = 2 \cdot 85 = \underline{170}$$

$a_1 = 3$
$q = 2$
$n = 6$
$S_6 = ?$

$$S_n = a_1 \frac{q^n - 1}{q - 1} = 3 \cdot \frac{2^6 - 1}{1} = 3 \cdot 63 = \underline{\underline{189}}$$

$a_1 = 20$
$q = \frac{3}{2}$
$S_n = 263{,}\frac{3}{4}$

$a_n = ?$
$n = ?$

$$S_n = a_1 \frac{q^n - 1}{q - 1}$$

$$\frac{1055}{4} = 20 \cdot \frac{\left(\frac{3}{2}\right)^n - 1}{\frac{3}{2} - 1}$$

$$\frac{1055}{80} \cdot \frac{1}{2} + 1 = \left(\frac{3}{2}\right)^n$$

$$\frac{1055}{160} + 1 = \left(\frac{3}{2}\right)^n$$

$$\frac{1215}{160} = \left(\frac{3}{2}\right)^n$$

$$\frac{243}{32} = \left(\frac{3}{2}\right)^n$$

$$\underline{\underline{n = 5}}$$

$$a_n = a_1 \cdot q^{n-1}$$

$$a_n = 20 \cdot \left(\frac{3}{2}\right)^4$$

$$a_n = 20 \frac{81}{16} = \frac{5 \cdot 81}{4}$$

$$a_n = \frac{405}{4}$$

$$a_n \doteq 101{,}25$$

a 2. dolg. példái.:

I.1 $4x^5-11x^3-11x^2+4=0$

$4(x^5+1)-11x^2(x+1)=0$

$x_1=-1$

$(x_2=7)$

$4x^4-4x^3-7x^2-4x-4=0$

$7x^2-7x+7-43x=0$

$7x^2-50x+7=0$

$4\left(x^2+\frac{1}{x^2}\right)-4\left(x+\frac{1}{x}\right)-7=0$

$4(u^2-2)-4(u+7)=0$

$4u^2-4u-15=0$

$u_1=\frac{5}{2}$

$u_2=-\frac{3}{2}$

$u=x+\frac{1}{x} \rightarrow$ $x_2=2$, $x_3=\frac{1}{2}$, $x_4=\frac{-3+i\sqrt{7}}{4}$, $x_5=\frac{-3-i\sqrt{7}}{4}$

II.1 $d=20$

$a_n=280$

$n=10$

$a_n=a_1+(n-1)\cdot d \rightarrow a_1=100$

$280=a_1+9\cdot 20$

$S_n=\frac{a_1+a_n}{2}\cdot n$

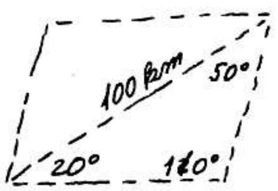
$S_n=\frac{100+280}{2}\cdot 10$

$S_n=(50+140)\cdot 10$

$S_n=1900$

A.

III.1



$\sin\alpha:\sin\gamma=a:c$

$a=\frac{\sin\alpha\cdot c}{\sin\gamma}$

$\sin 110=s.70\doteq 0{,}94$

$a=0{,}766\cdot 100:0{,}9397$

$a=\frac{76{,}6}{0{,}94}=81{,}5\,km$

$\sin\alpha:\sin\beta=a:b$

$b=0{,}342\cdot 81{,}5:0{,}766$

$b=36\,km$

1967. IV. 26 79. óra

$$a_s = a_1 \cdot q^{s-1}$$
$$a_r = a_1 \cdot q^{r-1}$$

$$\frac{a_s}{a_r} = \frac{a_1 \cdot q^{s-1}}{a_1 \cdot q^{r-1}}$$

$$a_s : a_r = q^{(s-1)-(r-1)}$$

$$a_s : a_r = q^{s-r}$$

$$a_s = a_r \cdot q^{s-r}$$

$$\underline{\underline{a_s = a_r \cdot q^{s-r}}}$$

A mértani sorozat első n tagjának összege 189.
Hány tagja van a sorozatnak ha $a_1 = 3$ és $q = 2$

$$S_n = a \cdot \frac{q^n - 1}{q - 1}$$

$$189 = 3 \cdot (2^n - 1)$$
$$63 = 2^n - 1$$
$$64 = 2^n$$
$$2^6 = 2^n$$
$$\underline{n = 6}$$

Négy szám mértani sorozatot alkot. A II. és IV.
tag összege 30. Az I. és III. tagé 15. Melyek a számok.

$$\left.\begin{aligned} a_1 + a_1 \cdot q^2 = 15 &\rightarrow a_1(1+q^2) = 15 \rightarrow a_1(1+q^2) = 15 \\ a_1 \cdot q + a_1 \cdot q^3 = 30 &\rightarrow a_1(q+q^3) = 30 \rightarrow a_1(1+q^2) = \frac{30}{q} \end{aligned}\right\} \rightarrow \underline{\underline{q = 2}}$$

$$a_1 + a_1 \cdot q^2 = 15 \qquad a_1(1+2^2) = 15 \rightarrow a_1 = \frac{15}{5} = 3$$

a_1	a_2	a_3	a_4
3	6	12	24

37/33 38/38

$a_1 = 4 \quad a_2 = \frac{4}{3} \quad \rightarrow q = ?$

$a_2 = a_1 \cdot q \rightarrow q = \frac{a_2}{a_1} = \frac{\frac{4}{3}}{\frac{4}{1}} = \frac{4}{12} = \frac{1}{3} = q$ $\quad 4, \frac{4}{3}, \frac{4}{9}, \frac{4}{27}, \frac{4}{81}$

$a_3 = -1 \quad a_4 = \sqrt{2}$

$q = \frac{a_4}{a_3} = \frac{\sqrt{2}}{-1} = -\sqrt{2}$ $\quad -\frac{1}{2}, \frac{1}{\sqrt{2}}, -1, \sqrt{2}, -2$

$a_2 = 5 \quad a_5 = \frac{5}{2}$

$a_1 q = a_2 = 5$
$a_1 q^4 = a_5 = \frac{5}{2}$

$q^3 = \frac{\frac{5}{2}}{\frac{5}{1}} = \frac{5}{10} = \frac{1}{2} = q^3$

$\left(a_1 = \frac{5}{q} = \frac{5}{\frac{1}{2}} = 10\right)$

$(10 \quad 5 \quad \frac{5}{2})$

$a_1 = \frac{5}{\frac{1}{\sqrt[3]{2}}} = \frac{5 \cdot \sqrt[3]{2}}{1} = 5 \cdot \sqrt[3]{2}$

$5 \cdot \sqrt[3]{2};\ 5;\ \frac{5 \cdot \cancel{\sqrt[3]{2}}}{\sqrt[3]{2}};\ \frac{5}{\sqrt[3]{4}};\ \frac{5}{2}$

$a_2 = 2 \quad a_3 = -4 \qquad q = \frac{a_3}{a_2} = -\frac{4}{2} = -2$

$-1;\ 2;\ -4;\ 8;\ -16$

$a_2 = \sqrt{3} \qquad a_5 = 2 \cdot \sqrt{3}$ $\qquad \frac{\sqrt{3}}{\sqrt[3]{4}};\ \frac{\sqrt{3}}{\sqrt[3]{2}};\ \sqrt{3};\ \sqrt{3} \cdot \sqrt[3]{2};\ \sqrt{3} \cdot \sqrt[3]{4}$

$\left.\begin{array}{l} a_2 = a_1 q = \sqrt{3} \\ a_5 = a_1 q^4 = 2\sqrt{3} \end{array}\right\} q^3 = 2$

$q = \sqrt[3]{2}$

38/38

$a_{10} = 15 \quad q = -3$

$$a_{10} = a_1 \cdot q^9$$
$$15 = a_1 \cdot (-3)^9$$
$$a_1 = \frac{15}{-(3)^9}$$
$$a_1 = -\frac{15}{19\,683} = -\frac{5}{6561}$$

$$\underline{-\frac{5}{6561} ; \frac{15}{6561} ; -\frac{45}{6561} ; \frac{135}{6561} ; -\frac{405}{6561}}$$

$a_6 = 8 \quad q = 2$

$$a_6 = a_1 \cdot q^5$$
$$a_1 = \frac{8}{32} = \frac{1}{4}$$

$$\underline{\frac{1}{4} , \frac{1}{2} , 1 , 2 , 4}$$

1967. IV. 27.

80. óra

Melyik az a 4 tagú mértani sorozat, melyben $a_1 + a_4 = 18 \quad a_2 + a_3 = 12$.

$$a_1 + a_1 \cdot q^3 = 18 \qquad a_1(1+q^3) = 18 \longrightarrow a_1 = \frac{18}{1+q^3}$$
$$a_1 \cdot q + a_1 q^2 = 12 \qquad a_1 q(1+q) = 12 \longrightarrow a_1 = \frac{12}{q+q^2}$$

$$18(q+q^2) = 12(1+q^3)$$

$$\frac{a_1(1+q^3)}{a_1 q(1+q)} = \frac{18}{12}$$
$$\frac{1-q+q^2}{q} = \frac{3}{2}$$
$$2 - 2q + 2q^2 = 3q$$
$$2q^2 - 5q + 2 = 0$$

$$q_{1,2} = \frac{5 \pm \sqrt{25 - 16}}{4} = \frac{5 \pm \sqrt{9}}{4}$$

$$\underline{q_1 = 2} \qquad \underline{q_2 = \frac{1}{2}}$$

$$a_{1_1} = \frac{18}{1+q^3} = \frac{18}{1+8} = 2 \qquad a_{11} = 2$$

$$a_{1_2} = \frac{18}{1+q^3} = \frac{18}{1+\frac{1}{8}} = \frac{18}{\frac{9}{8}} = \frac{144}{9} = 16$$

2, 4, 8, 16, 32.

16, 8, 4, 2, 1

80. Ó. 1967. IV. 27.

Valamely 3 tagú mértani sorozat első tagja 75. S_n 1575. Mennyi q és a_3?

$$a_1 + a_1 \cdot q + a_1 q^2 = 1575 \qquad q_{12} = \frac{-1 \pm \sqrt{1+80}}{2} = \frac{-1 \pm 9}{2}$$

$$a_1(1+q+q^2) = 1575$$

$$q^2+q+1 = 21 \qquad q_1 = 4 \qquad q_2 = -5$$

$$q^2+q-20 = 0$$

75.16
450

$$a_3 = a_1 \cdot q^2 = 75 \cdot 16 = 1200 = a_3$$

75.25
150
375

$$a_3 = a_1 \cdot q^2 = 75 \cdot 25 = 1875 = a_3$$

Vál. 3 tagú mért. sor. utolsó tagja 288. Összege 416. Mennyi a hányados és a_1?

$$a_1(1+q+q^2) = 416 \qquad a_3 = 288$$

$$416 = a_3 + \frac{a_3}{q} + \frac{a_3}{q^2} = \frac{a_3 q^2 + a_3 q + a_3}{q^2} = \frac{a_3(q^2+q+1)}{q^2} = 416$$

$$416 q^2 = 288 q^2 + 288 q + 288$$

$$128 q^2 - 288 q - 288 = \cancel{q} = 0 \qquad q_{12} = \frac{9 \pm \sqrt{81+144}}{8} = \frac{9 \pm 15}{8}$$

$$16 q^2 - 36 q - 36 = 0$$

$$q_1 = 3 \qquad q_2 = -\frac{3}{4}$$

$$4q^2 - 9q - 9 = 0$$

$$a_1 = \frac{a_3}{q^2} = \frac{288}{9} = 32$$

$$a_1 = \frac{a_3}{q^2} = \frac{288}{-\frac{3}{4}} = -\frac{1152}{3} = -384$$

Melyik az mértani sor, melyben $a_4 + a_5 = 72$
$a_4 - a_6 = -72$.

$$\left.\begin{aligned} a_4 + a_4 q &= 72 \\ a_4 - a_4 q^2 &= -72 \end{aligned}\right\} \quad \left.\begin{aligned} a_4(1+q) &= 72 \\ a_4(1-q^2) &= -72 \end{aligned}\right\} \quad \begin{aligned} 1-q &= -1 \\ q &= 2 \end{aligned}$$

$$a_4 = \frac{72}{3} = 24$$

$$a_3 = \frac{a_4}{q} = \frac{24}{2} = 12$$

$$a_2 = \frac{a_3}{q} = \frac{12}{2} = 6$$

$$a_1 = \frac{a_2}{q} = \frac{6}{2} = 3$$

3 6 12 24 48 96 ...

3 és 729 közé ikt. 4 számot úgy, hogy mértani sor keletkezzen:

$$Q = \sqrt[m+1]{d} = \sqrt[5]{726}$$

$$\log Q = \frac{1}{5} \cdot \log 726$$
$$\log Q = \frac{1}{5} \cdot 2,8609$$
$$\log Q = 0,5722$$
$$Q = 3,735$$

3 11,205 41,44 157,

$11,2 . 3,7$
37
74

$41,4 . 3,8$
152
152

$3 . 3,7 = 11,1$ $11,1 . 3,7 = 41,07$
37
37
37

$, 3,7$

$$Q = \sqrt[m+1]{q} = \sqrt[5]{243}$$
$$Q = 3$$

3, 6, 12, 24,

3, 9, 27, 81, 243, 729

Valamely víztárolóból 12 ízben bocsátva ki vizet, minden egyes alkalommal a benn levő vízmennyiségnek ugyanannyiad részét eresztik ki.
A 12 kibocsátás után a víztároló félig kiürült. A mindenkori víztartalomnak hányadát engedték ki esetenként?

$M = 2A \quad (\mathrm{m}^3)$

$\frac{2A}{q} \cdots \frac{2A}{q^2} \cdots\cdots \frac{2A}{q^{12}} \qquad \frac{2A}{q^{12}} = A \rightarrow 2A = A \cdot q^{12}$

$$A q^{12} = 2A \quad / \cdot \frac{1}{A}$$

$$q^{12} = 2$$

$$12 \cdot \log q = \log 2$$

$$\log q = \frac{1}{12} \cdot \log 2$$

$$\log q = \frac{1}{12} \cdot 0{,}30103$$

$$\log q \doteq 0{,}02508$$

$$q = 1{,}02$$

$1 : 1{,}01 \doteq 0{,}9901$

$1 - 0{,}9901 \doteq 0{,}01$

$\frac{1}{0{,}01} = 100$

Megközelítőleg a vízmennyiség 100-ad részét engedik ki.

$1 : 1{,}02 \doteq 0{,}98 \qquad 1 - 0{,}98 = 0{,}02$